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The U-line Assembly Line Balancing Problem*

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ABSTRACT

The traditional line or straight line assembly line balancing problem considers a production line in which stations are arranged consecutively in a line. A balance is determined by grouping tasks into stations while moving forward through a precedence diagram. However, as a consequence of introducing the just-in-time (JIT) production principle it has been recognized that arranging the stations in a U-line has several advantages over the traditional configuration. In this paper the U-line assembly line balancing problem is introduced. It is more complex than the straight line assembly line balancing problem because tasks can be assigned by moving forward, backward, or simultaneously in both directions through the precedence diagram. We also calculate simple problems to show that the U-line configuration frequently improves the line efficiency compared to traditional lines.

Keywords : U-line assembly line balancing problem (UALBP), straight line assembly line balancing problem, Just in time (JIT)

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Introduction

An assembly line consists of a sequence of *m* (work) stations through which the product units proceed. Each station performs a subset of the *n* operations (tasks) necessary for manufacturing the products. Due to the steady or intermittent movement of the line, each product unit remains at each station for a fixed time span, the cycle time *c*. In traditional assembly lines stations are consecutively arranged in a straight line. Each product unit proceeds along this line and visits each station once.

Assembly line balancing is the process of allocating a set of tasks to an ordered sequence of stations in such a way that some performance measures (e.g. cycle time, number of stations, line efficiency) are optimized subject to the precedence relations among the tasks. This problem is known as the simple assembly line balancing problem (SALBP) which can be stated as follows (e.g. Baybars,1986; Scholl and Klein,1999) ;

- A single product is manufactured in large quantities. Performing the tasks $j = 1,\ldots, n$ takes deterministic operation times t_j. The sum of all operation times is denoted by $\sum t$.
- The tasks are partially ordered by precedence relations as directed arcs. An arc (*i*,*j*) means that task *i* must be finished before task *j* can be started. Figure 1 shows an example of a precedence diagram with n = 11 tasks an operation time as node weights.
- Each task must be assigned to exactly one station. The set of task S_k assigned to station $k = 1,\ldots,m$ are called station loads; stations are numbered consecutively along the line.
- The total operation time of tasks assigned to a station *k*, called station time $t(S_k)$, must not exceed the cycle time :

$$t(S_k) = \sum_{j \in Sk} t_j \leq c \quad k = 1,\ldots, m \tag{1}$$

- The precedence relations must be observed. When a task *j* is assigned to a station *k*, each task *i* which precedes *j* in the precedence network must be assigned to one of stations $1,\ldots,k$.
- The objective consists of maximizing the line efficiency E which is defined as :

$$E = \sum t \,/\, (m \times c) \times 100\% \tag{2}$$

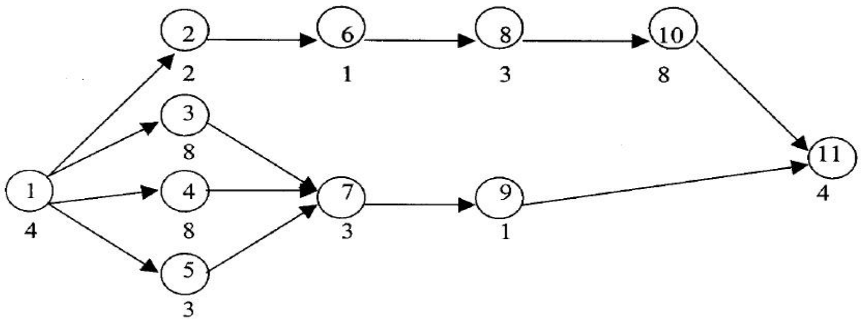
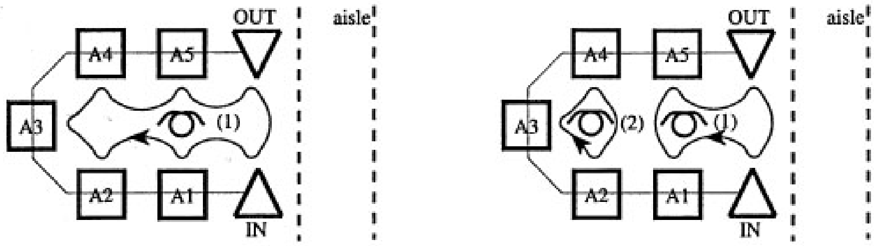
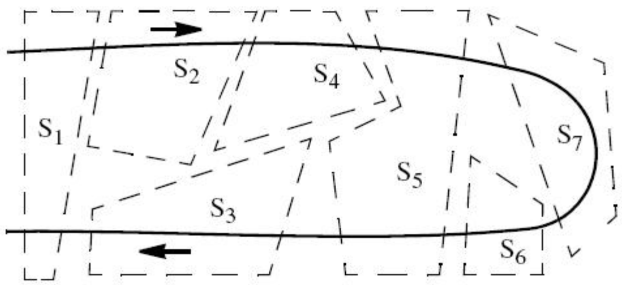


Figure 1. *Precedence diagram and the task times of the Jackson's problem (Jackson,1956)*

Largely due to pressures of just-in-time (JIT) manufacturing, many assembly lines are now being designed as U-shaped assembly lines (figure 2). When compared to straight lines they typically have, better balancing, improved visibility and communications, fewer work stations, more flexibility for adjustment, minimization of operation travel, and easier material handling (Chen, 2003).



a. 1-operator U-line (Miltenburg, 2001) *b. 2-operator U-line (Miltenburg, 2001)*



c. U-line configuration (Scholl and Klein,1999)

Figure 2. *U-shaped assembly line*

Literature review

Single model and mixed model straight line assembly line balancing have been thoroughly researched since the first published work in 1955. However, the first published work on U-shaped lines was not until 1994. In comparison to the well studied straight assembly line balancing problem, there are many areas in U-line assembly line balancing which require further research(Chen, 2003).

The first UALBP study in the literature was by Miltenburg and Wijngaard (1994), who developed a DP formulation for the single-model U-line to minimize the number of stations. The authors presented a Ranked Positional Weight Technique (RPWT)-based heuristic for larger size problems (111-tasks problems). Later, Miltenburge and Sparling (1995) developed three exact algorithms to solve the UALBP. The first was based on a reaching DP formulation, whereas the other two were breadth- and depth-first branch-and bound (B&B) algorithms.

Later, Urban(1998) developed an integer linear programming formulation to solve small- to medium-sized of UALBP with up to 45 tasks. Scholl and Klein (1999) developed a branch-and-bound procedure to solve, either optimally or suboptimally, problem with up to 297 tasks. Mixed-model U-lines were studied by Sparling and Miltenburg (1998). They developed a heuristic procedure for the U-line by which different products were assembled simultaneously. Their approximate solution algorithm that merges each model's precedence diagram into a single precedence diagram solved problems with up to 25 tasks. Miltenburg (1998) proposed a DP formulation for a U-line facility that consisted of numerous U-lines connected by multiline stations. Sparling (1998) developed heuristic solution procedures for a U-line facility consisting of individual U-lines operating at the same cycle time and connected with multiline stations. Ajenblit and Wainwright (1998) developed a genetic algorithm, and Erel et al. (2001) proposed simulated annealing as solution methodologies for larger U-line. In this paper, the U-line assembly line balancing problem is introduced and we also calculate simple problems to show that the U-line configuration frequently improves the line efficiency compared to traditional lines.

The problem of UALBP

The U-line assembly line balancing problem (UALBP) is an extension of simple assembly line balancing problem (SALBP) which is based on a U-shaped assembly line instead of a serial line. As in the case with SALBP, it can define three problem versions of UALBP (CF. Miltenburg and Wijngaard (1994)) as well as Scholl and Klein (1999)

• UALBP-1 : Given the cycle time (*c*), minimize the number of station (*m*)
• UALBP-2 : Given the number of stations (*m*), minimize the cycle time (*c*).
• UALBP-E :Maximize the line efficiency (*E)* for *c* and *m* being variable.

Since models for UALBP differ from those for SALBP only with respect to the precedence constraints. In SALBP **all (direct and indirect) predecessors** of a task *j* performed at a station *k* must be assigned to one of the stations *1,…,k.*

In UALBP, each task in principle can share a station with any of its predecessors or successors. However, **all predecessors or (and) all successors** of a task *j* performed at a station *k* must be assigned to one of the station *1,…,k.* In many cases, a higher efficiency is possible with UALBP. Note that increasing the line efficiency has the further positive effect of smoothing the levels of station utilization, i.e., the stations get more equally loaded.

The simple U-line assembly line balancing problem defined by Miltenburg and Wijngaard (1994) is given as follows : Miltenburg and Wijngaard's (1994) definition follows from that given by Gutjahr and Nemhauser (1964) for the traditional line balancing problem.

Given set of tasks $F = \{i \mid i = 1,2,\ldots, n\}$, a set of precedence constraints $P = \{(x,y) \mid$ task x must be completed before task $y\}$, a set of task times $T = \{t_i \mid i = 1,2,\ldots,n\}$, cycle time c and a number of workstation m, find a collection of subsets of F, $(S1, S2,\ldots,Sn)$ where $S_k = \{i \mid$ task i is done at a workstation $k\}$, that satisfy the following conditions:

$$\bigcup_{k=1}^{m} S_k = F \tag{3}$$

$$S_k \bigcap_{k \neq j} S_j = \varnothing \tag{4}$$

$$\sum_{i \in S_k} t_i \leq c, \quad k = 1,2,\ldots,n \tag{5}$$

For each task y ,
if $(x,y) \in P$, $x \in S_k$, $y \in S_j$, then $k \leq j$, for all x; or (6)
if $(y,z) \in P$, $y \in S_j$, $z \in S_i$, then $i \leq j$, for all z.

$$\left[mc - \sum_{k=1}^{m} \sum_{i \in S_k} t_i \right] \text{ is minimized.} \tag{7}$$

Condition 3 ensures that all tasks are assigned to a workstation. As a result of condition 4, each task is assigned only once. Condition 5 ensures that the work content of any workstation does not exceed the cycle time. Condition 6 ensures that the precedence constraints are not violated on the U-line. As a result of the objective function, the number of workstations will be minimized (Miltenburg and Wijngaard, 1994).

Illustration examples

Precedence diagrams, processing (or task) times and the calculation of 2 examples of U-line balancing problems were given in figure 3 and 4 respectively.

Ex. 1

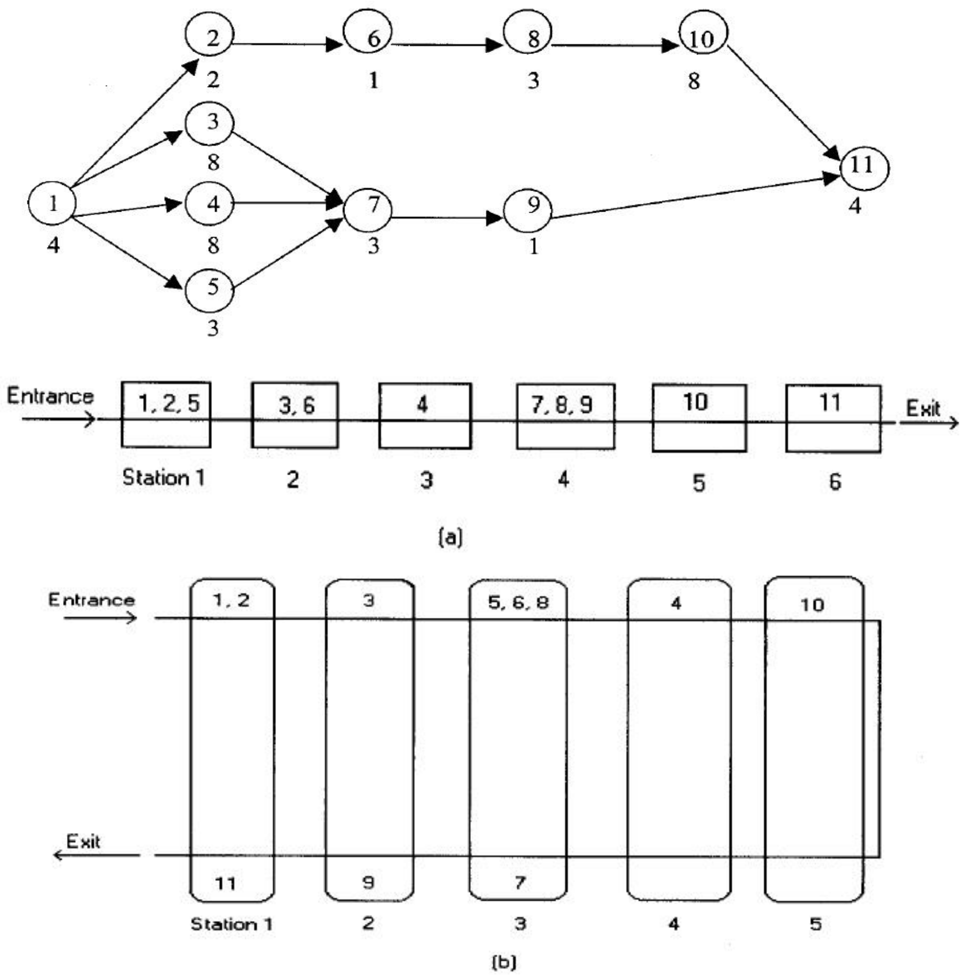


Figure 3. *Precedence diagram and the task times of the Jackson's problem (Jackson, 1956) Schematic view of (a) straight line and (b) U-line configurations*

Assuming a cycle time = 10 , total tasks time = 45

(a) Straight line m = 6 stations

$$E = \frac{\Sigma t}{m \times c} \times 100 = 45/6(10) \times 100 = 75\%$$

Balance delay = 100-E = 100-75 = 25%

(b) U-Line m = 5 stations

E = 45/5(10) x 100 = 90%

Balance delay = 100-90 = 10%

Ex. 2

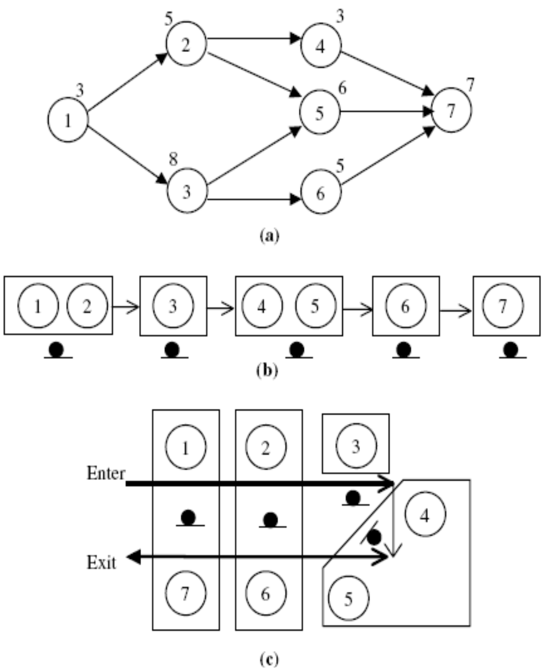


Figure 4. *Precedence diagram and the task times of example problem*
Schematic view of (a) precedence diagram (b) straight line and (c) U-line configurations
(Hadi Gokcen et al.,2005)

total tasks time = 37 , assume c = 10 ,

(a) Straight line		(b) U-Line	
	m = 5 stations		m = 4 stations
	E = 37/5(10)x100		E = 37/4(10) x 100
	= 74%		= 92.5%
Balance delay	= 100-E = 100-74 = 26%	Balance delay	= 100-92.5 = 7.5%

Consider the example problem of figure 4(a) and assume the cycle time to be c = 10. Figure 4(c) illustrates an optimal solution of UALBP-1 with four stations. As a convention, which will be maintained throughout the paper, stations are numbered consecutively from left to right. Let us examine station 2 which performs the tasks 2 and 6, i.e. S_2 = {2,6}. Task 2 is executed whenever a product unit crosses the station for the first time (from left to right) after its predecessor has been performed in station 1. When the product unit returns to station 2 (from right to left) all predecessors of task 6 have already been performed (in stations 1-4) and task 6 can be performed. Afterwards, the successor of task 6 (task 7) is executed in station 1.

In figure 4(b), illustrates the solution of SALBP-1 with five stations: S_1 = {1,2}, S_2 = {3}, S_3 = {4,5}, S_4 = {6}, S_5 = {7}. Due to $\sum t$ = 37, the line efficiency of the U-line is 92.5% whereas the line efficiency of the straight line is only 74%.

From the example problems calculation, it shows that the U-line configuration frequently improves the line efficiency and has fewer work stations compared to traditional lines.

Conclusion

Recently, U-line layouts have been utilized in many production lines in place of the traditional straight- line configuration due to the use of just-in-time principles. The shape of U-line improves visibility and allows the construction of stations containing tasks on both sides of the line. This arrangement, combined with cross-trained operators, provides greater flexibility in station construction than is available on a comparable straight production line. In this paper, the U-line assembly line balancing problem was introduced. From the problems calculation to show that the U-line configuration frequently improves the line efficiency compared to traditional lines. However, there are many areas in U-line assembly line balancing which require further research that is necessary to find more flexible solution approaches which provide a good compromise with respect to finding good feasible solutions early and saving enumeration effort.

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