An Improved Linear Combination of Two Estimators for Reducing the Mean Squared Error in a Sample Survey under Simple Random Sampling

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Received: 24 November 2021; Revised: 8 February 2022; Accepted: 9 March 2022
Published online: 27 June 2022

Abstract

The objective of this paper is to improve the efficiency of a linear combination of two estimators for estimating the population mean using auxiliary information in a sample survey. We also study some properties of the new estimator by using the concept of large-sample approximations and comparing them with some existing estimators through the numerical study. To achieve this, three data sets are used to support the performance of the new estimator. It has been shown that the new estimator is equivalent in terms of efficiency as compared to usual linear regression and it is better than other existing estimators under consideration in the terms of Mean Squared Error (MSE) and Percent Relative Efficiencies (PREs).

Keywords: Study variable, Auxiliary variable, Population mean, Sample survey, Estimator

I. INTRODUCTION

It is a well-known fact that several researchers use the information of auxiliary to enhance the efficiency of their estimators in estimating the population parameters. The main aim of studies is to find out more efficient estimators than recently proposed estimators and make inferences about the unknown population parameters such as population total, population mean, population proportion, or population variance. And one of the population parameters that are widely studied and used, is the population means. For this reason, several researchers have proposed ratio, product, exponential, and regression estimators taking the usefulness of the auxiliary information for estimating the population means in recent years.

Consider a finite population $Z=\{Z_1,Z_2,...,Z_N\}$ of size N distinct identifiable units. Let Y and X denote the study and auxiliary variables having values y_i and x_i, $i=1,2,...,N.$ Let X is correlated with Y and is used to estimate the unknown population mean $\bar{Y}$. When the mean of auxiliary variable X is available. Therefore, we can calculate the population mean $\bar{X}$. In this paper, we are interested in estimating the population mean $\bar{Y}$ of study variable Y under simple random sampling without replacement (SRSWOR), then we take a sample of size n without replacement from the population Z. We use the notation of $\bar{y}$ and $\bar{x}$ denote the sample means of bivariate (y,x), which are unbiased estimator of the population means of $\bar{Y}$ and $\bar{X}$. Therefore, the variance or MSE of $\bar{y}$ under SRSWOR is given by

$$Var(\bar{y})=MSE(\bar{y})=\lambda\bar{Y}^2C_y^2 \tag{1}$$

where $\lambda=(1-f)/n,\quad f=n/N,\quad \bar{Y}=\sum_{i=1}^{N}y_i/N,$

$$C_y=S_y/\bar{Y},\quad S_y^2=(N-1)^{-1}\sum_{i=1}^{N}(y_i-\bar{Y})^2.$$

In the situation when the correlation between study and auxiliary variables is highly positive and the regression line of study variable on an auxiliary variable is linear and nearly through or passes the origin, the ratio estimator is suitable. The first well-known ratio estimator proposed by Cochran [1] is defined as:

$$\hat{\bar{y}}_1=\bar{y}\left[\frac{\bar{X}}{\bar{x}}\right];\ \bar{x}\neq 0 \tag{2}$$

where $\bar{y}=\sum_{i=1}^{n}y_i/n,\quad \bar{X}=\sum_{i=1}^{N}x_i,\quad \bar{x}=\sum_{i=1}^{n}x_i/n.$

On the other hand, the product estimator is preferable in the situation of a highly negative correlation between study and auxiliary variables, while the regression line of its is linear. The initial product estimator defined by Robson [2] and later revisited by Murthy [3] is given by

$$\hat{\bar{y}}_2=\bar{y}\left[\frac{\bar{x}}{\bar{X}}\right];\ \bar{X}\neq 0 \tag{3}$$

However, in many situations, the regression does not pass or pass through a point away from the origin, and in such situations, the regression estimator is generally more efficient than the ratio and product estimators. Watson [4] first applied the highly correlated auxiliary variable and proposed the usual linear regression estimator for estimating the population means, which has a formula as below:

$$\hat{\bar{y}}_3=\bar{y}+\beta(\bar{X}-\bar{x}) \tag{4}$$

where β is the sample regression coefficient.

Later, Singh *et al.* [5] also extended the estimators of Cochran [1] Robson [2] and Murthy [3] by using the exponentiation method to propose an alternative ratio-cum-product type class of estimators for estimating the population mean. The estimator of Singh *et al.* [5] is given as follows:

$$\hat{\bar{y}}_4=\bar{y}\left[\frac{a\bar{X}+b}{a\bar{x}+b}\right]^{2g-1} \tag{5}$$

where $a(a\neq 0)$ b and g are real constants or the functions of known parameters of auxiliary variable such as standard deviation (S_x^2), coefficient of variation (C_x), correlation coefficient (ρ_{yx}), and so on.

To the first degree of approximation, the MSEs of $\hat{\bar{y}}_1$, $\hat{\bar{y}}_2$, $\hat{\bar{y}}_3$, and $\hat{\bar{y}}_4$ are respectively given by

$$MSE(\hat{\bar{y}}_1) = \lambda \bar{Y}^2 \left\lfloor C_y^2 + C_x^2(1-2C) \right\rfloor \quad (6)$$

$$MSE(\hat{\bar{y}}_2) = \lambda \bar{Y}^2 \left\lfloor C_y^2 - C_x^2(1-2C) \right\rfloor \quad (7)$$

$$MSE(\hat{\bar{y}}_3) = \lambda \bar{Y}^2 C_y^2 (1-\rho_{yx}^2) \quad (8)$$

$$MSE(\hat{\bar{y}}_4) = \lambda \bar{Y}^2 \left\lfloor C_y^2 + (2g-1)\theta C_x^2\{(2g-1)\theta - 2C\} \right\rfloor \quad (9)$$

where $C_x = S_x / \bar{X}$, $S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$, $K = \bar{X}/\bar{Y}$, $\rho_{yx} = S_{yx}/S_y S_x$, $C = \rho_{yx} C_y / C_x$, $\theta = a\bar{X}/(a\bar{X}+b)$, $S_{yx} = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})(y_i - \bar{Y})$.

More detailed discussion about the publication on the estimation of the population mean based on auxiliary information in SRSWOR scheme can be found in Khoshnevisan *et al*. [6], Singh and Agnihotri [7] Pandey and Dubey [8], Sisodia and Dwivedi [9], Upadhyaya and Singh [10], Singh and Tailor [11], Kadilar and Cingi [12], Yan and Tian [13], Abid *et al.* [14], Subzar *et al.* [15], Yadav *et al.* [16], Dansawad [17]–[18], Sabo *et al.* [19], Lurdjariyaporn and Dansawad [20]–[21], Ahmad *et al.* [22], Jerajuddin and Kishun [23], and so on.

The enthusiasm behind this paper is to propose a modified ratio-cum-product estimator of a population mean in the form of linear combination of two estimators by extending the estimators of Watson [4] and Singh *et al.* [5] under SRSWOR scheme when some information of auxiliary variables is known. It was also created to be compact and to be easy for calculating. We expect that the new estimator is considerably easier to use than recently other proposed estimators.

II. OBJECTIVES

1. The main objective of this study is to estimate the population mean in the form of linear combination of two estimators by extending the estimators of Watson [4] and Singh et al. [5] when some information of auxiliary variables is known under SRSWOR scheme.

2. This study also aims to evaluate some properties of the new estimator in the terms of MSE, and minimum MSE.

3. This study aims to perform a numerical study to assess the performance of the new estimator with the other existing estimators using Mean Squared Error (MSE) and Percent Relative Efficiencies (PREs).

III. METHODOLOGY

In this section, a modified ratio-cum-product estimator in the form of linear combination of two estimators for estimating the population mean can be defined by combining $\hat{\bar{y}}_3$ in Equation (4) with $\hat{\bar{y}}_4$ in Equation (5), Therefore the new estimator is given as:

$$\hat{\bar{y}}_5 = \left[\bar{y} + \beta(\bar{X} - \bar{x})\right]\left[\frac{a\bar{X}+b}{a\bar{x}+b}\right]^{2g-1}. \quad (10)$$

To obtain the MSE of the $\hat{\bar{y}}_5$ in Equation (10), we consider

$$\bar{y} = \bar{Y}(1+e_0), \text{ and } \bar{x} = \bar{X}(1+e_1) \quad (11)$$

then, we have $E(e_0) = E(e_1) = 0$, $E(e_0^2) = \lambda C_y^2$, $E(e_1^2) = \lambda C_x^2$, and $E(e_0 e_1) = \lambda C C_x^2$.

Now, rewriting Equation (10) in terms of e_0 and e_1 from (11), then Equation (10) turns into the Equation (12) as follows:

$$\hat{\bar{y}}_5 = \left[\bar{Y}(1+e_0) - \beta \bar{X} e_1\right]\left[1+\theta e_1\right]^{-(2g-1)}. \quad (12)$$

Expanding the right-hand side of Equation (12) to the first degree of approximation and retaining the terms of e_0 and e_1, up to the second power, we get

$$\hat{\bar{y}}_5 - \bar{Y} \approx \bar{Y}\left[\begin{array}{l} 1-(2g-1)\theta e_1 + g(2g-1)\theta^2 e_1^2 \\ +e_0 - (2g-1)\theta e_0 e_1 \end{array}\right] - \beta \bar{X}\left[e_1 - (2g-1)\theta^2 e_1^2\right] - \bar{Y}. \quad (13)$$

Squaring both sides of Equation (13) and then taking expectation, therefore the MSE of the $\hat{\bar{y}}_5$ up to the first degree of approximation, as follows

$$MSE(\hat{\bar{y}}_5) = \lambda \left\lfloor \bar{Y}^2 C_y^2 + A C_x^2 (A - 2\bar{Y}C) \right\rfloor \quad (14)$$

where $A = \varphi K + \bar{Y}(2g-1)\theta$, $\varphi = S_{yx}/S_x^2$.

To obtain the value of g that minimizes the MSE, we take partial derivative of Equation (14) with respect to g and equate it to zero as follows:

$$\frac{\partial MSE(\hat{\bar{y}}_5)}{\partial g} = \lambda \left| \begin{matrix} (\varphi K + \bar{Y}(2g-1)\theta)C_x^2 \\ ((\varphi K + \bar{Y}(2g-1)\theta) - 2\bar{Y}C) \end{matrix} \right| = 0$$

$$\Rightarrow g = \frac{\bar{Y}C - \varphi K + \bar{Y}\theta}{2\bar{Y}\theta}. \quad (15)$$

Replacing the value of g in Equation (14) with the optimal value of g in Equation (15), we get the minimum MSE of $\hat{\bar{y}}_5$ as

$$\min.MSE(\hat{\bar{y}}_5) = \lambda \bar{Y}^2 C_y^2 (1-\rho_{yx}^2) = MSE(\hat{\bar{y}}_3). \quad (16)$$

From Equation (16), we can see that it is equal to the MSE of usual linear regression based on the auxiliary variable under SRSWOR scheme.

To substitute the values for $a, b,$ and g in the given Equation (5) and (10), a few members of the estimators $\hat{\bar{y}}_4$ and $\hat{\bar{y}}_5$ can be displayed as follows in Table 1.

Table 1: A few members of estimators $\hat{\bar{y}}_4$ and $\hat{\bar{y}}_5$

A few members of $\hat{\bar{y}}_4$		Real constants or the functions		
		a	b	g
$\hat{\bar{y}}_{4(1)} = \bar{y}\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$	Sisodia and Dwivedi [9]	1	C_x	1
$\hat{\bar{y}}_{4(2)} = \bar{y}\left[\frac{\bar{X}}{\bar{x}}\right]^{2C_x-1}$	Singh *et al.* [5]	1	0	C_x
$\hat{\bar{y}}_{4(3)} = \bar{y}\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]^{2C_x-1}$	Singh *et al.* [5]	1	C_x	C_x
$\hat{\bar{y}}_{4(4)} = \bar{y}\left[\frac{\bar{X}+\rho_{yx}}{\bar{x}+\rho_{yx}}\right]$	Singh and Tailor [11]	1	ρ_{yx}	1
$\hat{\bar{y}}_{4(5)} = \bar{y}\left[\frac{\bar{X}}{\bar{x}}\right]^{2\rho_{yx}-1}$	Singh *et al.* [5]	1	0	ρ_{yx}
$\hat{\bar{y}}_{4(6)} = \bar{y}\left[\frac{\bar{X}+\rho_{yx}}{\bar{x}+\rho_{yx}}\right]^{2\rho_{yx}-1}$	Singh *et al.* [5]	1	ρ_{yx}	ρ_{yx}
A few members of $\hat{\bar{y}}_5$		Real constants or the functions		
		a	b	g
$\hat{\bar{y}}_{5(1)} = \left[\bar{y}+\beta(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]$	Kadilar and Cingi [12]	1	C_x	1
$\hat{\bar{y}}_{5(2)} = \left[\bar{y}+\beta(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}}{\bar{x}}\right]^{2C_x-1}$	Singh *et al.* [5]	1	0	C_x
$\hat{\bar{y}}_{5(3)} = \left[\bar{y}+\beta(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+C_x}{\bar{x}+C_x}\right]^{2C_x-1}$	Singh *et al.* [5]	1	C_x	C_x
$\hat{\bar{y}}_{5(4)} = \left[\bar{y}+\beta(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+\rho_{yx}}{\bar{x}+\rho_{yx}}\right]$	Kadilar and Cingi [12]	1	ρ_{yx}	1
$\hat{\bar{y}}_{5(5)} = \left[\bar{y}+\beta(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}}{\bar{x}}\right]^{2\rho_{yx}-1}$	Singh *et al.* [5]	1	0	ρ_{yx}
$\hat{\bar{y}}_{5(6)} = \left[\bar{y}+\beta(\bar{X}-\bar{x})\right]\left[\frac{\bar{X}+\rho_{yx}}{\bar{x}+\rho_{yx}}\right]^{2\rho_{yx}-1}$	Singh *et al.* [5]	1	ρ_{yx}	ρ_{yx}

IV. NUMERICAL STUDY

In this section, we examine the efficiencies of the proposed estimator $\hat{\bar{y}}_5$ with the other existing estimators under three natural population data sets. The descriptions of the parameters and constants under this study are described as follows in Table 2.

Table 2: The population data sets

Population	N	n	$\bar{Y}$	$\bar{X}$	C_y	C_x	ρ_{yx}
I: Murthy [3] y: Fixed capital x: Output	80	20	11.264	51.826	0.750	0.354	0.941
II: Cochran [24] y: The number of persons per block x: The numbers of rooms per block	10	4	101.100	58.800	0.145	0.128	0.652
III Yadav *et al*. [16] y: Yield of peppermint oil x: Cultivation area	150	40	33.460	4.200	0.760	0.730	0.910

In comparing the efficiency of the proposed estimator over other existing estimators in this present study, we have used the MSE and percent relative efficiencies (PREs) as criteria for comparison with respect to unbiased the estimator $\bar{y}$ which details are presented in the following Table 3.

Table 3: MSE and PREs of all existing estimators

Estimator	Population					
	I		II		III	
	MSE	PRE	MSE	PRE	MSE	PRE
$\bar{y}$	2.6763	100.0000	10.4574	100.0000	11.8555	100.0000
$\hat{\bar{y}}_1$	0.8952	298.9715	6.6624	156.9611	2.0682	573.2206
$\hat{\bar{y}}_2$	4.4575	60.0413	14.2523	73.3730	21.6428	54.7781
$\hat{\bar{y}}_3$	0.3065	873.2175	6.0119	173.9445	2.0380	581.7336
A few members of $\hat{\bar{y}}_4$						
$\hat{\bar{y}}_{4(1)}$	0.9032	296.3017	6.6565	157.1010	2.1377	554.6029
$\hat{\bar{y}}_{4(2)}$	3.4214	78.2240	24.3060	43.0238	4.6364	255.7075
$\hat{\bar{y}}_{4(3)}$	3.4160	78.3476	24.2826	43.0654	5.4134	219.0050
$\hat{\bar{y}}_{4(4)}$	0.9165	292.0167	6.6326	157.6658	2.2102	536.4042
$\hat{\bar{y}}_{4(5)}$	1.0433	256.5254	7.5059	139.3226	2.2155	535.1198
$\hat{\bar{y}}_{4(6)}$	1.0643	251.4643	7.5198	139.0648	2.8557	415.1525
A few members of $\hat{\bar{y}}_5$						
$\hat{\bar{y}}_{5(1)}$	0.8991	297.6633	6.6617	156.9787	2.1291	556.8415
$\hat{\bar{y}}_{5(2)}$	3.4119	78.4406	24.2784	43.0727	4.5917	258.1951
$\hat{\bar{y}}_{5(3)}$	3.4065	78.5647	24.2550	43.1143	5.3624	221.0853
$\hat{\bar{y}}_{5(4)}$	0.9123	293.3538	6.6377	157.5450	2.1988	539.1745

Table 3: MSE and PREs of all existing estimators (cont.)

Estimator	Population					
	I		II		III	
	MSE	PRE	MSE	PRE	MSE	PRE
A few members of $\hat{\bar{y}}_5$						
$\hat{\bar{y}}_{5(5)}$	1.0387	257.6595	7.4980	139.4688	2.2040	537.9200
$\hat{\bar{y}}_{5(6)}$	1.0596	252.5695	7.5119	139.2111	2.8307	418.8151

It appears from Table 3 that:

(i) The estimator $\hat{\bar{y}}_3$ (usual linear regression) has the smallest MSE value and the largest PRE value among all estimators considered in the same population groups. And from Equation (16), we can see that the proposed estimator $\hat{\bar{y}}_5$ (at its optimal value) has MSE values equal to the estimator $\hat{\bar{y}}_3$. Therefore, the estimators $\hat{\bar{y}}_3$ and $\hat{\bar{y}}_5$ (at its optimal value) perform the best among the mentioned estimators as it gives the smallest MSE value and the largest PRE value.

(ii) Moreover, when comparing the performance between all members of $\hat{\bar{y}}_4$ and $\hat{\bar{y}}_5$ by pairwise comparison, we found that all members of $\hat{\bar{y}}_5$ are more efficient than all members of $\hat{\bar{y}}_4$. Because they gave smaller MSE and bigger PRE in each population groups.

(iii) Besides, the estimators $\hat{\bar{y}}_3$ and $\hat{\bar{y}}_5$ (at its optimal value), the estimator $\hat{\bar{y}}_1$ is also an appropriate choice among the estimators in this population group.

V. RESULTS AND CONCLUSION

When considering the Equation (16), it is observed that the MSE of the proposed estimator in the minimum case has the same expression as the MSE of usual linear regression estimator proposed by Watson [4], which is known to be more efficient than the ratio and product estimators. The numerical results present in Table 3 in the situation of positive correlation between the study and auxiliary variables show that the MSE of the estimators $\hat{\bar{y}}_3$ (usual linear regression) and $\hat{\bar{y}}_5$ (at its optimal value) are consistently less than the other existing estimators under consideration for all three population. While, the value of PRE of the estimators $\hat{\bar{y}}_3$ and $\hat{\bar{y}}_5$ (at its optimal value) are persistently higher than other ones and all ratio estimators ($\hat{\bar{y}}_1$, $\hat{\bar{y}}_4$, $\hat{\bar{y}}_5$) are consistently better in the terms of MSE and PRE than product estimator ($\hat{\bar{y}}_2$). Therefore, besides the estimator $\hat{\bar{y}}_3$, we recommend using the proposed estimator $\hat{\bar{y}}_5$ as an alternative to the usual linear regression estimator for estimating the population mean. Because it has the same precision as the usual linear regression estimator and routinely outperforms the other existing estimators in terms of MSE and PRE under consideration.

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