Engineering and Applied Science Research April – June 2020;47(2):137-144 Research Article

Engineering and Applied Science Research
https://www.tci-thaijo.org/index.php/easr/index
Published by the Faculty of Engineering, Khon Kaen University, Thailand

A safety-based evaluation of strut-and-tie methods for shear design of RC deep beams in accordance with international concrete codes

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Received 26 August 2019
Revised 5 November 2019
Accepted 8 November 2019

Abstract

On the basis of strut-and-tie models (STMs) in accordance with international concrete codes such as ACI 318-14, AASHTO LRFD, CSA A23.3, Eurocode2 and *fib* MC 2010, a safety-based evaluation of shear design methods for RC deep beams was performed and is reported in this paper. The variability in load actions and member resistances consisting of uncertainty in material characteristics, fabrication tolerances and modeling uncertainties were taken into account as random variables. Using Rackwitch-Fiessler's procedure with typical ranges for normal and high strength concrete and an extensive range of live-to-dead load ratios, the reliability or safety indices used to measure the safety level were investigated. It was found that the deep beams made from normal strength concrete and designed using STMs following the international concrete codes considered here provided a satisfactory safety level. Finally, for each of the STMs for design of deep beams, probability-based reduction factors are suggested to fulfill the target reliability index by greater than 3.5.

Keywords: Deep beam, Strut-and-tie model, Structural concrete, Reliability analysis, Structural Safety

1. Introduction

Reinforced concrete (RC) deep beams have been used as structural members to transmit loads in buildings, bridges and many types of construction. Owing to the differences in behavior of deep and slender beams, the approaches for analysis and design of these two types of beams are significantly dissimilar. As the shear span-to-depth ratio (*a*/*d*) of a deep beam is smaller than about 2, the strut-and-tie action governs its shear strength, i.e., the loads can be transmitted from the applied points to the supports directly. Accordingly, a simple and rational approach, i.e., strut-and-tie model (STM), has been specified in international concrete codes of practice for designing deep beams. The concept of the STM is described briefly below. The compressive and tensile stress paths in an RC deep beam subjected to two point loads can be represented by the STM as shown in Figure 1. It contains diagonal compression struts and tension ties, connected together at nodal zones. Struts and ties are designed as compression and tension members in a truss structure Numerous researchers have conducted experimental programs [1-4] and have proposed the STMs [5-9] to describe the behavior of RC deep beams for more than half a century. Moreover, The STM approach is also applied for analysis and design of other RC members with discontinuity regions [10-12]. Continuing interest is demonstrated in this research area.

From an extensive literature search, only a few researchers [7-8] have examined the safety of RC deep beams designed according to ACI 318-08 and AASHTO LRFD. In particular, most of the existing understrength factors for the shear design of deep beams using the STM according to current concrete code provisions have not been achieved using a probability-based procedure, i.e., reliability analysis, but have been taken equal to the factors specified for the shear design method for slender beams. Consequently, it is questionable whether the use of these understrength factors for the design of RC deep beams provides an appropriate safety level. Currently, reliability analysis has been applied to evaluate the safety level of structural members and develop their understrength factors, as well as other applications in civil engineering. Numerous research studies have been performed in this subject area, such as developing the reliability methods [13], target reliability [14], reliability of concrete structures [7, 8, 15-18], and code calibration [19-20].

To further extend the previous work by Chetchotisak et al. [7-8] so that it is more reliable and rational, a database for developing model uncertainties was greatly extended. The safety levels of the STM for design of deep beam according to ACI 318-14 [21], AASHTO LRFD [22], CSA A23.3 [23], Eurocode2 [24] and *fib* MC 2010 [25] were investigated, and probability-based strength reduction factors are also recommended.

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doi: 10.14456/easr.2020.14

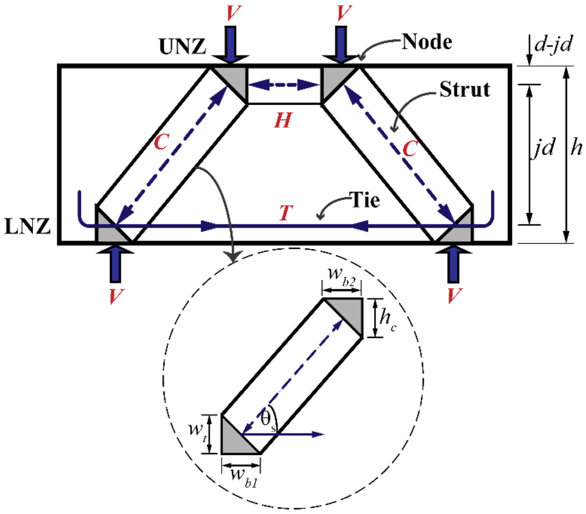

Figure 1 An STM for an RC deep beam

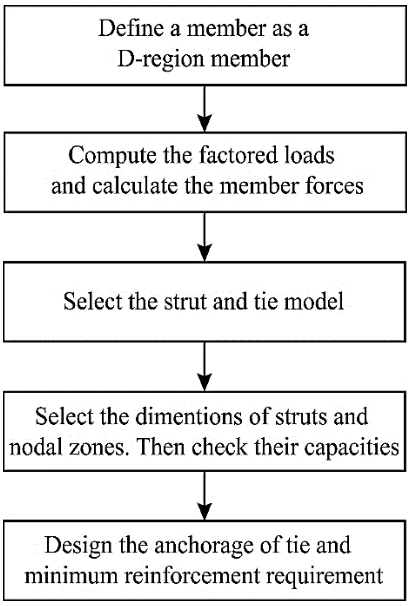

Figure 2 Flow diagram for design of RC deep beams

2. Strut-and-tie model for shear design of reinforced concrete deep beams

An STM consisting of diagonal concrete struts, tension ties and nodal zones is used to idealize the load transfer mechanism of an RC deep beam, as shown in Figure 1. Concrete struts and tension ties are used to represent the concrete portions in compression and steel reinforcement, respectively. Guidance for the design of RC deep beams and other D-region members using STM is shown in Figure 2. From the equilibrium condition, a diagonal force in a concrete strut C and a tension force T in a tie member can be computed as:

$$C = V / \sin\theta_s \tag{1}$$

$$T = H = V / \tan\theta_s \tag{2}$$

where V and H are the vertical and horizontal forces. θ_s is the inclined angle of the diagonal compression strut with respect to the horizontal plane, and can be obtained from

$$\theta_s = \tan^{-1}\frac{jd}{a} \tag{3}$$

where a is the shear span while jd represents the flexural lever arm, i.e., the distance between the resultants of compressive and tensile forces, expressed as:

$$jd = h - w_t/2 - h_c/2 \tag{4}$$

where w_t is the effective depth of the concrete tie at the lower nodal zone (LNZ) and h_c is the depth of the upper nodal zone (UNZ), which will be described later. To determine the nominal shear strength of an RC deep beam, in general, the minimum strength among struts, ties and nodal zones is considered to be the strength of the entire member. With reference to the code-based STMs, the capacity of a concrete strut F_{ns}, nodal region F_{nn} as well as steel tension tie F_{nt} can be expressed as:

$$F_{ns} = f_{ce}A_{str} \tag{5}$$

$$F_{nn} = f_{ce}A_{nz} \tag{6}$$

$$F_{nt} = f_y A_{ts} \tag{7}$$

where f_y is the yield strength of the steel reinforcement, while f_{ce} represents the effective strength of a compression strut or nodal region provided by each of the concrete codes, i.e., ACI 318-14, AASHTO LRFD, CSA A23.3 , Eurocode2 and *fib* MC 2010, as summarized in Table 1. A_{str}, A_{nz} and A_{ts} are the cross-sectional areas of a compression strut, nodal region and steel tie, respectively. Due to a relatively high certainty in strength prediction of RC beams failing in flexure and a low variability in material properties of steel reinforcement, this study is limited to only the shear design of RC deep beams. According to Eqs. (1) and (5), the shear strength of an RC deep beam, V_n, can generally be assumed to be governed by the load-carrying capacity of the diagonal compression strut, and can be expressed as

$$V_n = f_{ce}A_{str}\sin\theta_s \tag{8}$$

Next, considering the dimensions of UNZ and LNZ in Figure 1, A_{str} can be calculated as:

$$A_{str} = \min(w_t\cos\theta_s + w_{b1}\sin\theta_s, h_c\cos\theta_s + w_{b2}\sin\theta_s)b_w \tag{9}$$

where b_w is the width of the deep beam, w_{b1} and w_{b2} are the width of bearing plate at support and loading points, respectively. Additionally, w_t and h_c can be computed based on Eq. (6), i.e., $A_{nz} = b_w w_t$ for LNZ and $A_{nz} = b_w h_c$ for UNZ.

3. Reliability analysis

3.1 Load and resistance factor design

In the case of only dead and live loads, the general design format in load and resistance factor design (LRFD) can be expressed as:

$$\phi R_n \geq \gamma_D D_n + \gamma_L L_n \tag{10}$$

Table 1 Effective strengths of concrete struts and nodal zones used in conjunction with STMs

Code/Standard	**Effective strength**
ACI 318-14	Prismatic strut: $f_{ce} = 0.85\beta_s f_c'$, $\beta_s = 1.0$ Bottle- shaped strut: $\beta_s = 0.60$ Nodal Zone: $f_{ce} = 0.85\beta_n f_c'$ CCC: $\beta_n = 1.0$ CCT: $\beta_n = 0.60$
AASHTO-LRFD and CSA A23.3	Strut: $f_{ce} = \frac{f_c'}{0.8+170\varepsilon_1} \leq 0.85f_c'$, $\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002)/\tan^2\theta_s$ Nodal Zone: CCC: $f_{ce} = 0.85f_c'$ CCT: $f_{ce} = 0.65f_c'$
Eurocode2	Strut without transverse tension: $f_{ce} = \sigma_{Rd,\ \max} = 0.85f_{ck}$ Strut with transverse tension: $f_{ce} = \sigma_{Rd,\ \max} = 0.6(1\text{-}f_{ck}/250) \cdot f_{ck}$ Nodal Zone CCC: $f_{ce} = 1.0\cdot(1\text{-}f_{ck}/250) \cdot f_{ck}$ CCT: $f_{ce} = 0.75\cdot(1-f_{ck}/250) \cdot f_{ck}$
fib MC2010	$f_{ce} = k_c f_{ck}$ $\eta_{fc} = \left(\frac{30}{f_{ck}}\right)^{1/3} \leq 1.00$ Diagonal Strut $k_c = 0.55\eta_{fc}$ Nodal Zone: CCC: $k_c = 1.00\eta_{fc}$ CCT: $k_c = 0.75\eta_{fc}$

$f_{ck} = f_c' - 16$ = characteristic cylinder strength [26]
ε_1 denotes transverse tensile strain of the diagonal strut
ε_s denotes tensile strain in the tension tie
CCC node = nodes bounded by compression or bearing
CCT node = nodes anchoring one tie

Table 2 Partial safety factors used for different design methods

Code/Standard	γ_D	γ_L	ϕ
ACI318-14	*1.2	1.6	0.75
AASHTO-LRFD	1.25	1.75	0.70
CSA A23.3	*1.25	1.5	0.65
Eurocode2	1.35	1.5	0.67
fib MC2010	1.35	1.5	0.67

* for only dead load considered, $\gamma_d = 1.4$

where R_n, D_n and L_n are the nominal values of the shear strength, dead load and live load, respectively. Additionally, ϕ, γ_D and γ_L are the strength reduction factor, the dead and live load factors, respectively. These partial safety factors are applied to control unforeseeable variability in load effects and shear strength of members, and listed for each of the codes of practice in Table 2.

3.2 Safety index

To assess the safety level of a structural design approach, the safety index β was employed using reliability analysis. The reliability analysis technique is also used to develop the load factors and understrength factors. As is well-known, the safety index has an inverse relation with the probability of failure (p_F). It is defined as the likelihood that a structural member will be unsafe, as follows:

$$\beta = -\Phi^{-1}(p_F) \tag{11}$$

$$p_F = \Pr(g \leq 0) \tag{12}$$

where Φ^{-1} is the inverse standard normal distribution function. The term, $g = R - Q$, represents the limit state function explaining the margin of safety of a structural component. R and Q are random variables of resistance and load effect, respectively. However, for this investigation, only dead D and live loads L were taken into account, ($Q = D + L$). For the linear limit state function based on the safety margin concept, i.e., the resistance minus the applied load and the assumption of no significant correlation, the reliability index β can be defined as:

$$\beta = \frac{\mu_R - \mu_D - \mu_L}{\sqrt{\sigma_R^2 + \sigma_D^2 + \sigma_L^2}} \tag{13}$$

where μ_R, μ_D and μ_L are the mean value of resistance, applied dead and live loads, respectively, while σ_R, σ_D and σ_L are their corresponding standard deviations. In Eq. (13), the mean value, being a product of bias factor and nominal value, can be used as follows: $\mu_R = \lambda_R R_n$, $\mu_D = \lambda_D D_n$ and $\mu_L = \lambda_L L_n$, where λ_R, λ_D, and λ_L are bias factors (i.e., ratios of mean to nominal value) of resistance, dead load and live load, respectively. Additionally, the standard deviations in Eq. (13) are obtained as products of means and the related COVs: $\sigma_R = \mu_R\delta_R$, $\sigma_D = \mu_D\delta_D$ and $\sigma_L = \mu_L\delta_L$.

3.3 Member resistance model

According to Nowak and Collins [27], the member resistance, R can be taken into account as a product of the nominal member resistance, R_n and three uncertainty factors: material variability, M, variability in fabrication tolerances, F, and modelling error, P as:

$$R = R_n M\ F\ P \tag{14}$$

The first two factors are based on the statistical data shown in Table 3, and were obtained by Monte Carlo simulations (MCS). Since the precision of MCS is well known to depend strongly on the number of samples used [27-28], a larger sample size performs better than a small one. For example, Ang and Tang [28] and Baji and Ronagh [29] showed that the use of 100,000 samples is appropriate for MCS. Therefore, this study was performed with such a sample size. The last factor, i.e., the modeling error used to measure the level of precision and consistency of a design method, is expressed in terms of statistical parameters and probabilistic distribution of the ratios of experimental-to-nominal predicted capacity without any understrength factors.

A database of 573 tests of RC deep beams extended from Chetchotisak et al. [7-8] (408 test data) and further assembled from Reineck and Todisco [30] and Todisco et al. [31], was utilized in this analysis. Using the Kolmogorov-Smirnov goodness-of-fit test at the 5 percent significance level, the bias value λ_p and the COV δ_p as well as the type of probability distribution of the modeling uncertainty were obtained. Here, some commonly used distributions, e.g. Gaussian, Lognormal, Weibull and Gumbel distributions, were applied.

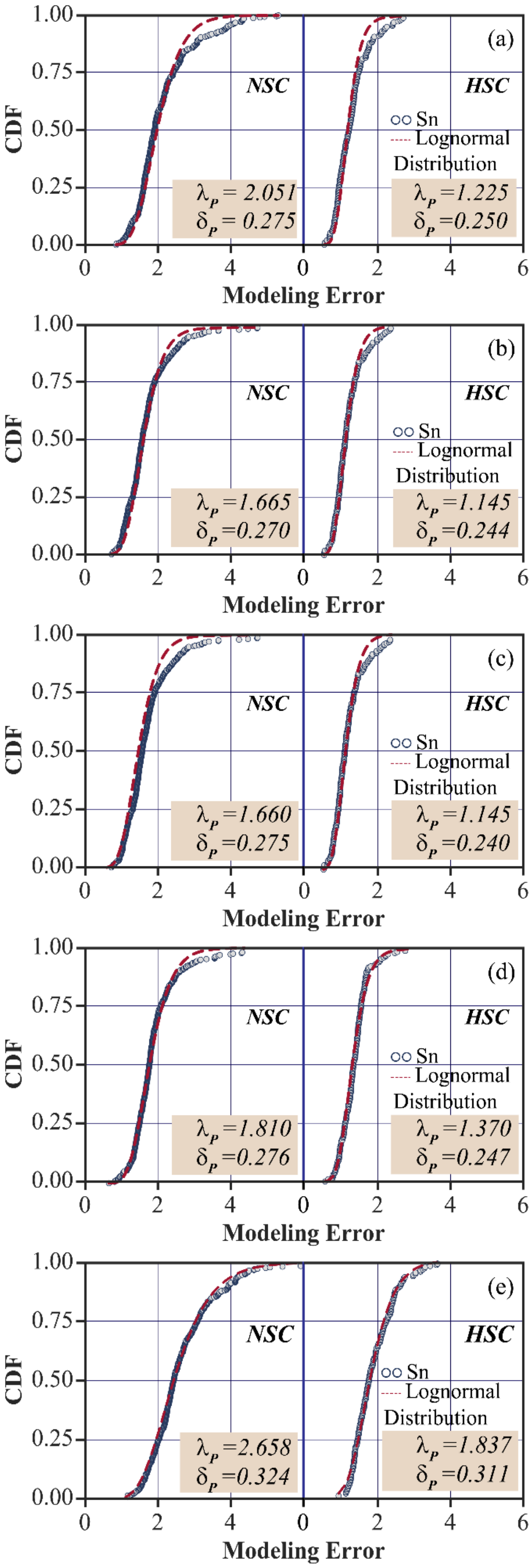

Figure 3 CDF for the K-S goodness-of-fit test of modeling error of deep beam design methods: (a) ACI 318-14; (b) AASHTO-LRFD; (c) CSA A23.3; (d) Eurocode2 and (e) *fib* MC2010

Table 3 Random variable data used in this study

Random variable	Bias λ	COV δ	Type of distribution
*Dead load	1.05	0.10	Normal
*Live load	1.00	0.18	Normal
Modeling error	--------Following Fig. 3 -------		
*f_c'= 20 MPa	1.38	0.155	Normal
*f_c'= 30 MPa	1.25	0.135	Normal
*f_c'= 40 MPa	1.18	0.120	Normal
*f_c'= 50 MPa	1.11	0.11	Normal
*f_c'= 60 MPa	1.10	0.11	Normal
*f_c'= 80 MPa	1.08	0.11	Normal
**b_w	1.01	0.04	Normal
**d	0.99	0.04	Normal

*Rakoczy and Nowak [32], **Applied from the properties of RC beams from Nowak et al. [33]

Figure 3 shows comparisons between the cumulative probability distribution function (CDF) of observations (S_n, circles) and the assumed lognormal distribution achieved by best fit (dashed curve) of the modeling error. The statistical parameters λ_p and δ_p are also shown. It was found that no probability models fitted by lognormal distributions were rejected at the 5% significance level. Therefore, it can be concluded that the selected parameters and the lognormal distributions were appropriate for all design methods. Additionally, on the basis of the central limit theorem [27], the bias factor λ_R and COV, δ_R of the resistance are given by:

$$\lambda_R = \lambda_{MF}\lambda_P \tag{15}$$

$$\delta_R = \sqrt{\delta_{MF}^2 + \delta_P^2} \tag{16}$$

where λ_{MF} and δ_{MF} are bias factors of material-fabrication and the related COVs, respectively. These statistical parameters were obtained from MCS as previously described. It should be noted that each of the random variables used for this analysis had different probability distributions, e.g., Gaussian and lognormal distributions, etc. However, the equivalent normal distribution approach, e.g., Rackwitch-Fiessler's method [27], can be applied.

4. Reliability analysis results

4.1 Safety evaluation and calibration

In this study, the criterion applied to assess and calibrate a structural design method in terms of safety or reliability is that such a design method should have satisfactory and consistent safety, covering the usual range of design parameters [34]. Here, a safety index of 3.5 (equivalent to an approximate probability of failure of 1/5,000) is used as an indication to ensure an acceptable level of safety for structural concrete that is susceptible to failure in shear [22, 35] At first, a concrete strength of 30 MPa for the case of normal strength concrete (NSC) and 60 MPa for the case of high strength concrete (HSC) was used. A live-to-dead load ratio *L/D* of 0.3 and safety index, β, obtained for various code-based STMs for shear design of RC deep beams were investigated, as shown in Figure 4. The statistical parameters employed are described above.

As shown from this figure, it was found that all STMs made from NSC had relatively greater safety indices compared to those made from HSC. As a result, it can be

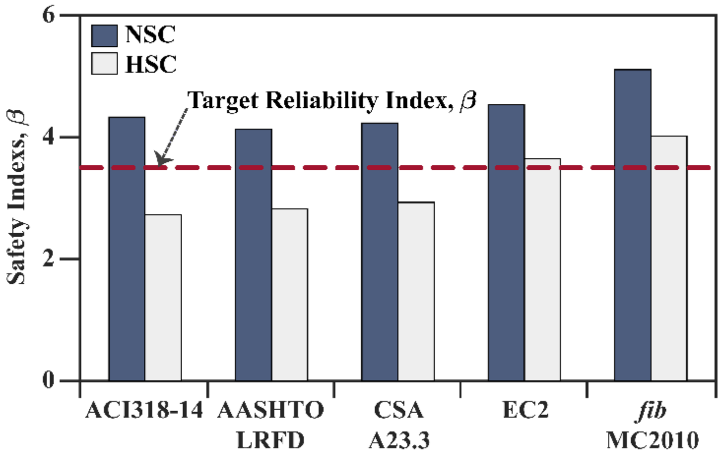

Figure 4 Safety indices evaluated for various design methods

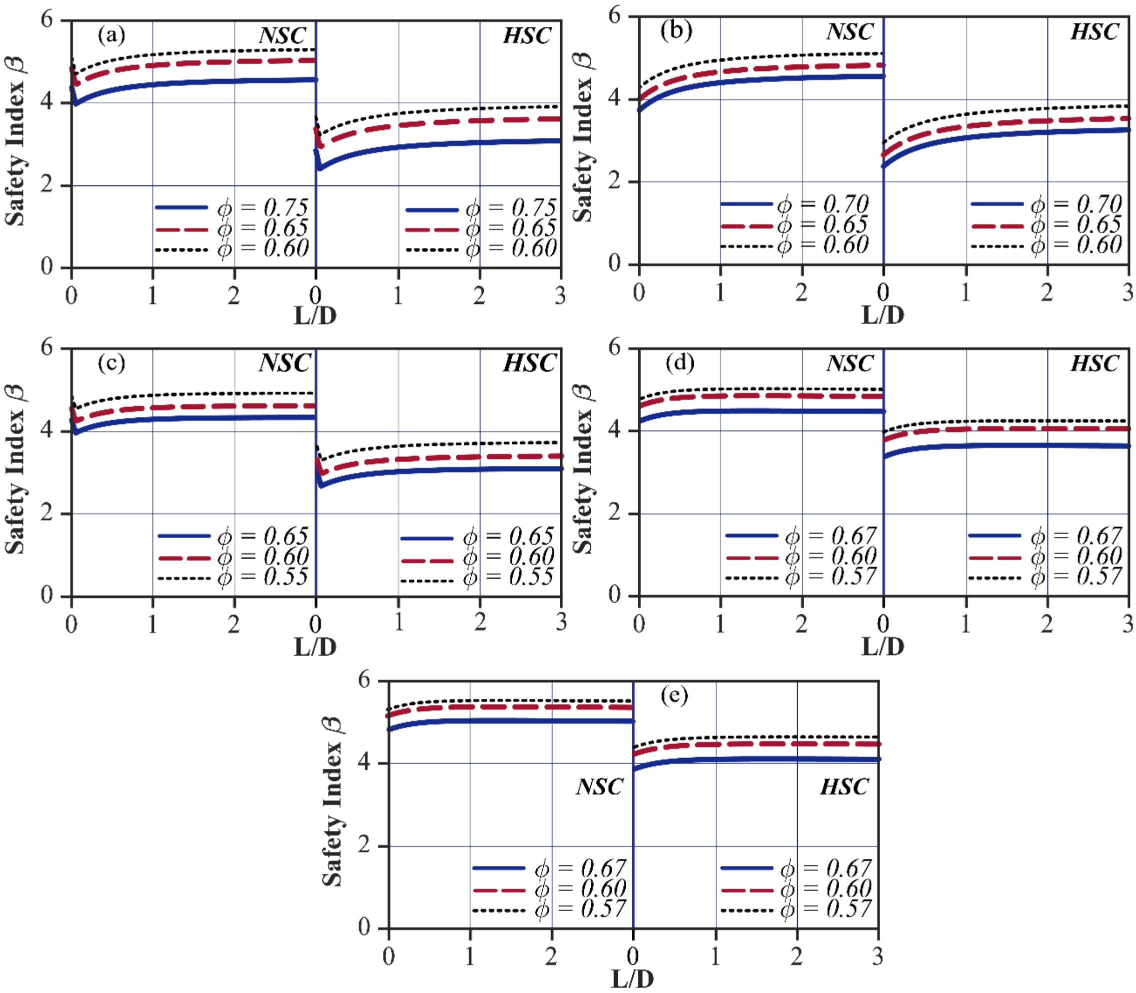

Figure 5 Variation of load ratio on safety indices with various ϕ values for five shear design methods: (a) ACI 318-14, (b) AASHTO-LRFD, (c) CSA A23.3, (d) Eurocode2 and (e) *fib* MC2010

concluded that the code-based STMs for deep beams made from NSC result in a greater degree of safety than those made from HSC. This seems to agree with the findings of Chetchotisak et al. [15] in the case of punching shear design methods. Additionally, as described by Zhang and Hsu [36], Kaufmann and Marti [37] and Zwicky and Vogel [38], the strength of a concrete strut decreases with increasing concrete compressive strength. Among concrete codes considered here, Eurocode2 and *fib* MC 2010 consider the strength of a concrete strut as an inverse relationship between the concrete compressive strength and the strength of concrete strut, while other codes do not. Therefore, the North American codes are insufficient in terms of safety when using HSC, while the European codes are more reasonable. This is consistent with the previous work of Ismail et al. [4].

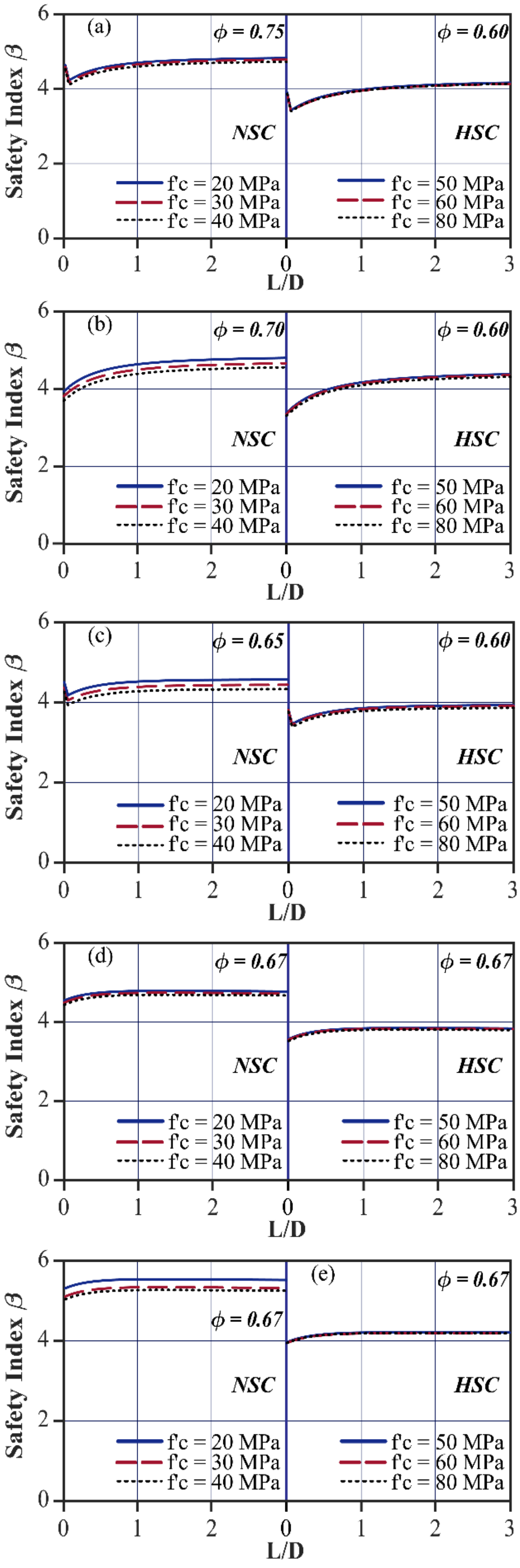

Figure 6 Variation of load ratio on safety indices with different concrete strengths for five shear design methods: (a) ACI 318-14, (b) AASHTO-LRFD, (c) CSA A23.3, (d) Eurocode2 and (e) *fib* MC2010

Table 4 Proposed strength reduction factors for various design methods

Code/Standard	NSC	HSC
ACI 318-14	0.75	0.60
AASHTO-LRFD	0.70	0.60
CSA A23.3	0.65	0.60
Eurocode2	0.67	0.67
fib MC2010	0.67	0.67

4.2 Sensitivity of the safety index to the design parameters

According to Szerszen and Nowak [19], the ratio of live-to-dead load (*L/D*) is considered to be the most important design parameter affecting the safety index, and is selected for this study. Figure 5 shows the effect of *L/D* ratio on the safety indices for an RC deep beam designed according to various codes. For this case, a concrete cylinder strength of 30 MPa was assumed. In general, the safety indexes are found to increase with the *L/D* ratio, and are relatively uniform when the *L/D* ratio is larger than about unity.

Additionally, due to variation in the material properties of concrete [32-33], the strength of concrete is another significant parameter influencing the safety index. The sensitivity of the safety indices with the change in concrete strength according to different codes was also investigated. This was based on the statistical data for construction materials proposed by Rakoczy and Nowak [32], as shown in Figure 6. These results suggest that the safety indices increase with a decrease in the compressive strength of the concrete. This is mainly attributed to the fact that concrete materials with a lower cylinder compressive strength provide a higher bias value in their strengths [32-33].

4.3 The new proposed understrength factors

STMs in accordance with international concrete codes have been used practically for the design of RC deep beams and other D-region members for several decades. However, the partial safety factors for the code-based STMs for shear design have not been developed using a probabilistic method as mentioned before. Taking into account the safety indices computed for various STMs for shear design of deep beams in Figures 5 and 6, and based on the target safety indices of 3.5 for all and 3.8 for Eurocode2 and *fib* MC 2010, new understrength factors are recommended in Table 4.

5. **Conclusions**

A safety assessment of RC deep beam shear design approaches in accordance with international concrete code provisions is presented in this paper. The uncertainties in load demand and member strength of RC deep beams arising from variability in material properties and fabrication tolerances, as well as error of design methods, have been considered. A database of 573 deep beam test results having concrete strength varying from 14 to 120 MPa and shear span-to-depth ratios from 0.27 to 2.5, was used in this study. A safety index was used as an indication of the safety level of the design approach. Based on the analysis results obtained in the present study, the principal findings are summarized as follows:

1. All code-based STMs considered here provide a sufficient level of safety for design of RC deep beams made from NSC. Alternatively, by applying an inverse relationship between concrete strength and the strength of diagonal struts, only designs

using Eurocode2 and *fib* MC 2010 satisfied the target level of safety in the case of both NSC and HSC.

2. From the usual ranges of NSC and HSC, the ratio of live-to-dead load varying from 0 to 3 and the target safety index of more than 3.5, the strength reduction factors are recommended as exhibited in Table 4.
3. The authors recommend that for the case of HSC, the strength reduction factors specified in the concrete code provisions such as ACI 318-14, AAHTO RLFD, and CSA A23.3 should be revised to 0.60.

6. Acknowledgements

The authors would like to gratefully acknowledge the support and funding of the Department of Civil Engineering, Khon Kaen University and Rajamangala University of Technology Isan, Khon Kaen Campus under Grant No. ENG 26/63.

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