



An investigation of min- max method problems for RSSI- based indoor localization: Theoretical and experimental studies

Apidet Booranawong^{*1)}, Kiattisak Sengchuai¹⁾, Nattha Jindapetch¹⁾ and Hiroshi Saito²⁾

¹⁾Department of Electrical Engineering, Faculty of Engineering, Prince of Songkla University, Songkhla 90112, Thailand

²⁾Division of Computer Engineering, The University of Aizu, Aizu-Wakamatsu 965-8580, Japan

Received 8 January 2020

Revised 26 March 2020

Accepted 1 April 2020

Abstract

A study of limitations of a min-max or a bounding-box method for received signal strength indicator (RSSI)-based indoor localization is introduced in this paper. The main goal of our study is to clearly understand how the widely used min-max method determines an unknown target position, and to investigate its significant limitations. For this purpose, we provide both theoretical and experimental studies. The theoretical study first gives an understanding of min-max theoretical limitations, while an experimental study then reveals more limitations. Experiments were done in an indoor environment, a laboratory room, where we employed an LPC2103F with a CC2500 RF module as a wireless node. Our results indicate that the min-max method can be efficiently used to estimate an unknown target's position. However, such a method has limitations in several cases. First, it produces a significantly high estimation error when the unknown target is located outside an internal zone, the area within reference node positions. Second, fluctuations of measured RSSI signals in an obstacle environment is a major problem that produces significantly more estimation errors. Various effects in this case are detailed in the paper. Our information will be useful to develop more efficient min-max methods.

Keywords: RSSI, Bounding-box, Localization, Theoretical study, Experiment

1. Introduction

Indoor target localization is an essential subject in the context of indoor wireless networks [1-2] because position data can be used in several applications, including tracking people in buildings, or during emergency events [1-2], patient tracking in hospitals [3-4], rescue robot tracking [5], industrial robot guidance [6], position detection of products stored in warehouses [2, 6], worker tracking in above-ground and underground construction sites [7-8], and automated control of devices (e.g., HVAC systems, lights, and cameras) [9-11], among many others. Therefore, one of the major challenges in indoor wireless networks is target localization. To determine an unknown target position, RSSI information about the power level of a received signal is widely used [12-14]. This is because most wireless devices have RSSI circuits built into them. Thus, no additional hardware is required. This can directly help to reduce hardware costs, computational complexity, and power consumption of the system [12].

According to our literature review, several localization methods have been introduced. The well-known bounding-box or RSSI-based min-max method [15] is widely used because its algorithm is simple. It also provides high estimation accuracy as well as low computational complexity. Hence, a min-max algorithm is easy to implement on existing hardware [12, 16].

In the research literature, many studies used this method for position estimation. In [17], a wireless sensor network for RF-based indoor localization was introduced. The min-max method was implemented on a real hardware platform and used for position estimation in a laboratory scenario. Here, the min-max method provided good performance. In [18], the min-max and well-known multilateration localization methods were used to track a moving target inside a faculty building. Experimental results showed that they were able to track a target path with good accuracy and low computational effort. The authors also reported that the min-max algorithm was the simplest among the accurate methods. In [16], an experimental comparison of the maximum likelihood method, the trilateration localization method and the min-max method in low-power IEEE 802.15.4 networks was presented. In [12], the authors studied the performance of RSSI-based methods. The min-max method, the ring overlapping circle RSSI method, the trilateration method, and the maximum likelihood method were tested. Here, the work in [12] and [16] concluded that the min-max method gave better results than other methods both in terms of accuracy and lower computational complexity when small numbers of reference nodes were applied. We note that to determine an unknown target location, the min-max method needs to know the actual reference node positions. We will describe this issue in the next section. In [7], a comparative evaluation of RSSI-based indoor localization methods for

*Corresponding author. Tel.: +6684 581 7864

Email address: apidet.boo@gmail.com

doi: 10.14456/easr.2020.34

construction jobsites was introduced. Four localization methods including the min-max, the maximum likelihood, the ring overlapping circle RSSI, and the K-nearest neighbor methods were evaluated. Also, the experimental results confirmed that the performance of the min-max was better than others.

In [19], an experimental comparison of the trilateration and the min-max methods for indoor scenarios was introduced. The results demonstrated that the min-max method provided higher accuracy than another algorithms. Moreover, the authors also summarized min-max method limitations observed during the experiments. Unfortunately, [19] did not describe the theoretical support to reveal more limitations of the method. Compared to [19], in this paper, we describe further details of min-max method limitations from both theoretical and experimental perspectives. Finally, [20] and [21] extended min-max methods for wireless sensor networks. The accuracy of the traditional the min-max method was improved by applying a weighted method that gave different weights to the four sides of a rectangular overlapping region (i.e., a definition zone as presented in Section 2). Here, an estimated target position could be located at any point inside the overlapping region, and estimation accuracy was improved. Simulation results confirmed the accuracy of the extended min-max methods. However, in the works of [20] and [21], only the simulation approach was studied, and the proposed methods were designed to handle one case of the traditional method limitation. We note that as recommended in [12], the experimental approach showed much worse performance than the simulation.

In this paper, an exploration of min-max method problems is studied. The main objective of our study is to investigate how the widely used min-max method determines an unknown position, and to explore limitations of this method. Both theoretical and experimental studies are provided. Results demonstrate that the min-max is an efficient method that can be used to estimate a target position in an indoor scenario. However, the traditional min-max method still has limitations that will be detailed and reported in the paper. We believe that our information can be used to further develop more appropriate min-max algorithms.

The structure of this paper is as follows. Section 2 describes an RSSI-based localization system including the min-max method. Sections 3 and 4 describe the theoretical and experimental studies including setup, results, and discussion. The conclusions are presented in Section 5.

2. An RSSI-based localization system with the min-max method

The localization system in this work is developed and tested below in Section 4. Three reference nodes are stationary at known positions. A target node is located at an unknown position at a test location. Each reference node continually broadcasts a packet to the target node in every time interval. Upon receiving the beacon packet, the target node reads the RSSI value using its radio circuitry. Simultaneously, the target node transfers the measured RSSI data as it is read from each reference node to a base station node connected to a central computer. At the computer, the RSSI is then converted to a distance using the path-loss equation (i.e., the radio propagation model), and the min-max method [15, 22] is used to determine the unknown position.

In this work, the widely used path-loss equation is used [7, 16, 23]. The path-loss equation describes the relationship between the measured RSSI value and the distance value of a test field. It is expressed in Equation (1), where $RSSI_d(dBm)$ is a mean RSSI value at a distance d (i.e., a distance between a transmitter and a receiver). The parameter, $RSSI_{d_0}(dBm)$, is a mean received power at a reference distance from the transmitter (d_0) of 1 m, and α is called the path loss exponent. It measures the rate at which the received signal strength decreases with distance [23]. The parameter α is determined from a test field. It depends on the specific propagation environment. The parameters $RSSI_{d_0}(dBm)$ and α can be determined by measuring and collecting RSSI data from the test field. This is described in Section 4.

$$RSSI_d(dBm) = RSSI_{d_0}(dBm) - \left[10 \times \alpha \times \log_{10} \left(\frac{d}{d_0} \right) \right] \quad (1)$$

The min-max method, also known as the bounding-box method, is employed to determine an unknown target position [15]. Figure 1 illustrates the min-max concept. RSSI values from all reference nodes are measured by an unknown target, where the reference nodes are located at x_i and y_i (i refers to the reference number). The measured RSSI is converted to a distance (d_i) by applying the path-loss equation.

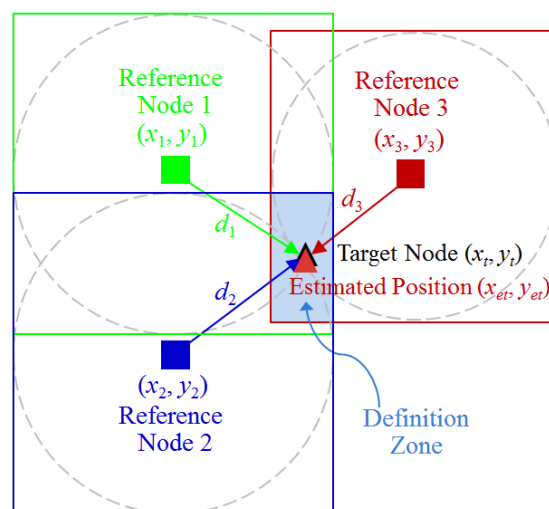


Figure 1 Estimation of a target node position using the min-max method [7, 15, 19, 22]

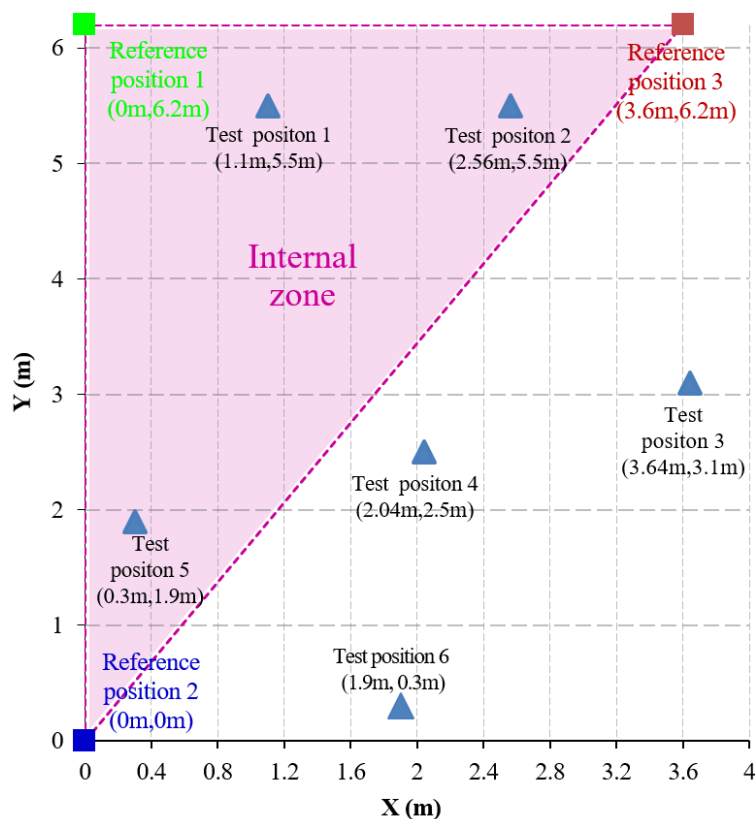


Figure 2 The test scenario corresponding to a real layout in a laboratory as discussed in Section 4, and the internal zone is the area defined by three reference node positions

Then, a bounding-box around the reference is constructed. Here, the reference position is at the center (with an edge length of $2d_i$). An intersection region as an area within x_{min} , x_{max} , y_{min} , and y_{max} is determined, as expressed by Equations (2-5). It is notable that the intersection area is computed by finding the maximum of all the lowest values of the coordinates and the minimum of all maximum values. The center of the intersection region is the estimated target position (x_{et} , y_{et}), as determined by Equations (6) and (7). In [20] such an intersection region is called the definition zone, and the area within three reference node positions (i.e., within (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) , as shown in Figure 1) is called the internal zone. According to Equations (2–7), the min-max method requires only a few arithmetic operations. Consequently, this method is simple with low computational complexity [12, 16].

$$x_{min} = \max_{1 \leq i \leq N} (x_i - d_i) \quad (2)$$

$$x_{max} = \min_{1 \leq i \leq N} (x_i + d_i) \quad (3)$$

$$y_{min} = \max_{1 \leq i \leq N} (y_i - d_i) \quad (4)$$

$$y_{max} = \min_{1 \leq i \leq N} (y_i + d_i) \quad (5)$$

$$x_{et} = \frac{(x_{min} + x_{max})}{2} \quad (6)$$

$$y_{et} = \frac{(y_{min} + y_{max})}{2} \quad (7)$$

3. Theoretical study

In this section, our theoretical study is presented. The test scenario (with fixed positions for reference and unknown target nodes), which corresponds to the real experiment presented in Section 4, is defined. Here, the distances between target nodes and reference nodes are known. This means that, for the theoretical study, we do not include the effect of the RSSI variation (i.e., Equation (1) is ignored). The actual distance from the reference node is directly used for the min-max calculation of the unknown position. This will give an understanding of the fundamental theoretical limitations. Details are described below.

3.1 Theoretical setup

The test scenario is shown in Figure 2. The experiment was conducted in a laboratory at the EE Department, Prince of Songkla University [19]. The actual room size is 4.54 m \times 7.40 m (actual measurements), and the reference nodes are placed inside the room. We define that three reference nodes are placed in the corners of the room, reference position 1 ($x_1 = 0.00$ m, $y_1 = 6.20$ m), reference position 2 ($x_2 = 0.00$ m, $y_2 = 0.00$ m), and reference position 3 ($x_3 = 3.60$ m, $y_3 = 6.20$ m). Additionally, we also define that there are six unknown targets at test position 1 ($x_t = 1.1$ m, $y_t = 5.5$ m), test position 2 ($x_t = 2.56$ m, $y_t = 5.5$ m), test position 3 ($x_t = 3.64$ m, $y_t = 3.1$ m), test position 4 ($x_t = 2.04$ m, $y_t = 2.5$ m), test position 5 ($x_t = 0.3$ m, $y_t = 1.9$ m), and test position 6 ($x_t = 1.9$ m, $y_t = 0.3$ m). As discussed in Section 2, test positions 1, 2, and 5 are in the internal zone, while the test positions 3, 4, and 6 are located outside the internal zone. In this way, we can study how well the min-max

Table 1 Theoretical results

Test position		Actual distance from the reference node			Estimated target position							Error distance (m)
x_t (m)	y_t (m)	Distance from ref. 1 (or d_1)	Distance from ref. 2 (or d_2)	Distance from ref. 3 (or d_3)	x_{min} (2)	x_{max} (3)	y_{min} (4)	y_{max} (5)	x_{et} (6)	y_{et} (7)		
1.100	5.500	1.304	5.609	2.596	1.004*	1.304	4.896	5.609	1.154	5.253	0.253	
2.560	5.500	2.654	6.067	1.254	2.346	2.654	4.946	6.067	2.500	5.507	0.060	
3.640	3.100	4.781	4.781	3.100	0.500	4.781	3.100	4.781	2.641	3.941	1.306	
2.040	2.500	4.225	3.227	4.015	-0.415	3.227	2.185	3.227	1.406	2.706	0.667	
0.300	1.900	4.310	1.924	5.420	-1.820	1.924	1.890	1.924	0.052	1.907	0.248	
1.900	0.300	6.198	1.924	6.140	-1.924	1.924	0.060	1.924	0.000	0.992	2.022	
Average											0.759	
Standard deviation (SD)											0.763	
95% confidence interval (CI)											0.610	

*Example: $x_{min} = \max(x_1 - d_1, x_2 - d_2, x_3 - d_3)$, then $x_{min} = \max(0 - 1.304, 0 - 5.609, 3.600 - 2.596)$, so $x_{min} = \max(-1.304, -5.609, 1.004) = 1.004$

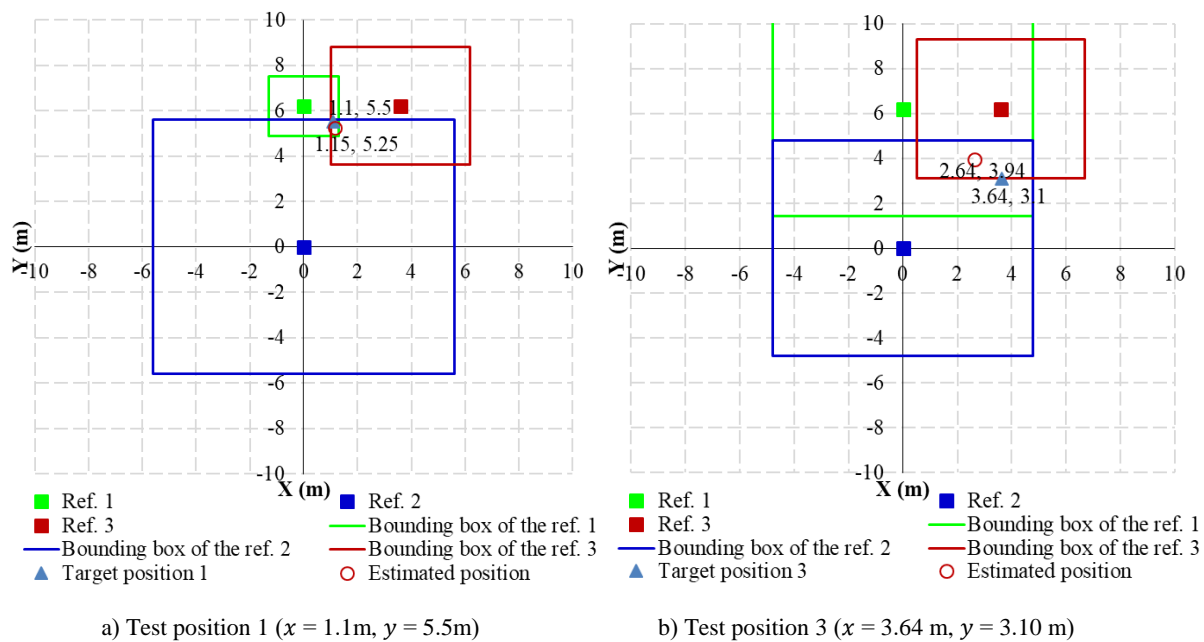


Figure 3 Illustrations of the estimated position in the theoretical study, a) and b) are test positions 1 and 3, respectively.

method estimates unknown target positions when they are placed at various locations.

3.2 Theoretical results and discussion

Based on the calculation using Equations (2-7), the estimated target positions determined by the min-max method are presented in Table 1. It is notable that the actual distances (between the references and the targets, d_1 , d_2 , and d_3) can be calculated using $d_i = \sqrt{(x_i - x_t)^2 + (y_i - y_t)^2}$. An error distance is shown in the last column of Table 1. It is defined as the difference between the actual position and the estimated position. This distance can be calculated as $error\ distance = \sqrt{(x_t - x_{et})^2 + (y_t - y_{et})^2}$.

From the theoretical results in Table 1, we can observe that the min- max estimation accuracy directly depends on the unknown target positions. When the tested positions are within the internal zone (i.e., the area within three reference node positions), test positions 1, 2, and 5, the estimated positions are

close to the actual target positions. Alternatively, for test positions 3, 4, and 6, the target nodes are placed outside the internal zone and the error distances are consequently larger. The error distance increases when the target location is far from the internal zone. For position 6, the error distance is higher than for test positions 3 and 4. This is because when the target location is far from the internal zone, the definition zone is also bigger. Since the min-max method tries to estimate the target position at the definition zone center, as can be seen in Figure 1 and Equations (6-7), if the actual target is not located at the center, an estimation error results. Figures 3(a) and (b) also illustrate the estimated positions for test positions 1 and 3.

To confirm the findings described above, we also show the performance of the min-max method when 650 target nodes (examples) are tested in the same scenario. Here, the 650 target nodes are placed in a grid topology. The error distance of this case is presented in Figure 4. The average error distance from 650 target nodes is equal to 0.845 m, with a standard deviation of 0.812. The minimum and maximum error distances are 0 m

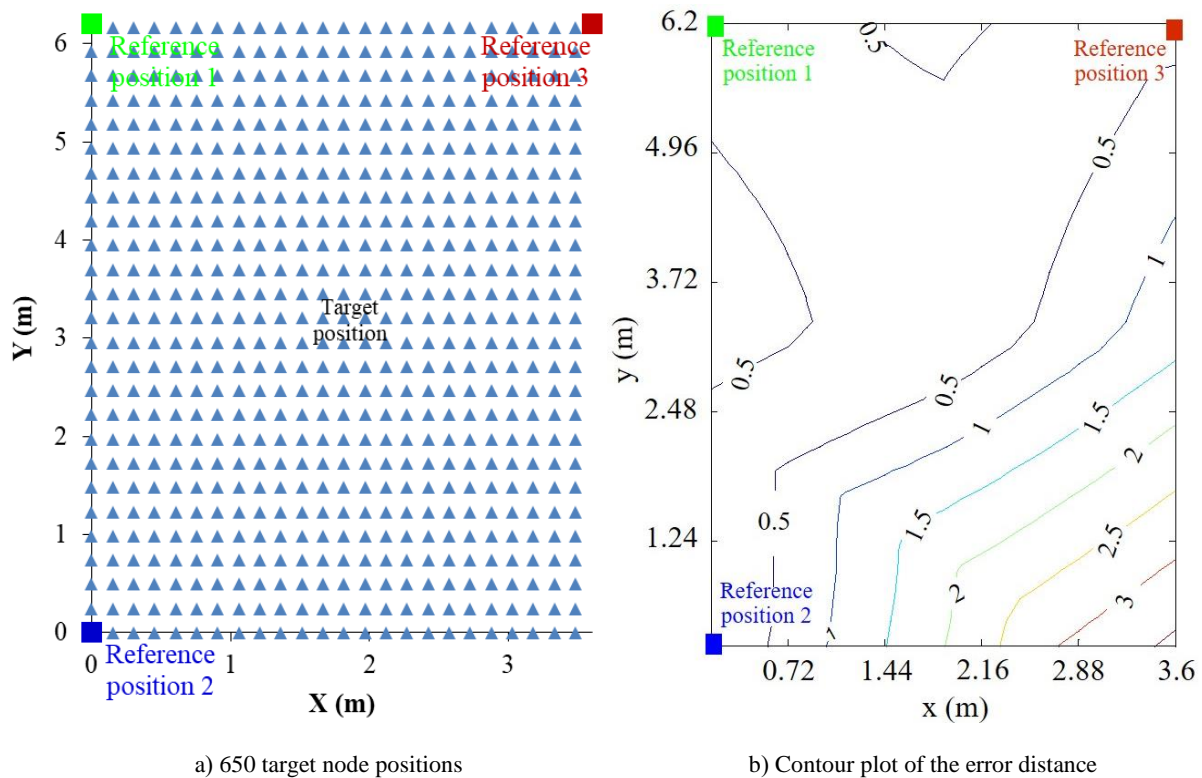


Figure 4 a) 650 target node positions and b) a contour plot of the error distance

and 3.607 m, respectively. The result in Figure 4 confirms that the min-max method produces a large estimation error when the target is located outside the internal zone. The error distance significantly increases when target nodes are placed on the corner (lower right) of the room.

As reported in [2, 12, 16], the level of the error distance can be reduced by adding more references to the test area. Therefore, we also show the theoretical results when more reference nodes are added in the same scenario. Details are presented in the Appendix A. However, it is notable that when using more references, the complexity and cost of the system are increases significantly [2, 12, 16]. Decreasing the complexity [24] and increasing the estimation accuracy with a small number of references are required.

4. Experimental study

In this section, our experimental study of the min-max method is presented. As discussed above, the test scenario in this case is the same as the scenario introduced in the theoretical study. Experimentally, the effect of RSSI variation on the accuracy of the min-max can be studied. In this way, more min-max method limitations will be revealed. Experimental details are described below.

4.1 Experimental setup

Experiments were done in a laboratory in the EE Department of Prince of Songkla University [19]. The test field is shown in Figure 5. In this test field, there are obstacles including book cabinets, computers, machines, chairs, tables, walls, and people (during the test). As discussed in Section 3, we placed three reference nodes at the corners of the room at reference position 1 ($x_1 = 0.00$ m, $y_1 = 6.20$ m), reference position 2 ($x_2 = 0.00$ m, $y_2 = 0.00$ m), and reference position 3

($x_3 = 3.60$ m, $y_3 = 6.20$ m). We did not place the reference node at $x = 3.60$ m, $y = 0.00$ m (i.e. the lower right corner of the room) because there is a big cabinet at this location. Six different target nodes are also placed at the test position 1 ($x_t = 1.1$ m, $y_t = 5.5$ m), test position 2 ($x_t = 2.56$ m, $y_t = 5.5$ m), test position 3 ($x_t = 3.64$ m, $y_t = 3.1$ m), test position 4 ($x_t = 2.04$ m, $y_t = 2.5$ m), test position 5 ($x_t = 0.3$ m, $y_t = 1.9$ m), and test position 6 ($x_t = 1.9$ m, $y_t = 0.3$ m). The reference nodes and the targets are placed 1 m above the floor. As stated in Section 2, after the target node receives the packet (including the RSSI), the measured RSSI is sent to the base station, which is connected to a computer via an RS232 interface. Here, the measured RSSI can be transferred to the base station with a one-hop communication step. At the computer, the min-max method is applied to determine the target position. In the position estimation, 1,000 RSSI samples are collected by the target node.

We use a LPC2103F with a CC2500 (microcontroller with radio module) for the wireless node, as illustrated in Figure 6. Here, the CC2500 is a low-power wireless radio module operating in the 2400-2483.5 MHz ISM/SRD band developed by Texas Instruments [25]. The data rate of this module is set at 250 kbps. It is connected and communicated with the LPC2103F board via an SPI (serial peripheral interface), where the LPC2103F and the CC2500 are the master and slave, respectively [26-28].

As discussed in Section 2, the RSSI to distance conversion is performed using Equation (1). To determine the path-loss equation of the test field in Figure 5, in the beginning, one transmitter and one receiver are used to measure the RSSI at five distances: 1 m, 2 m, 3 m, 4 m, and 5 m. We move the devices from 1 to 5 m distant from each other with steps of 1 m. At each distance, 10,000 RSSI samples are collected by the receiver. Figure 7 is a plot of the average RSSI (dBm) vs. the distance (meters, logarithmic scale). The path-loss equation

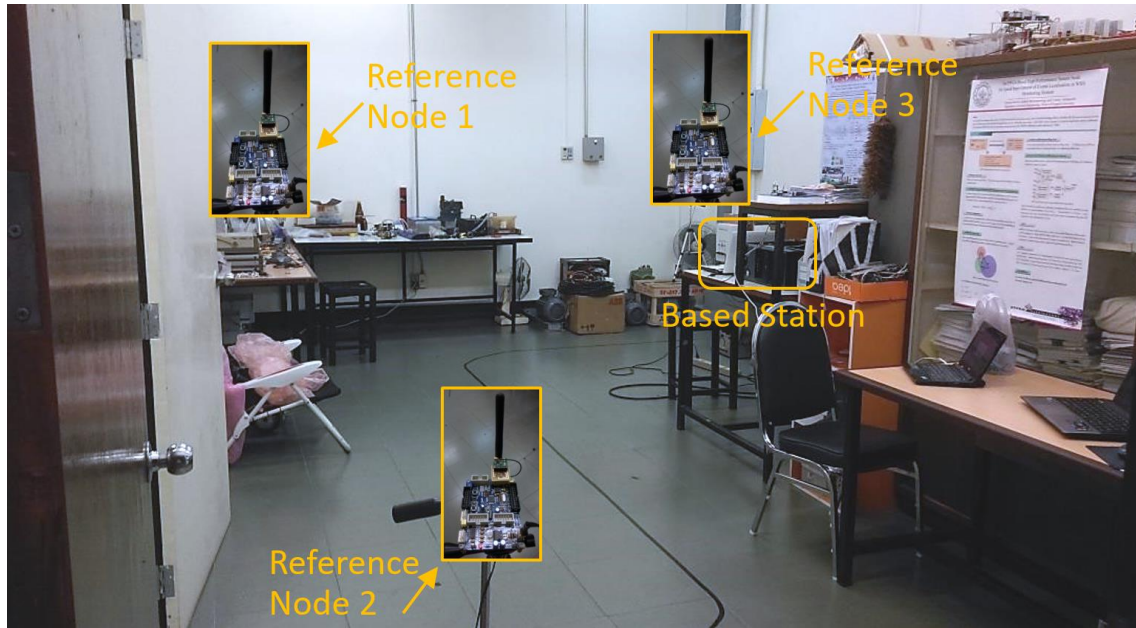


Figure 5 Test field in a laboratory room

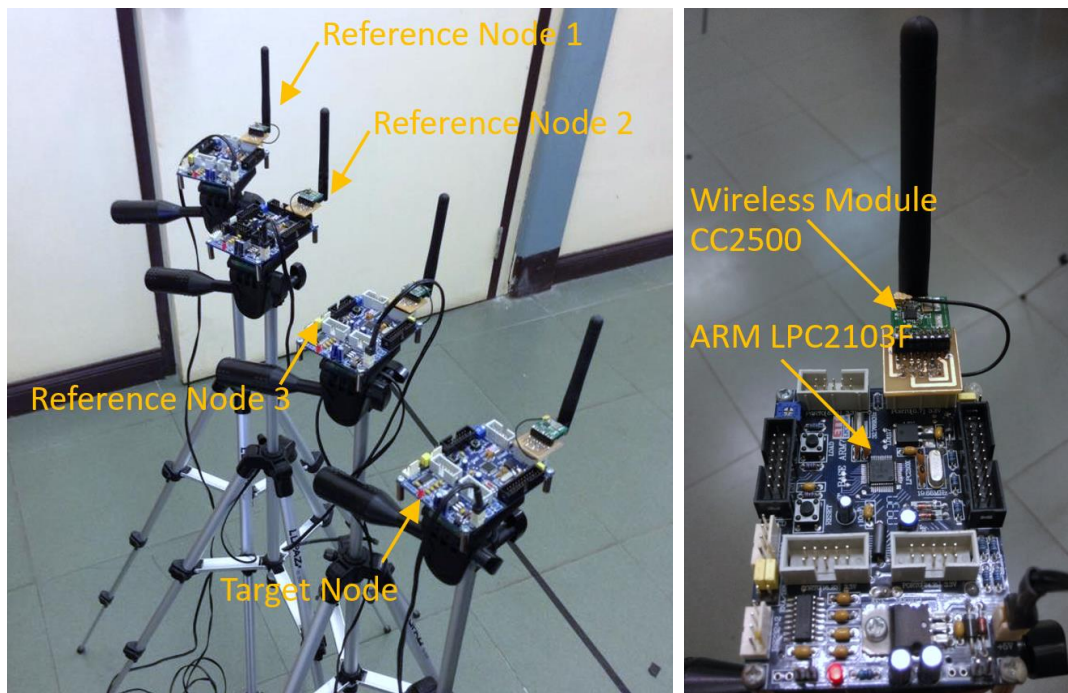


Figure 6 Wireless nodes based on a LPC2103F with a CC2500

can be determined by applying linear curve fitting, where $RSSI_{d_0}$ (dBm) is equal to -41.818, and α is equal to 3.6146 (note: $RSSI_d = RSSI_{d_0} - [10 \times \alpha \times \log_{10}(d/d_0)]$).

To evaluate the experimental data, the average RSSI from each reference, average distance from each reference, average error distance, and average estimated position are selected as performance indices. They are expressed as Equations (8-11). N is the number of RSSI samples (1,000 samples), (x_{et_i}, y_{et_i}) is the estimated position of the sample i , and (x_t, y_t) is the test position. $RSSI_i$ is the measured RSSI value, and D_i is the distance value converted from the measured RSSI. The 95% CI is also provided for each average result.

$$\text{Average estimated position} = \left(\frac{1}{N} \sum_{i=1}^N x_{et_i}, \frac{1}{N} \sum_{i=1}^N y_{et_i} \right) \tag{8}$$

$$\text{Average error distance} = \frac{1}{N} \sum_{i=1}^N \sqrt{(x_t - x_{et_i})^2 + (y_t - y_{et_i})^2} \tag{9}$$

$$\text{Average RSSI} = \frac{1}{N} \sum_{i=1}^N RSSI_i \tag{10}$$

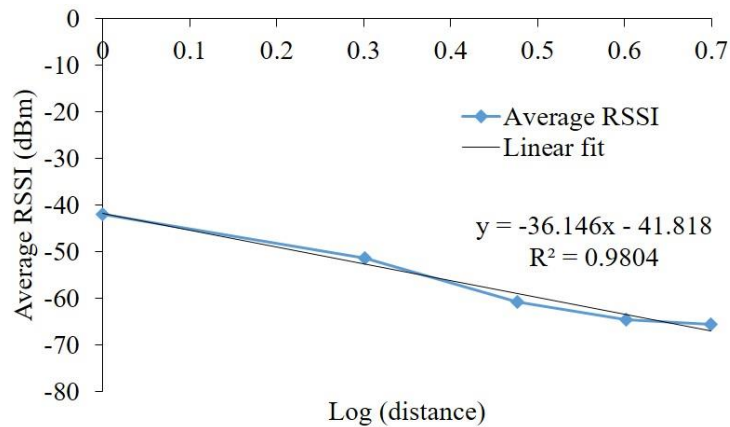


Figure 7 Average RSSI value versus distance, x is a distance on a logarithm scale, while y is an average RSSI value

Table 2 Experimental results showing the estimated position of the error distance

Test point		Average estimated position		Average error distance(m)	95% CI.
x_t (m)	y_t (m)	x (m)	y (m)		
1.100	5.500	1.525	4.055	1.506	0.004
2.560	5.500	2.116	3.940	1.621	0.007
3.640	3.100	2.135	4.903	2.350	0.007
2.040	2.500	1.642	2.974	0.620	0.001
0.300	1.900	0.812	2.347	0.682	0.004
1.900	0.300	1.221	2.521	2.324	0.002
				Average	1.518
				SD	0.756
				95% CI.	0.605

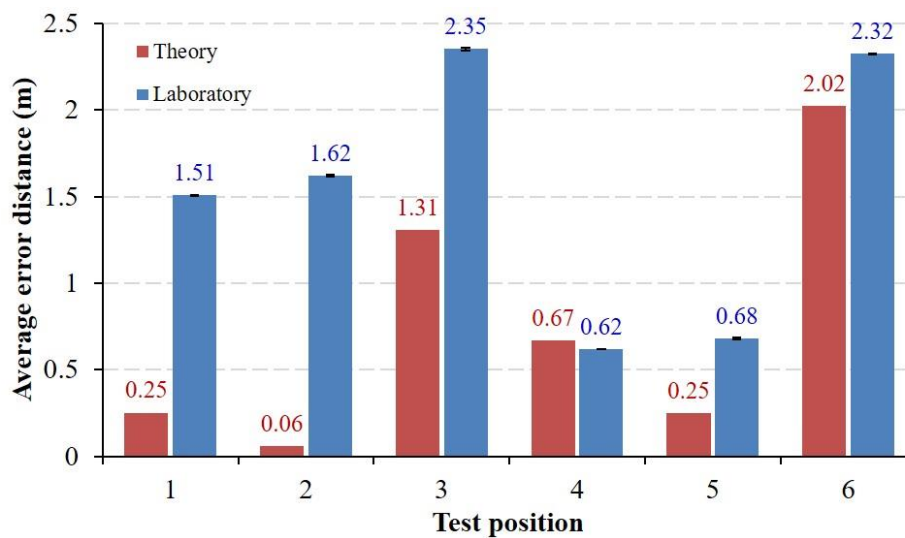


Figure 8 Average error distance at each test position, theoretical versus experimental

$$Average\ distance = \frac{1}{N} \sum_{i=1}^N D_i \tag{11}$$

4.2 Experimental results and discussions

The estimation position and the error distance (average values) with 95% CI are demonstrated in Table 2 and Figure 8. The results reveal that the min-max shows the lowest estimation error when the unknown target is placed at test position 4 ($x_t=2.04m, y_t=2.5m$). Here, the average error distance is equal to 0.620 m. Alternatively, the min-max method gives the highest estimation error when the unknown target is placed at

test position 3 ($x_t=3.64m, y_t=3.1m$). The average error distance is equal to 2.350 m. We note that by comparison, in the results from the theoretical study in Table 1, the min-max provides the lowest estimation error at test position 2 ($x_t=2.56m, y_t=5.5m$) with an average error distance of 0.060 m. The highest estimation error is at test position 6 (i.e., $x_t=1.9m, y_t=0.3m$) with an average error distance of 2.022 m. The average error distance from all test positions in the cases of the theoretical and experimental studies are equal to 0.759 m and 1.518 m, respectively, as shown in Tables 1 and 2. Here, the results from the theoretical and experimental

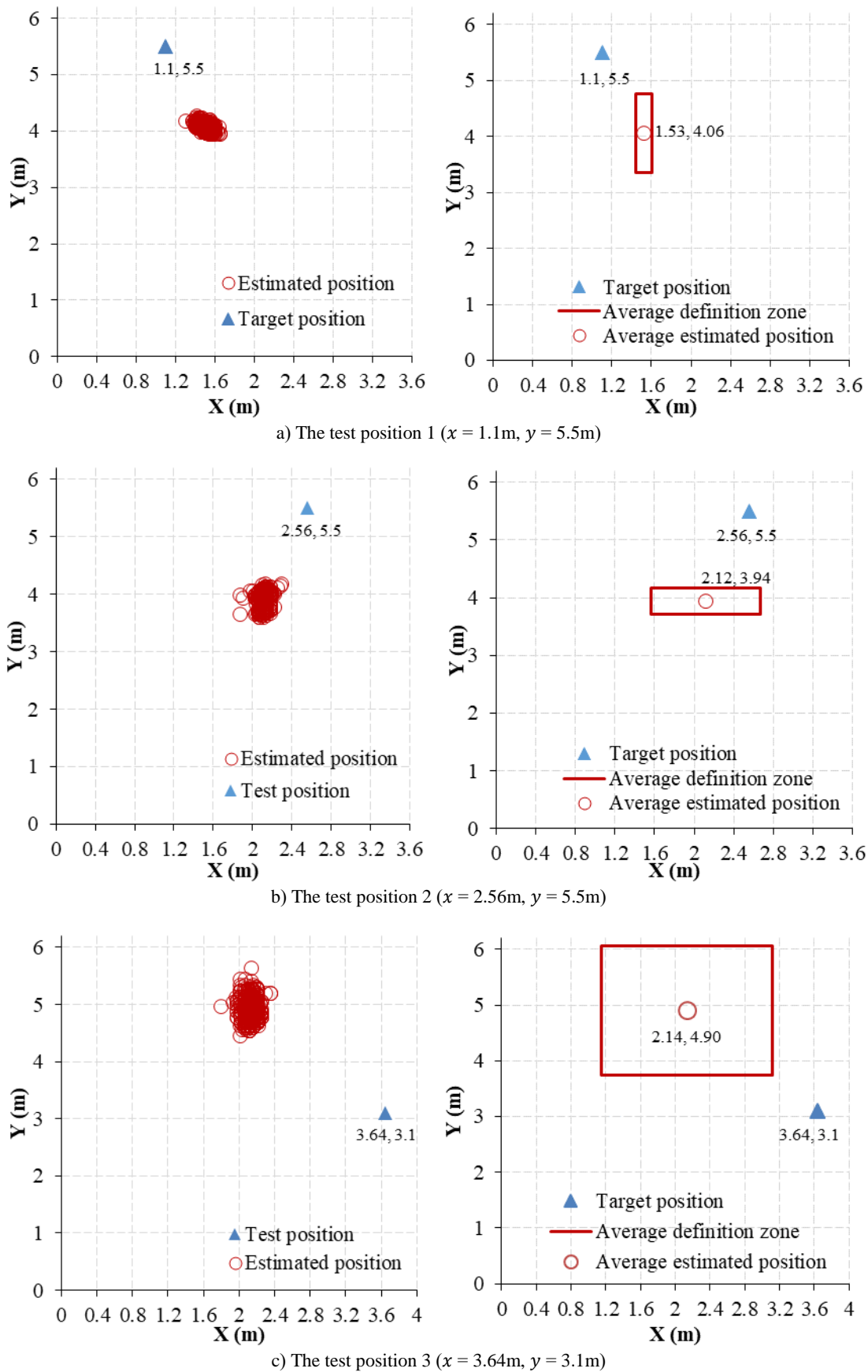


Figure 9 Illustrations of estimated positions and the average result with the average bounding box (or the average definition zone), a-f) are test positions 1, 2, 3, 4, 5, and 6, respectively

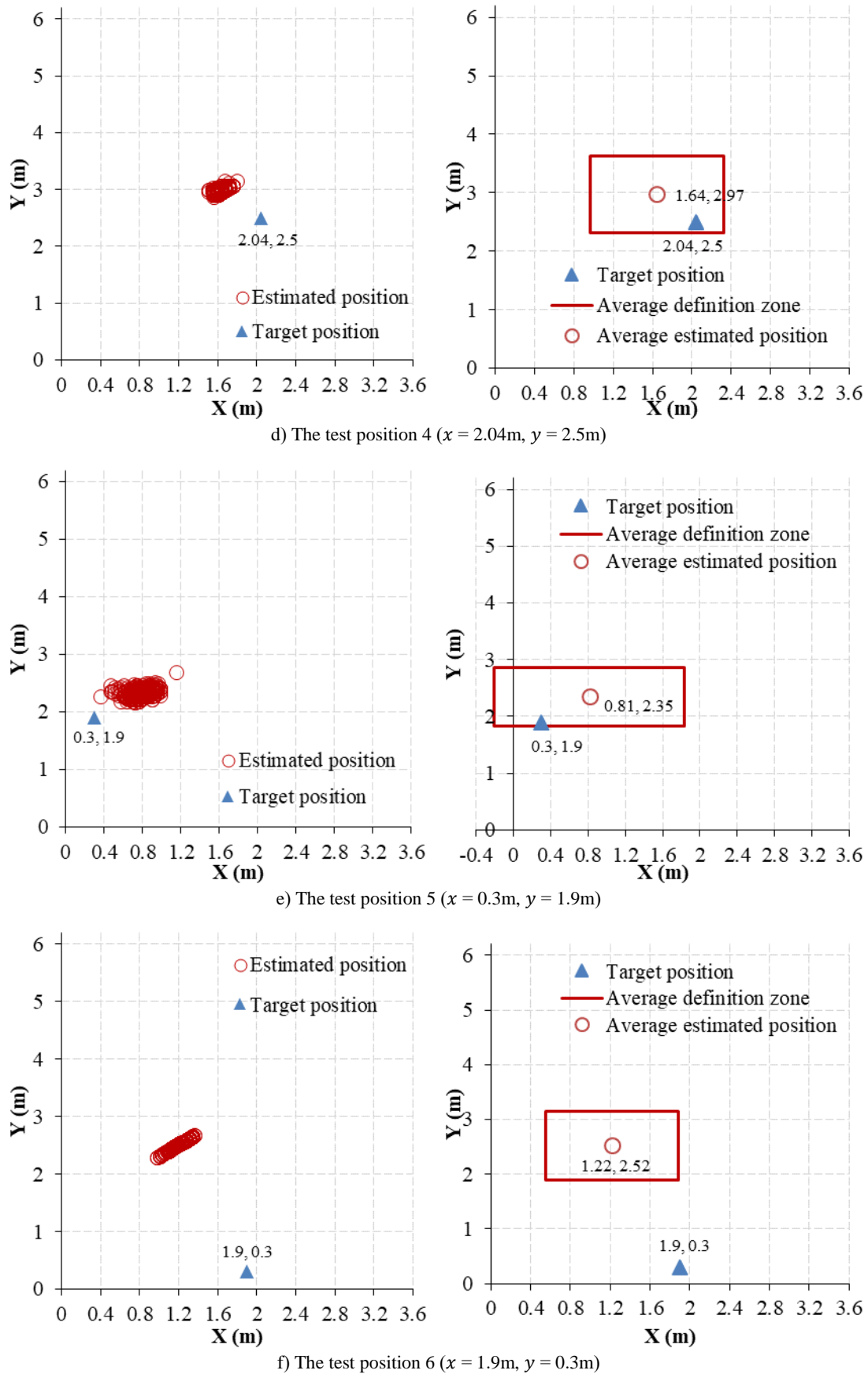


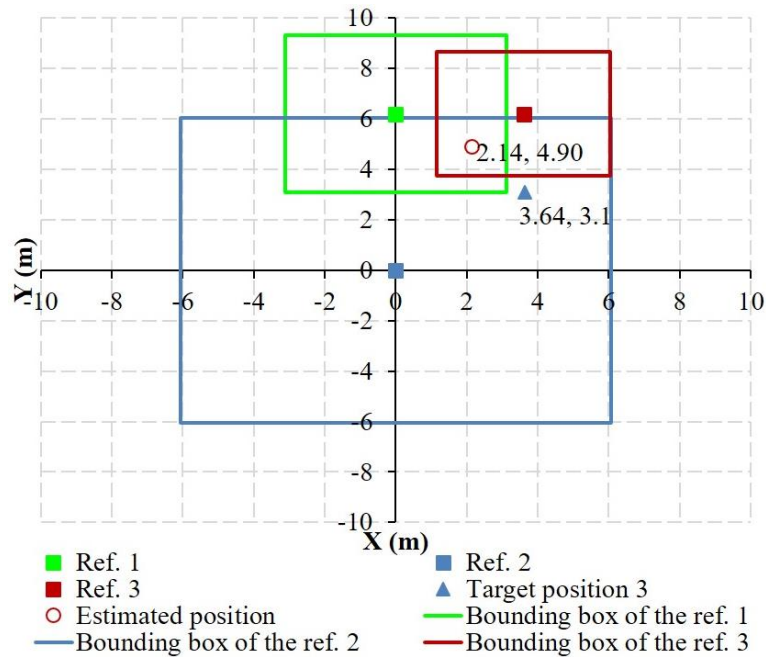
Figure 9 (Continued) Illustrations of estimated positions and the average result with the average bounding box (or the average definition zone), a-f) are test positions 1, 2, 3, 4, 5, and 6, respectively

Table 3 The average RSSI and the average distances from the reference node at each test position (experimentally derived)

Test position		Average RSSI and average distance											
x_t (m)	y_t (m)	RSSI from ref 1 (dBm)	SD	Distance from ref 1	SD	RSSI from ref 2 (dBm)	SD	Distance from ref 2	SD	RSSI from ref 3 (dBm)	SD	Distance from ref 3	SD
1.100	5.500	-47.502	0.972	1.442	0.087	-60.640	0.382	3.353	0.083	-52.545	0.312	1.992	0.040
2.560	5.500	-57.072	0.345	2.665	0.058	-62.201	0.984	3.713	0.225	-52.851	0.371	2.031	0.048
3.640	3.100	-59.529	0.385	3.122	0.077	-69.815	0.798	6.057	0.313	-55.762	0.395	2.450	0.063
2.040	2.500	-56.617	0.358	2.588	0.060	-54.926	0.365	2.321	0.055	-56.897	0.389	2.635	0.066
0.300	1.900	-60.561	0.359	3.336	0.077	-51.218	0.643	1.830	0.074	-62.596	0.558	3.803	0.140
1.900	0.300	-63.129	0.653	3.937	0.017	-51.698	0.576	1.887	0.068	-59.144	0.440	3.045	0.086

Table 4 Difference between the average distance in Table 3 and the actual distance in Table 1

Test position		Distance error between the average distance and the actual distance		
x_t (m)	y_t (m)	Distance error 1 (m)	Distance error 2 (m)	Distance error 3 (m)
1.100	5.500	$ 1.442-1.304 = 0.138$	$ 3.353-5.609 = 2.256$	$ 1.992-2.596 = 0.604$
2.560	5.500	$ 2.665-2.654 = 0.011$	$ 3.713-6.067 = 2.354$	$ 2.031-1.254 = 0.777$
3.640	3.100	$ 3.122-4.781 = 1.659$	$ 6.057-4.781 = 1.276$	$ 2.450-3.100 = 0.695$
2.040	2.500	$ 2.588-4.225 = 1.637$	$ 2.312-3.227 = 0.915$	$ 2.635-4.015 = 1.380$
0.300	1.900	$ 3.336-4.310 = 0.974$	$ 1.830-1.924 = 0.094$	$ 3.803-5.420 = 1.617$
1.900	0.300	$ 3.937-6.198 = 2.261$	$ 1.887-1.924 = 0.037$	$ 3.045-6.140 = 3.095$

**Figure 10** Illustration of the experimentally estimated position 3, where the bounding boxes of references 1 and 2 do not cover the target

studies are different. Also, the estimation accuracy of the min-max significantly decreases in the experiment case. Additionally, as discussed in Section 3, that the min-max produces a small estimation error and an error when the target is inside or outside the internal zone. This is not always true experimentally.

In Figures 9 (a-f), the estimated positions are presented to show the variation of estimated positions with the average result with the average bounding box (or the average definition zone). It is notable that the average bounding box is determined by averaging x_{min} , x_{max} , y_{min} , and y_{max} from all samples. This result reveals that at test positions 1, 2, 3, and 6, the bounding box do not cover the target. Since the min-max determines the target position inside the box, the estimation error is larger. For test positions 4 and 5, the bounding box can

cover the target position. However, the target is located on the border region. It still has an estimation error, since the min-max determines the position at the center of the box.

The reason why the min-max method shows the limitations depicted in Figure 9 can be explained using the results in Tables 3 and 4. Table 3 shows the average RSSI and the average distance from the reference node at each test position. Table 4 shows the difference between the average distance shown in Table 3 and the actual distance shown in Table 1 (i.e., in the second column). Here, we can see that the average experimental distance is different from the theoretical distance. For example, at test position 3 ($x_t = 3.64\text{m}$, $y_t = 3.1\text{m}$), the distance errors are 1.659 m (for Ref. 1), 1.276 m (for Ref. 2), and 0.695 m (for Ref. 3). It is odd that the distances are quite different. In the test environment presented in Figure 5, there

are many obstacles, i.e., chairs, tables, book cabinets, computers, electrical machines, walls, and people. Therefore, measured RSSI signals can fluctuate over time due to multipath fading (caused by the reflection, diffraction, and scattering of the radio signals in obstacle encumbered environments), interference, and noise effects. Here, radio signals are unstable and unpredictable [12, 23]. Also, in this work, the simple and inaccurate path-loss Equation (1) is used. Due to the RSSI variation effect in the test field, this path-loss equation cannot efficiently create an optimal relationship between the measured RSSI and the distance. Consequently, errors also are present while setting the path-loss equation. The RSSI signal variation can lead increased error during the RSSI-distance conversion and the box creation. Hence, the estimation error can be significant.

It is notable, as shown in Tables 1 and 4, that for test position 3 ($x_t = 3.64\text{m}$, $y_t = 3.1\text{m}$), actual distances from the references 1 and 2 are 4.781 m and 4.781 m. However, in Table 3, the average RSSI values of those cases are -59.529 dBm and -69.815 dBm, respectively, which correspond to 3.122 m and 6.057 m, respectively. This confirms the RSSI variation effect. Additionally, in this case, the results in Figure 3c and Figure 10 are different. Due to the RSSI variation effect, the bounding boxes of references 1 and 3 do not cover target position 3, as presented in Figure 10.

5. Conclusions

An investigation of RSSI-based min-max method problems for indoor scenarios is presented in this paper. We investigate how the widely used and well-known min-max method works and identify its limitations. Both theoretical and experimental studies are provided. The results demonstrate that the min-max method can be properly used to estimate an unknown target position. However, the min-max method has limitations. First, its estimation accuracy directly depends on the unknown target positions. The min-max gives large estimation errors when the unknown target is outside of the internal zone. Second, the measured RSSI signal fluctuation can produce significant estimation errors. RSSI signal variation can lead to errors during the RSSI to distance conversion and the bounding box creation. In this case, a) the bounding box does not cover the target, or b) the bounding box can cover the target, but the estimated position is not near the target since the min-max determines the position from the center of the box. Our information is useful for users and researchers to further develop more efficient min-max methods.

In future work, design and development of the min-max method to address the limitations presented in this work will be considered. Also, a balance between the estimation accuracy and the computational complexity of an extended min-max method should be provided.

6. Acknowledgments

This work was supported by Faculty of Engineering, Prince of Songkla University, Thailand.

7. References

- [1] Patwari N, Ash JN, Kyperountas S, Hero III AO, Moses RL, Correal NS. Locating the nodes: cooperative localization in wireless sensor networks. *IEEE Signal Process Mag.* 2005;22(4):54-69.
- [2] Liu H, Darabi H, Banerjee P, Liu J. Survey of wireless indoor positioning techniques and systems. *IEEE Trans Syst Man Cybern C Appl Rev.* 2007;37(6):1067-80.
- [3] Redondi A, Chirico M, Borsani L, Cesana M, Tagliasacchi M. An integrated system based on wireless sensor networks for patient monitoring, localization and tracking. *Ad Hoc Netw.* 2013;11(1):39-53.
- [4] He J, Geng Y, Pahlavan K. Modeling indoor TOA ranging error for body mounted sensors. *Proceedings of the IEEE 23rd International Symposium on Personal, Indoor, and Mobile Radio Communications (PIMRC)*; 2012 Sep 9-12; Sydney, Australia. USA: IEEE; 2012. p. 682-6.
- [5] Severino R, Alves M. Engineering a search and rescue application with a wireless sensor network-based localization mechanism. *Proceedings of IEEE International Symposium on a World of Wireless, Mobile and Multimedia Networks*; 2007 Jun 18-21; Espoo, Finland. USA: IEEE; 2007. p. 1-4.
- [6] Mautz R. Indoor positioning technologies [thesis]. Zürich: ETH Zurich; 2012.
- [7] Luo X, Brien WJ, Julien CL. Comparative evaluation of received signal-strength index (RSSI) based indoor localization techniques for construction jobsites. *Adv Eng Informat.* 2011;25(2):355-63.
- [8] Pei Z, Deng Z, Xu S, Xu X. Anchor-free localization method for mobile targets in coal mine wireless sensor networks. *Sensors.* 2009;9(4):2836-50.
- [9] Bjorkbom M, Nethi S, Eriksson LM, Jantti R. Wireless control system design and co-simulation. *Contr Eng Pract.* 2011;19(9):1075-86.
- [10] Booranawong A, Teerapabkajornmet W, Limsakul C. Energy consumption and control response evaluations of AODV routing in WSANs for building-temperature control. *Sensors* 2013;13(7):8303-30.
- [11] Booranawong A, Teerapabkajornmet W. An enhanced AODV routing protocol for wireless sensor and actuator networks. *Int J Electron Comm Eng.* 2013;7(12):1693-700.
- [12] Zanca G, Zorzi F, Zanella A, Zorzi M. Experimental comparison of RSSI-based localization algorithms for indoor wireless sensor networks. *Proceedings of the Workshop on Real-world Wireless Sensor Networks*; 2008 Apr 1; Glasgow, Scotland. New York: ACM press; 2008. p. 1-5.
- [13] He S, Hu T, Chan SHG. Contour-based trilateration for indoor fingerprinting localization. *Proceedings of the 13th ACM Conference on Embedded Networked Sensor Systems*; 2015 Nov 1-4; Seoul, South Korea. New York: ACM press; 2015. p. 225-38.
- [14] Hossain AKMM, Soh WS. Cramer-Rao bound analysis of localization using signal strength difference as location fingerprint. *Proceedings of the IEEE INFOCOM*; 2010 Mar 14-19; San Diego, USA. USA: IEEE; 2010. p. 1-9.
- [15] Savvides A, Park H, Srivastava M. The bits and flops of the N-hop multilateration primitive for node localization problems. *Proceedings of the First ACM International Workshop on Wireless Sensor Networks and Application*; 2002 Sep 28; Atlanta, USA. New York: ACM press; 2002. p. 112-21.
- [16] Goldoni E, Savioli A, Risi M, Gamba P. Experimental analysis of RSSI-based indoor localization with IEEE 802.15.4. *Proceedings of the 2010 European Wireless Conference*; 2010 Apr 12-15; Lucca, Italy. USA: IEEE; 2010. p. 71-7.
- [17] Kaseva VA, Kohvakka M, Kuorilehto M, Hannikainen M, Hamalainen TD. A wireless sensor network for RF-based indoor localization. *EURASIP J Adv Signal Process.* 2008;2008:1-27.

- [18] Kianoush S, Goldoni E, Savioli A, Gamba P. Low-complexity localization and tracking in hybrid wireless sensor networks. *ISRN Sens Network*. 2012;2012:1-7.
- [19] Rattanalert B, Jindamanepon W, Sengchuai K, Booranawong A, Jindapetch N. Problem investigation of min-max method for RSSI based indoor localization. *Proceedings of the 12th International Conference on Electrical Engineering/ Electronics, Computer, Telecommunications and Information Technology*; 2015 Jun 24-27; Hua Hin, Thailand. USA: IEEE; 2015. p. 1-5.
- [20] Robles JJ, Pola JS, Lehnert R. Extended min-max algorithm for position estimation in sensor networks. *Proceedings of the 9th Workshop on Positioning Navigation and Communication (WPNC)*; 2012 Mar 15-16; Dresden, Germany. USA: IEEE; 2012. p. 47-52.
- [21] Shi X, Zhang L. High-precision weighted bounding box localization algorithm for wireless sensor network. *Proceedings of the third on Information Science and Technology*; 2013 Mar 23-25; Yangzhou, China. USA: IEEE; 2013. p. 1110-3.
- [22] Langendoen K, Reijers N. Distributed localization in wireless sensor networks: a quantitative comparison. *Comput Network*. 2003;43(4):499-518.
- [23] Mao G, Anderson BD, Fidan B. Path loss exponent estimation for wireless sensor network localization. *Comput Network*. 2007;51(10):2467-83.
- [24] Booranawong A, Jindapetch N, Saito H. Adaptive filtering methods for RSSI signals in a device-free human detection and tracking system. *IEEE Syst J*. 2019;13(3):2998-3009.
- [25] Texas Instrument. Instrument T CC2500 datasheet [Internet]. 2017 [cited 2017 Nov 8]. Available from: <http://www.tij.co.jp/jp/lit/ds/swrs040c/swrs040c.pdf>.
- [26] Yang LD. Implementation of a wireless sensor network with EZ430-RF2500 development tools and MSP430FG4618/F2013 experimenter boards from Texas instruments [thesis]. Louisiana: Department of Electrical & Computer Engineering, Louisiana State University and Agricultural and Mechanical College; 2011.
- [27] Jindamanepon W, Rattanalert B, Sengchuai K, Booranawong A, Jindapetch N. A novel FPGA-based multi-channel multi-interface wireless node: implementation and preliminary test. In: Sulaiman H, Othman M, Othman M, Rahim Y, Pee N, editors. *Advanced Computer and Communication Engineering Technology, Lecture Notes in Electrical Engineering*. Cham: Springer; 2016. p. 1163-73.
- [28] Booranawong A, Jindapetch N, Saito H. A system for detection and tracking of human movements using RSSI signals. *IEEE Sensor J*. 2018;18(6):2531-44.

8. Appendix A: The min-max method with a varying number of reference nodes

In Section 3, the estimation error can be reduced by using more reference nodes in the test field [2, 12, 16]. Here, we illustrate theoretical results when more reference nodes are added in the scenario shown in Figure 2. Estimated position results by applying different numbers of reference nodes (i.e., 3, 8, and 32 nodes) with 231 unknown target nodes (examples) are shown in Figures 11 (a-c). The average error distances versus the number of reference nodes (i.e., 3, 4, 8, 16, and 32 nodes) are also shown in Figure 12. The results reveal that using 3 to 32 reference nodes, the average error distance (with the minimum and maximum values) are 0.85 m, 0.37 m, 0.17 m, 0.07 m, and 0.03 m, respectively.

As can be observed from Figure 12, although the average error distance can be reduced by using more reference nodes, the errors are not much different, as in the cases of 16 and 32 reference nodes (i.e., 0.07 m and 0.03 m). Moreover, using more reference nodes increases computational overhead. As illustrated in Figure 13, the number of times that the min-max method runs each mathematical operation is significantly increased when more reference nodes are applied. We note that the number of mathematical operations (summation (+), subtraction (-), division (/), and comparison) required by the min-max method, as presented in Equations (2-7), are determined, for example, using three reference nodes, (+), (-), (/), and comparison are 8, 6, 2, and 8 times more intensive, respectively. They correspond to $y=2n+2$, $y=2n$, $y=2$, and $y=4n-4$, respectively, where n is numbers of reference nodes. However, as discussed in Section 3, increasing the estimation accuracy, reducing the complexity, and using fewer reference nodes are very challenging. This is the design goal of the localization method.

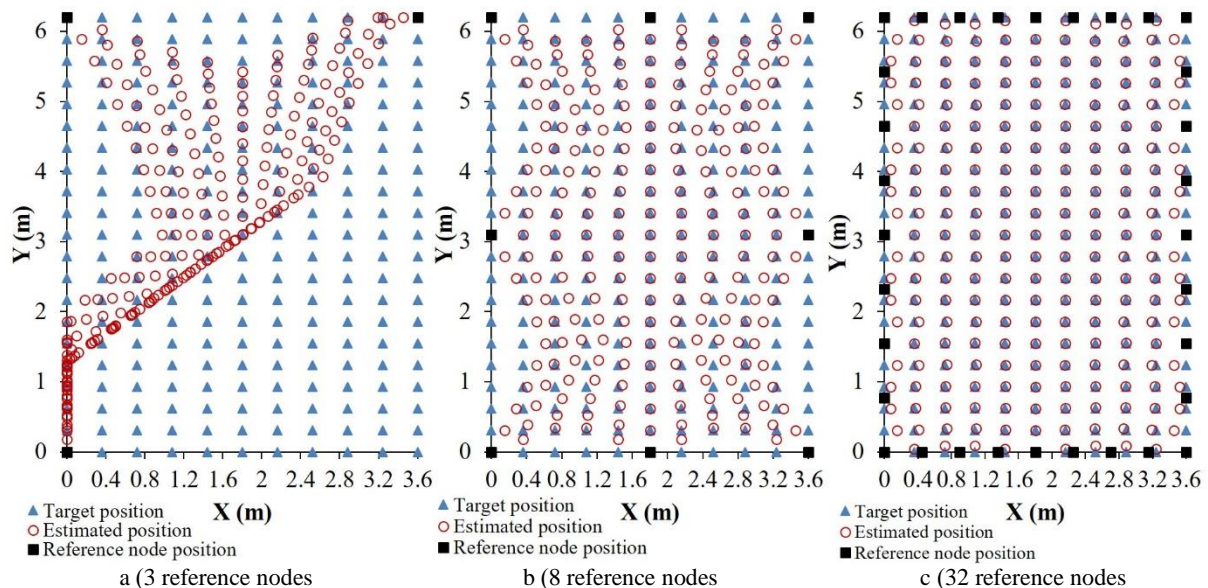


Figure 11 Estimated position results with different numbers of reference nodes, a-c use 3, 8, and 32 reference nodes, respectively

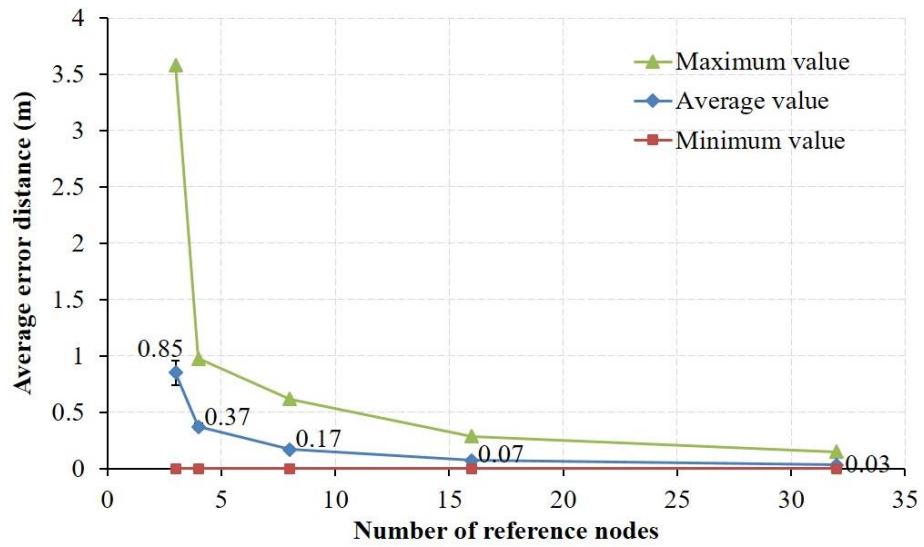


Figure 12 Average error distances versus the number of reference nodes

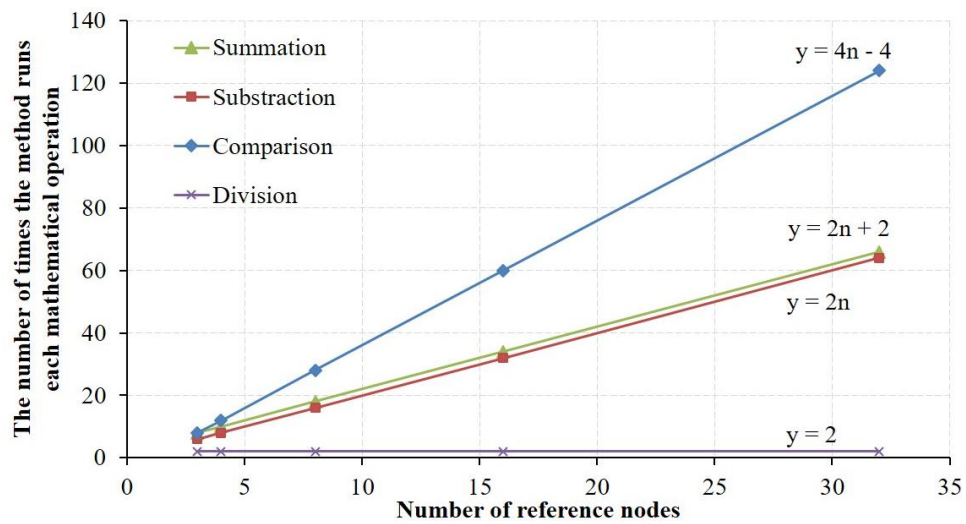


Figure 13 The number of times that the min-max method runs each mathematical operation, where n is numbers of reference nodes