Journal of Engineering and Digital Technology (JEDT)
Vol.13 No.1 January - June 2025

Effect of Cutout Eccentricity on the Elastic Stability of Square Perforated Plates

Ek-u Thammakornbunjut[1] Sombat Tamna[2] Pongkorn Meelapchotipong[3]
Jongsak Pakpia[4] Paiboon Limpitipanich[5*]

[1,3,4]AMM Research Laboratory, Faculty of Engineering, Thai-Nichi Institute of Technology, Bangkok, Thailand
[2]Department of Mechanical Engineering, Faculty of Engineering,
Rajamangala University of Technology Isan, Khon Kaen Campus, KhonKaen, Thailand
[5]Department of Mechanical Engineering, Faculty of Engineering, Burapha University, Chonburi, Thailand*

*Corresponding Author. E-mail address: paiboonl@eng.buu.ac.th

Received: 31 October 2024; Revised: 26 March 2025; Accepted: 25 April 2025
Published online: 27 June 2025

Abstract

This research investigates an elastic stability, in a form of buckling load, of a square plate with an eccentric square cutout by considering the effects of eccentricity and size of a cutout and loadings. Three size of square cutouts, three eccentric positions along x- and y-axes, and three type of loadings are considered. For boundary conditions, a simple support on all edges of the plates is prescribed. The problems are modeled and simulated by finite element program, Ansys. In most cases, the results show that larger holes decrease the stability of the plates, while loading the plate in compression along both axes causes the plate to be the most vulnerable. However, loading the plate in one axis and pulling it in the other results in the best stability or buckling capacity. It also shows that the eccentricity of the cutout in relation to the load affects the stability of the plate. In the case of uniaxial and biaxial compressive loads, the eccentric hole along the main compressive load increases plate stability. In contrast, the stability of the plates are decreased in the case of biaxial tension-compression load no matter direction of eccentricity along the loads. For the buckling mode, as expected, a higher mode, a higher buckling capacity can be achieved.

Keywords: Biaxial loading; Buckling; Eccentric cutout; Finite element method; Square plate

I. INTRODUCTION

Many structures in various areas of civil, marine, aerospace, mechanical, and automotive engineering have used plates in their design and construction. Many of those designs usually made an opening in the plate to reduce weight, pipe and wire passage, inspection, and maintenance purposes. Because of this perforation, the distribution of the membrane stress near the hole will be changed and a reduction in load-carrying capacity can then be expected. For practical design in which high safety and lowest cost are primary considerations, it is important to understand the behavior of perforated plates subjected to in-plane compressive forces. This kind of problem is buckling in which the plate loses stability of its equilibrium after the in-plane load reaches the critical value.

Numerous investigations into plate stability have been conducted and documented [1]–[3]. Stability criteria are usually associated with the load-carrying capacity of the structures. In the case of plate-type structures, this means the ability of a plate to withstand the compressive or shear loads that act parallel to its plane. Geometry of the plates are mainly influence to their stability or buckling loads. With a presence of a cutout or an opening, the problems are much more difficult to analyze in theoretical approach compared to experimental and numerical investigations. To author's knowledge, research on the buckling behavior of square or rectangular plates with cutouts dates back to 1947 [4]. Nemeth [5] and Shanmugam [6] have offered in-depth insights into the stability of perforated plates. Meanwhile, Narayanan and Chow [7], [8] and Paik [9], [10] have conducted significant studies on this topic, extending their research to the ultimate strength of square and rectangular plates.

Analyzing the buckling of perforated plates typically entails placing the holes at the plate's center and subjecting it to uniaxial, biaxial, or shear loads. Numerous experimental investigations and theoretical formulations have been employed for this purpose. However, handling biaxial loading conditions poses challenges in experimentation, and devising precise formulas or theoretical models becomes intricate. The accuracy of results often diminishes as the size of cutouts increases or when holes are misaligned. Consequently, researchers have increasingly relied on the computational methods such as finite element method to analyze this issue due to its widely acknowledged accuracy and precision.

Previous research has mainly focused on the effects of hole shapes, positions, and load characteristics [7]–[16]. Only a few have focused on eccentricity of a hole [17], [18]. However, the specific impact of hole eccentricity on the buckling load of plates under biaxial compression or biaxial tension-compression, especially with square holes, has not been clearly addressed. This research aims to assess how cutout eccentricity influences the buckling load of square plates with square cutouts under both uniaxial and biaxial in-plane loads, using the finite element analysis.

II. RESEARCH METHODOLOGY

A. Modeling of Perforated Square Plates

This study used the finite element software Ansys to compute the buckling loads of square plates with eccentric cutout under three uniform in-plane loads: uniaxial compressive load along the x-axis, biaxial compressive loads in both x- and y-axes, and biaxial compression and tension in the x- and y-axes, respectively. As shown in Figure 1, this research evaluated a square plate of 300×300 mm^2 with a modulus of elasticity of 211 GPa and a Poisson's ratio of 0.3. A square cutout with dimensions expressed as a ratio (d/b) of 0.1, 0.2, and 0.3 relative to the panel size were introduced. Regarding the eccentricity (e_x or e_y), they were defined as 15, 45, and 75 mm along x- and y-axes and were considered relative to the plate size (e_x/a or e_y/a).

Additionally, the boundary conditions were defined as simply supported on all edges.

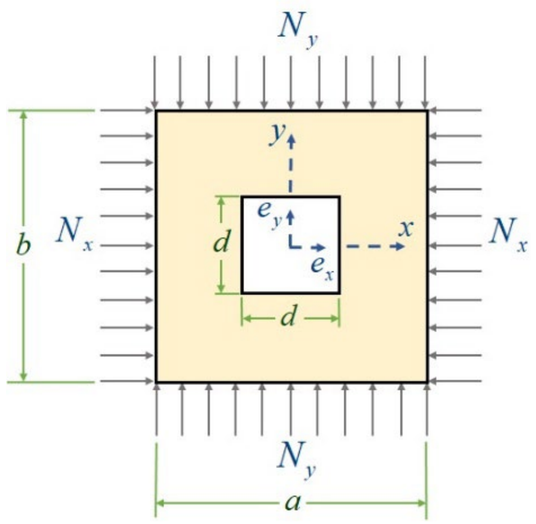


Figure 1: Plate and cutout geometries with load characteristic

B. Loading Conditions

The applied load consists of a uniformly distributed force acting along the edge of the plate, measured in kN/m and directed perpendicular to the edge. As shown in Figure 1, the buckling load is determined by the calculated force in the x-direction, which is always a compressive force. Meanwhile, the force in the y-direction will be zero, compression, or tension representing the uniaxial, compression-compression, and tension-compression cases, respectively. Note that the magnitude of the force on the y-axis will be equal to the one on the x-axis at the buckling state.

C. Numerical Analysis

In present work, a finite element program, Ansys, was employed for analysis. An element size of 0.01 m was specified at the distance from the hole and decreased to 0.002 m at the edge of the hole. The computations based on linear buckling assumption was performed using eigenvalue buckling analysis. A simply supported condition on all four edges was assigned.

The convergence of the solutions were tested by increasing the number of elements until the change in the calculated value is less than 1%. In each case, it was varied between 1369 and 2721, depending on the cutout size and the eccentricity. In addition, some case of plates without holes were examined by comparing with the experimental results to verify the accuracy of the program as shown in [15].

III. RESULTS AND DISCUSSION

At first, we consider the effects of size and eccentricity of the cutout under three loading conditions to the buckling loads. It is obviously, as shown in Figure 2 to 6, that the buckling loads were less as the hole size increased at any type of loads and eccentricity of the hole. Form Figure 2 and 3, when considering the effect of hole eccentricity in the case of uniaxial compression loading, it was found that the buckling load decreased slightly when the hole was slightly eccentric. However, when the hole was more eccentric and approached the edge of the plate, the buckling load increased in the case of hole eccentricity in the x-direction. While the eccentricity in the y-direction the buckling load did not increase significantly at hole sizes of d/b = 0.1 and d/b = 0.2, the buckling load decreased significantly at hole size of d/b = 0.3. This means that if the plate is necessary to make a perforation, especially in a square shape, a cutout position should be placed near the loading edge to remain buckling resistant. From this calculation result, further analysis should be conducted by increasing the hole size.

In Figure 4, in the case of a square plate subjected to biaxially compressive loads, the hole eccentricity in the x-direction and y-direction is the same. A larger hole eccentricity will result in a slightly larger buckling load of the plate.

In the case of biaxial tension-compression, as shown in Figure 5 and 6, the effect of hole eccentricity in either the x-direction or the y-direction has a similar trend, that is, the greater the eccentricity, the lower the buckling load, and the greater the decrease as the hole size increases. It can be said that in the case of biaxial tension-compression, the holes should not be offset from the center of the plate to provide the greatest resistance to buckling.

Apart from that, when considering the results of the three types of loading (see Figures 2–6), it can be seen that in the case of uniaxial loading, the buckling load will be in the range of 50–65 kN/m, in the case of biaxial compression-compression loading, the buckling load will be in the range of 25–35 kN/m, and in the case of biaxial tension-compression loading, the buckling load will be in the range of 70–140 kN/m. It can be said that if you want the perforated plate to resist buckling well, it should be designed to withstand tensile force in the direction perpendicular to the compression load. And if the structure of the perforated plate needs to withstand biaxial compression loads, special care should be taken when using it or a reinforcement may be added to make the panel stronger.

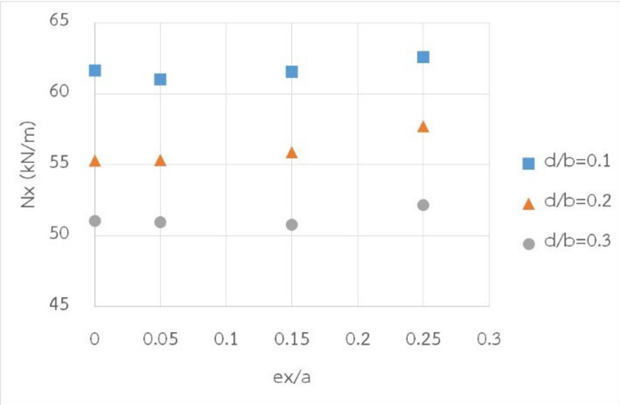


Figure 2: Buckling loads of plates with eccentric cutouts in x-direction subject to uniaxial loading

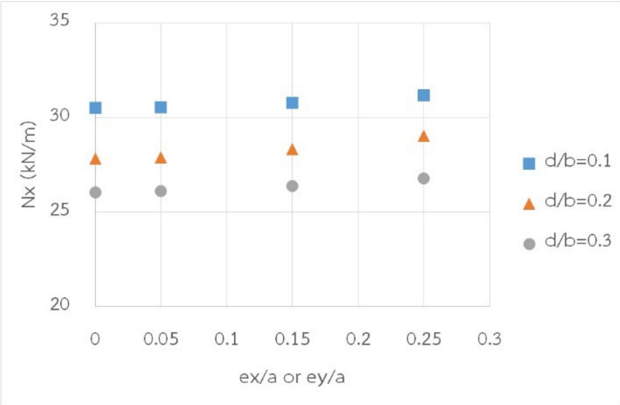


Figure 4: Buckling loads of plates with eccentric cutouts in x- or y-directions subject to biaxial compression-compression loading

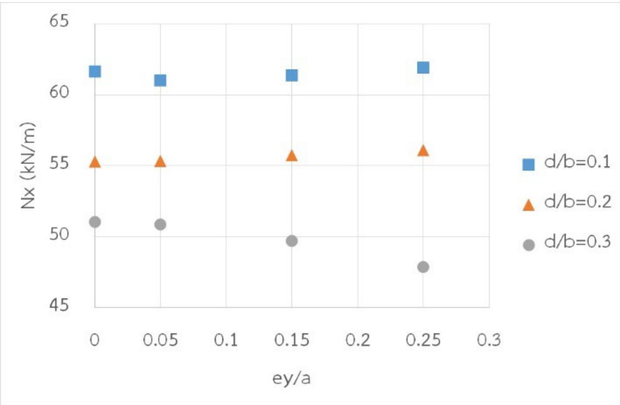


Figure 3: Buckling loads of plates with eccentric cutouts in y-direction subject to uniaxial loading

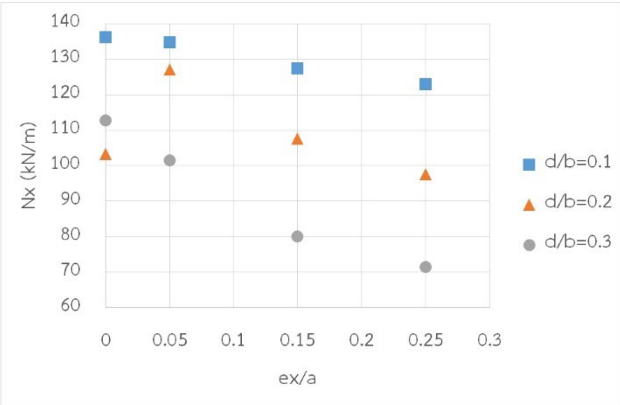


Figure 5: Buckling loads of plates with eccentric cutouts in x-directions subject to biaxial tension-compression loading

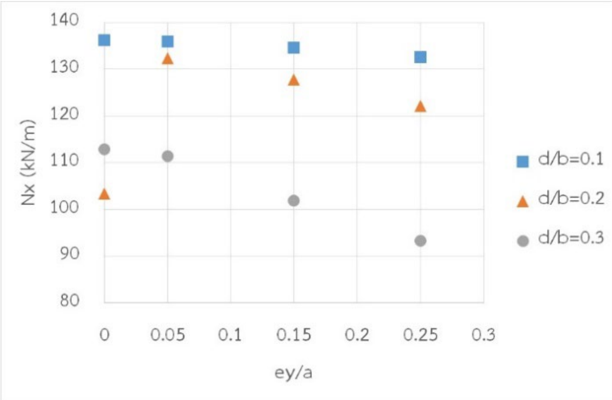


Figure 6: Buckling loads of plates with eccentric cutouts in y-directions subject to biaxial tension-compression loading

Unlike the buckling of columns, after the in-plane compressive force reaches the buckling load, the plate can still withstand higher loads, and bending behavior of the plate will change from mode 1 to higher modes in sequence. From Figures 7–12, which are the buckling loads of perforated plates at different modes receiving three types of loading, with the eccentricity of the holes in x- and y-directions, it can be seen that the buckling load increases along the mode, which tends to be consistent with the theory of non-perforated plates.

Figure 7: Buckling mode of plates with eccentric cutouts in x-direction at d/b = 0.1 subject to three loadings

Figure 8: Buckling mode of plates with eccentric cutouts in x-direction at d/b = 0.2 subject to three loadings

Figure 9: Buckling mode of plates with eccentric cutouts in x-direction at d/b = 0.3 subject to three loadings

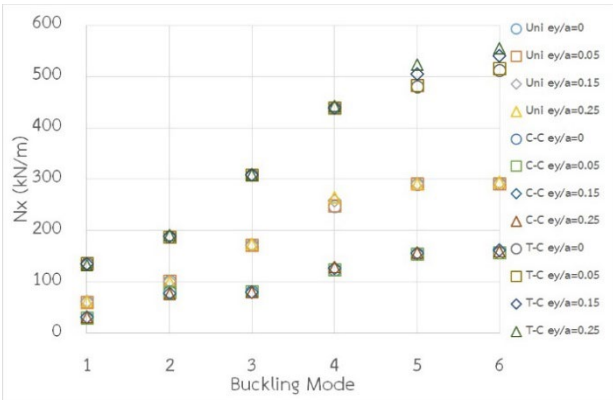


Figure 10: Buckling mode of plates with eccentric cutouts in y-direction at d/b = 0.1 subject to three loadings

Figure 11: Buckling mode of plates with eccentric cutouts in y-direction at d/b = 0.2 subject to three loadings

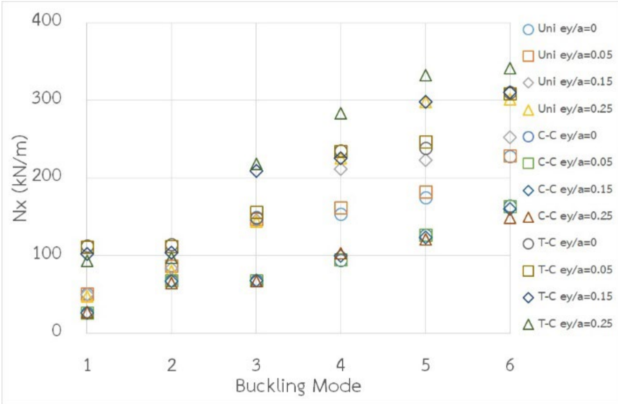
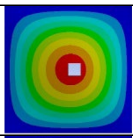
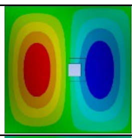
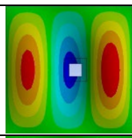
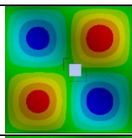
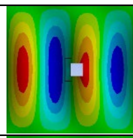
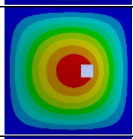
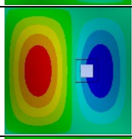
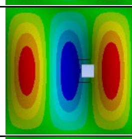
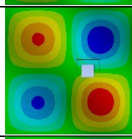
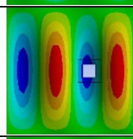
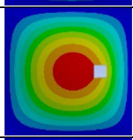
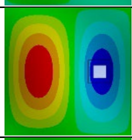
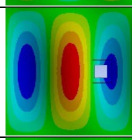
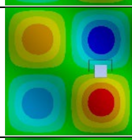
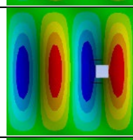
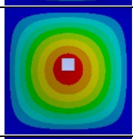
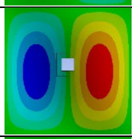
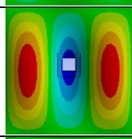
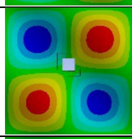
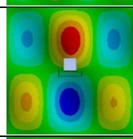
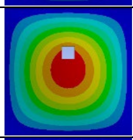
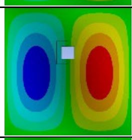
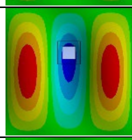
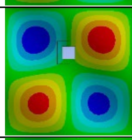
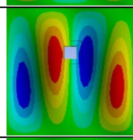
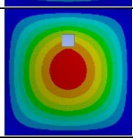
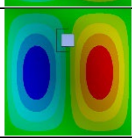
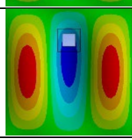
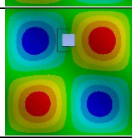
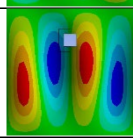


Figure 12: Buckling mode of plates with eccentric cutouts in y-direction at d/b = 0.3 subject to three loadings

Table 1: Buckling Modes of a Square Plate with an Eccentric Square Hole Subject to Uniaxial Load

Load Type	d/b	ex	ey	Mode Number				
				1	2	3	4	5
Uniaxial	0.1	15	0					
		45	0					
		75	0					
		0	15					
		0	45					
		0	75					

In addition, an out of plane deformations of 5 buckling modes of square plates with eccentricity at d/b = 0.1 subject to uniaxial load, biaxial compressive loads, and biaxial tension-compression loads are illustrated, just for example, in Table 1–3, respectively. In the case of biaxial tensile and compressive loads, the mode shapes in any cases look different from the other two load types. Due to the characteristics of deflection in each mode, which in some cases may be similar but in some cases are different, if a sensor is to be installed to detect the deflection of the perforated plate, the engineers should consider the appropriate location in order to be able to predict the value and mode of buckling correctly.

Table 2: Buckling Modes of a Square Plate with an Eccentric Square Hole Subject to Biaxial Compressive Load

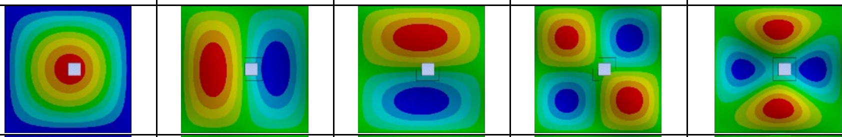
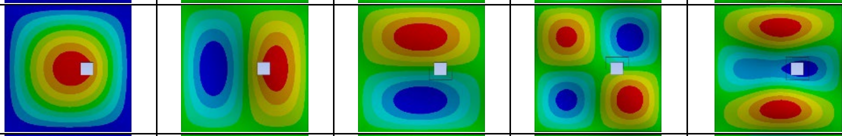
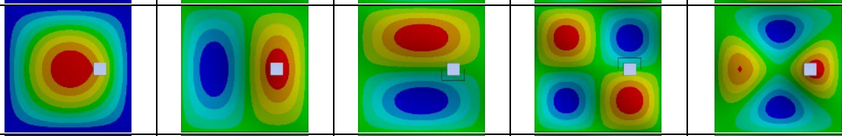
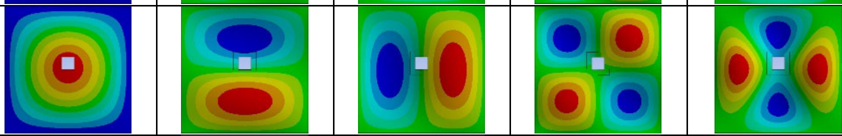
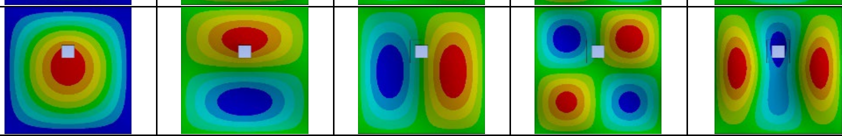
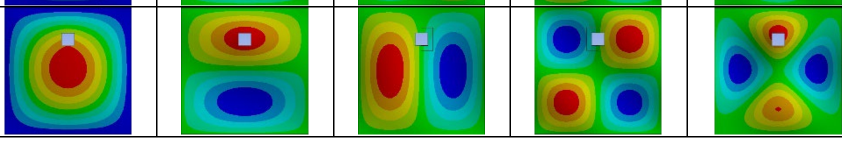
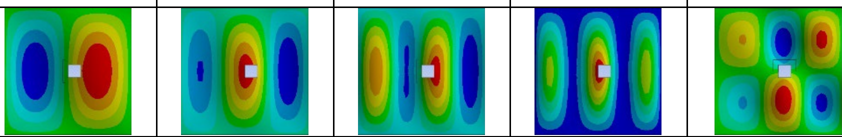
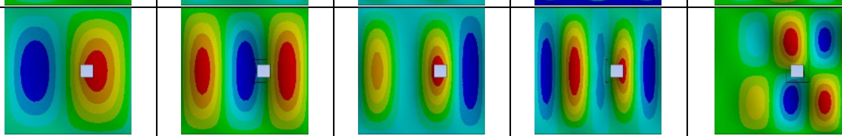
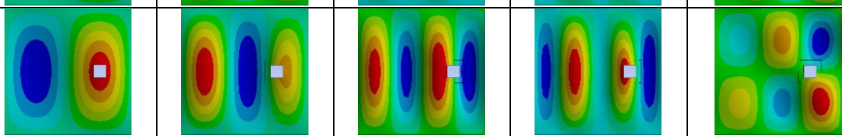
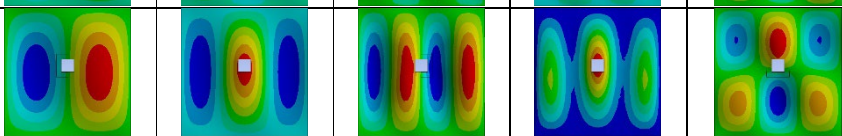
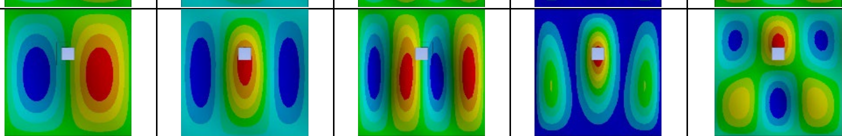
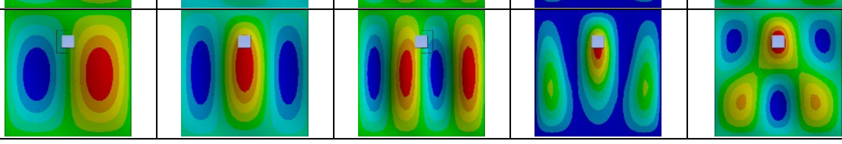
Load Type	d/b	ex	ey	Mode Number 1	2	3	4	5
Biaxial Compression-Compression	0.1	15	0					
		45	0					
		75	0					
		0	15					
		0	45					
		0	75					

Table 3: Buckling Modes of a Square Plate with an Eccentric Square Hole Subject to Tensile-Compressive Load

Load Type	d/b	ex	ey	Mode Number 1	2	3	4	5
Biaxial Tension-Compression	0.1	15	0					
		45	0					
		75	0					
		0	15					
		0	45					
		0	75					

IV. CONCLUSION

This work examines the buckling load of a square plate with an eccentric square cutout. By considering the effect of the eccentricity of the cutout in x- and y-axes, the size of the hole, and three different types of load, it can be concluded that the larger the hole size, the lower the buckling resistance of the plate. The load that compresses the plate on both axes will make the plate weakest. While applying one axis of tension and one axis of compression, the plate can resist buckling the best. In terms of eccentricity, it relates to the type of loading, that is, when the load is acting in a uniaxial direction and biaxial compression, eccentricity of the hole in a direction parallel to the load will increase the value of the buckling load. In other words, the plate has a higher buckling resistance. But when the plate is subjected to tensile and compressive loads perpendicular to each other the eccentricity of the hole in the direction parallel to the compression results in a reduction in the buckling resistance no matter which direction the eccentricity goes. In this work, the cutout size were limited to the d/b ratio of 0.3 which will be studied further. However, under this limitation, the cutout eccentricity should be placed in the direction of a main compressive load (x-axis), which will increase the buckling resistance in the case of uniaxial or compressive-compressive loads otherwise, the eccentricity is not recommended. In addition, buckling loads of the mode shapes up to the fifth mode are demonstrated, which show that a higher mode will be achieved with a higher buckling load. The mode shapes in the case of d/b = 0.1 are shown as examples for an opportunity to use plate's buckling mode shape as a sensor or an actuator in MEMs, mechatronics, and related works.

ACKNOWLEDGEMENT

This work was supported by the Faculty of Engineering, Burapha University, Thailand through research grant no. 29/2548 which is gratefully acknowledged.

REFERENCES

[1] P. S. Bulson, *The Stability of Flat Plates*. London, UK: Chatto & Windus, 1970.

[2] F. Bloom and D. Coffin, *Handbook of Thin Plate Buckling and Postbuckling*. New York, NY, USA: Chapman & Hall/CRC, 2001.

[3] R. M. Jones, *Buckling of Bars, Plates, and Shells*. Blacksburg, VA, USA: Bull Ridge Publishing, 2006.

[4] S. Levy, R. M. Woolley, and W. D. Kroll, "Instability of simply supported square plate with reinforced circular hole in edge compression," *J. Res. Nat. Bureau of Standards*, vol. 39, pp. 571–577, Dec. 1947.

[5] M. P. Nemeth, "Buckling and postbuckling behavior of laminated composite plates with a cutout," National Aeronautics and Space Administration, Washington, DC, USA, NASA Tech. Rep. 3587, 1996.

[6] N. E. Shanmugam, "Thin-walled structural elements containing openings," in *Proc. 3rd Int. Conf. Thin-Walled Struct.*, Cracow, Poland, Jun. 2001, pp. 37–51.

[7] R. Narayanan and F. Y. Chow, "Ultimate capacity of uniaxially compressed perforated plates," *Thin-Walled Struct.*, vol. 2, no. 3, pp. 241–264, 1984.

[8] F.-Y. Chow and R. Narayanan, "Buckling of plates containing openings," in *Proc. 7th Int. Specialty Conf. Cold-Formed Steel Struct.*, St. Louis, MO, USA, Nov. 1984, pp. 39–53.

[9] J. K. Paik, "Ultimate strength of perforated steel plates under edge shear loading," *Thin-Walled Struct*., vol. 45, no. 3, pp. 301–306, Mar. 2007.

[10] J. K. Paik, "Ultimate strength of steel plates with a single circular hole under axial compressive loading along short edges," *Ships and Offshore Struct.*, vol. 2, no. 4, pp. 355–360, 2007.

[11] M. Yoshiki, Y. Fujita, A. Kawamura, and H. Arai, "Instability of plates with holes (1st report)," (in Japanese), *J. Zosen Kiokai*, vol. 1967, no. 122, pp. 137–145, 1967.

[12] Y. Fujita, K. Yoshida, and H. Arai, "Instability of plates with holes (2nd Report)," (in Japanese), *J. Soc. Naval Architects Japan*, vol. 1969, no. 126, pp. 285–294, 1969.

[13] K. M. El-Sawy and A. S. Nazmy, "Effect of aspect ratio on the elastic buckling of uniaxially loaded plates with eccentric holes," *Thin-Walled Struct.*, vol. 39, no. 12, pp. 983–998, Dec. 2001.

[14] K. M. El-Sawy and M. I. Martini, "Elastic stability of bi-axially loaded rectangular plates with a single circular hole," Thin-Walled Struct., vol. 45, no. 1, pp. 122–133, Jan. 2007.

[15] E. Thammakornbunjut, P. Meelapchotipong, J. Pakpia, S. Tammapattaragul, and P. Limpitipanich, "Buckling of square plates with different central cutouts," *J. Eng. Digit. Technol. (JEDT)*, vol. 10, no. 2, pp. 13–20, 2022.

[16] A. Milazzo *et al*., "Buckling and post-buckling of variable stiffness plates with cutouts by a single-domain Ritz method," *Thin-Walled Struct.*, vol. 182, 2023, Art. no. 110282, doi: 10.1016/j.tws.2022.110282.

[17] J. Blesa Gracia and F. G. Rammerstorfer, "Increase in buckling loads of plates by introduction of cutouts," *Acta Mech.*, vol. 230, no. 8, pp. 2873–2889, 2019.

[18] K. S. Subash Chandra, T. Rajannac, and K. Venkata Rao, "A parametric study on the effect of elliptical cutouts for buckling behavior of composite plates under non-uniform edge loads," *Latin Amer. J. Solids Struct.*, vol. 17, no. 8, 2020, Art. no. e330.