Buckling of Square Plates with Different Central Cutouts

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Abstract

This research aims to study the effects of cutout shape and size with different load combinations on buckling of perforated square plates. Loadings are divided into three types which are a uniaxial compressive force, biaxial compressive forces, and biaxial tensile and compressive forces. These loads are in-plane and uniformly distributed along the edges of the plates. In the case of biaxial loading, these forces are perpendicular to each other and to the edges of the plates. The models are analyzed as square plates with simple supports on all four edges. The finite element method is employed to determine the buckling loads in the direction of *x*-axis. From numerical results, increasing cutout size will reduce the buckling load in all cases. In addition, considering the buckling load of the plate subjected to uniaxial load as a reference case, it is found that the plates subjected to biaxial compression have the lowest buckling loads. The buckling load is also reduced as the ratio of compressive forces per unit length in the *y*- and *x*-axis is increased. In contrast, the plates subjected to biaxial tension and compression will have greater buckling loads. The above results are the same for both circular and square holes. However, when compared in terms of strength to weight ratio, plates with small cutouts produce slightly different results for both shapes. But for the larger cutouts, the square cutout will significantly provide better strength to weight ratio than a circular cutout at the same diameter to width ratio.

***Keywords*:** Biaxial load, Buckling, Cutout, Finite element method, Plate

I. INTRODUCTION

In the past, thin plate structures are widely used in various engineering structures such as ships, planes, and bridges. In particular, it is designed to support in-plane loads because the thin plate has better buckling resistance behavior than columns. That is when the panel is subjected to in-plane load until it buckles it can be handled to even higher loads by switched from the lowest mode to a higher mode. In contrast, the column which supports the axial force will suddenly fail after the force reached the first buckling load. Details of the basic theory and experiments of buckling can be found in the work of Timoshenko and Gere [1]. In addition, various cases of plate buckling can be studied from the work of Bulson [2].

In general, plates often have perforation for weight reduction, maintenance, and pipe access [3]. In the past, many research have been conducted on the buckling of perforated plates to ensure proper design and implementation. In the beginning, theoretical analysis was performed with experimental verifications [4]–[12]. Later, after a numerical technique namely finite element method was developed and has been improved to be a powerful tool for structural analysis it was widely used to solve this kind of problem. Complex loadings and boundary conditions have been solved and reported with high accuracy [13]–[22].

Most of the past research has studied the effects of cutout shapes, i.e., circle, square, and rectangle and its position subjected to uniaxial or biaxial loads. Most of them have not compared the buckling capacity between circular and square holes on plates that are subjected to biaxial loads especially in the combination of tension and compression. Therefore, this research aims to compare the difference between these two cutout shapes on the buckling load of the square plate. The applied forces are uniaxial, biaxial compression, and biaxial tension-compression loadings. The finite element method is carried out in the analyses by the ANSYS program. Incidentally, the shape of the plate, cutouts, and loads in this study are shown in Figure 1.

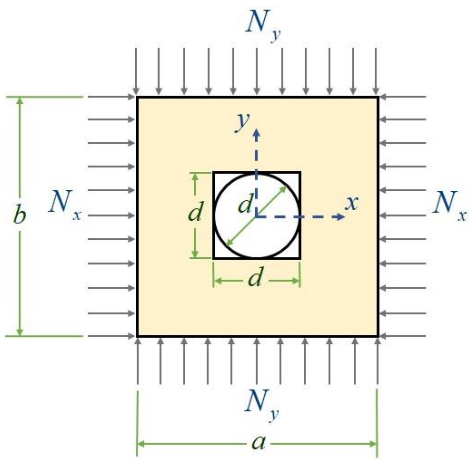


Figure 1: Geometry of plate, cutouts, and loads

II. RESEARCH METHODOLOGY

A. Geometric Modeling

The material of a plate is structural steel with Young's modulus of 200 GPa and a Poisson ratio of 0.3. Its geometry is square with a width of 300 mm and 2 mm thickness. We define the size of the hole by a ratio of the hole diameter and hole width to the plate width for the case of circular and square holes, respectively. The effect of cutout size is studied by varying this ratio as 0.1, 0.2, 0.3, 0.4, and 0.5. The plate support conditions are simple support on all four edges. The middle plane of the plate lies on the *x*-*y* plane [19].

B. Loading Conditions

The applied load is prescribed as uniformly in-plane forces acting on the edge of the plate in the *x*- and *y*-directions as shown in figure 1. However, by the constant thickness of the plate, it is convenient to use it as force per unit length and will simply call the force or load in the *x*- or *y*-axes (N_x and N_y) instead. There are three loading conditions which are uniaxial compression, biaxial compression, and biaxial tension and compression.

We will consider the compressive force in the *x*-axis as the main axis in determining the critical buckling load. In addition, the stress ratio of the *y*-axis to the x-axis (or the ratio of N_y to N_x) is defined as R = 0.5, 1.0, 1.5, and 2.0 to examine its effect on the different cutouts. The tensile force is positive while the compressive force is negative. Therefore, the case of biaxial tension and compression this ratio will be negative. Table I shows all of the cases to be analyzed including the non-cutout plate cases. For uniaxial loading, the stress ratio will be zero. Both plates with circular and square holes will be numerically analyzed with the same loading conditions.

C. Numerical Analysis

In this research, numerical results were carried out using the ANSYS program which uses the finite element method (FEM) as a calculation engine. Briefly, the principle of this method is to divide the problem geometry into subdomains, called elements. They are represented by rectangular or triangular elements for two-dimensional problems or hexagonal or tetrahedron elements for three dimensions whose mathematical equations govern their behavior consistent with the problem characteristics. Each element has nodes and an interpolation function to approximate the field variables within the element.

Table 1: Cutout and loadings to be analyzed

Cutout Shape	d/b	R = σ_y/σ_x (= Ny/Nx ; uniform thickness)				
		Uniaxial Loading	Biaxial Loading			
Circle and Square	0	0.0	Compression-Compression			
	0.1		0.5	1.0	1.5	2.0
	0.2		or			
	0.3					
	0.4		Tension-Compression			
	0.5		-0.5	-1.0	-1.5	-2.0

The equations of each element are then assembled into a system of equations in the form

$$[K]\{u\}=\{F\} \tag{1}$$

where $[K]$ is a stiffness matrix, $\{u\}$ is a nodal vector, and $\{F\}$ is a nodal force vector.

Buckling of plates is a bifurcation problem, usually call bifurcation buckling. It refers to an instability failure of structures. At the bifurcation point on the load-deformation curve, the deformations begin to grow in a new pattern which is quite different from the pre-buckling pattern. The buckling problem will be formulated as an eigenvalue problem. The stiffness matrix in Eq. (1) will consist of a linear or small deformation stiffness matrix K_0 at the pre-buckling state and a stress stiffness matrix K_σ which relates to the stress level of the plate. Therefore, Eq. (1) can be written as

$$[K_0+\lambda K_\sigma]\{u\}=\{F\} \tag{2}$$

where λ is an eigenvalue which will be calculated as a load multiplier. At the buckling point, the determinant of the stiffness matrix must equal to zero as

$$|K_0+\lambda K_\sigma|=0 \tag{3}$$

After solving the Eq. (3), the eigenvalues as a critical load multiplier will be provided and the corresponding buckling loads can then be determined from

$$P_{cr_i}=\lambda_i P \tag{4}$$

where P_{cr_i} is the buckling load of mode *i*, λ_i is the eigenvalue of mode *i*, and P is the initial applied load. In the case of the ANSYS program, the calculated results, based on linearly elastic assumption, will be eigenvalue or load multiplier. If the initial applied load is less than the buckling, load the load multiplier will be greater than 1, but oppositely, it will be less than 1. As shown in Eq. (4), by multiplying the initial applied load by the load multiplier, the buckling load could be obtained at each mode.

III. MODELING AND VERIFICATIONS

A. Finite Element Modeling

Modeling and analyzing problems with the finite element method by the ANSYS program can be divided into six steps as shown in Figure 2. The procedure starts with drawing the geometry of the problem and setting all dimensions for easy adjustment. Then prescribes its material properties and creates a proper mesh with convergence tests. In this step, type and size of the element should be carefully selected to fit the problem. They will affect the amount of the element and processing time. The next step is to provide the edge conditions and the applied loads. The analysis section for the buckling problem consists of two parts: the static analysis and the eigenvalue analysis, which must choose the required results to demonstrate i.e., stresses, deflections, and buckling load multipliers. In addition, as a post-processing step, the user can select other results to be carried out for further analysis.

This research uses SHELL181 element, a four-node element with six degrees of freedom at each node, in the meshing process. The program semi-automatically generates mesh in which smaller meshes are only created around the periphery of the cutout. Consequently, the number of elements is not constant in each case due to the size of the hole.

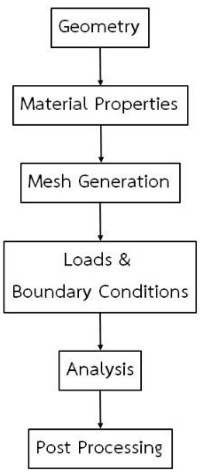


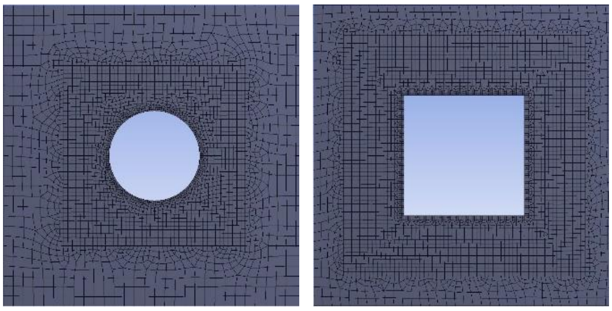
Figure 2: Problem modeling and analysis in FE program

They range approximately between 2,000 and 5,500 elements. Examples of mesh generations for circular and square holes with a relatively small element around the hole are shown in Figure 3.

Applying loads and boundary conditions are the same step in ANSYS program. In present study, the plate model lies on the x-y plane and the main load will apply on the x-axis. The applied load in the y-axis will be zero, compression, and tension for three loading conditions. To apply a simple support condition for all four edges, a zero displacement in z-direction is set on all four edges and two points at the corners on the edge of the plate parallel to the x-direction are restrained to translate only in the x-direction while the point between these two is set to move only in the y-direction.

B. Verification of the Numerical Results

To verify the correctness of the results, we have compared the buckling loads from the program with the available theoretical results [23], the case of non-perforated plates. All three loading conditions have been considered. Table II shows the comparison results of the buckling loads between ANSYS and theory which is obviously seen that the discrepancy is very low. As a result, the calculation of the buckling load of the perforated plates from the program should be accurate enough. In addition, some buckling modes are examined by comparing to the experiments as shown, as an example, in Figure 4. The experiments had been done as a senior project in 2007 [24]. Rectangular plates with some aspect ratio subjected to uniaxial loading had been tested. Shadow-Moire method had been used to demonstrate buckling modes. However due to the imperfection of the grips at the edge of the plates the out of plane displacement did not looks symmetry around the cutout in case of square plates, as shown in the Figure 4.



(a) (b)

Figure 3: Example of mesh generations for (a) circular cutout with d/b = 0.3 and (b) square cutout with d/b = 0.4

Table 2: Verification of the results from ANSYS

Loadings	R	Nx,cr (kN/m)		
		Theory	ANSYS	% Discrepancy
Uniaxial	0	64.271	64.272	0.00167
Biaxial Compression-Compression	0.5	42.847	42.848	0.00167
	1.0	32.135	32.136	0.00167
	1.5	25.708	25.709	0.00245
	2.0	21.424	21.424	0.00167
Biaxial Tension-Compression	-0.5	114.770	115.160	0.34023
	-1.0	133.898	134.280	0.28547
	-1.5	160.677	161.020	0.21327
	-2.0	200.847	201.060	0.10623

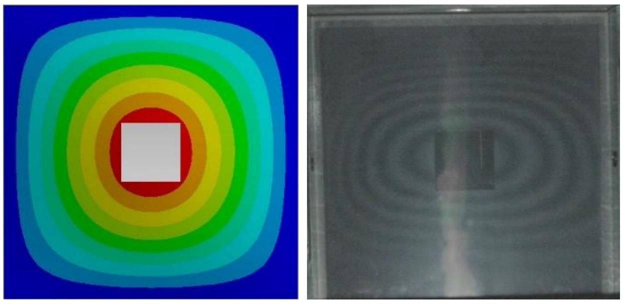


Figure 4: Buckling mode of a square plate with a square cutout of d/b = 0.2 subjected to uniaxial load

IV. RESULTS AND DISCUSSION

The calculation results will be in the form of force per unit length which is N/mm. In this research, we have changed them to an accustomed unit kN/m. In addition, it can also be converted to stress by multiply with 1000/t which results in the unit of MPa.

The characteristic considered in this analysis is the ability to resist buckling of the plates. Plates with higher buckling resistance will have higher buckling loads. The comparison between hole size, hole shape, and loadings are examined from buckling load values. The numerical results consist of the difference of loadings, i.e., uniaxial loading, biaxial compression, and biaxial tension and compression as shown in Figure 5–7, respectively. All of the results are compared the critical buckling loads in the *x*-axis by increasing hole size for both circular and square holes. In the case of uniaxial loading, the strength to volume ratio (S/V ratio) will also be considered here. Generally, this kind of analysis has used strength to weight ratio (S/W ratio) but this work did not vary materials properties, especially the density, so the S/V ratio could be carried out. However, if the S/W ratio is required we can multiply the S/V ratio by material density and gravitational acceleration.

A. Effect of Hole Size

From the calculation results, as shown in Figure 5-7, the hole size is considered in the form of hole diameter to plate width ratio. As the hole size increases, the buckling load decreases in a similar trend for the first two loading conditions, Figure 5 and 6. It seems reasonable because making a cutout at the center of the plate causes the plate to lose its material texture for force resistance which means that the buckling strength of plates will inevitably decrease. In the initial phase, d/b = 0 to d/b = 0.3, the buckling load decreases faster than in the case of d/b = 0.4 to 0.5. As for the last loading conditions, the buckling load at d/b = 0.1 is slightly higher than the case of d/b = 0. However, for the case of d/b = 0.2 to 0.5, the buckling load tends to decrease as expected. This is because the applied load is tension and compression and are perpendicular to each other by keeping the load ratio, R, constant. It means that the tensile force in the y-direction in the case of d/b = 0.1 is greater than the case of d/b = 0 therefore its buckling load is greater.

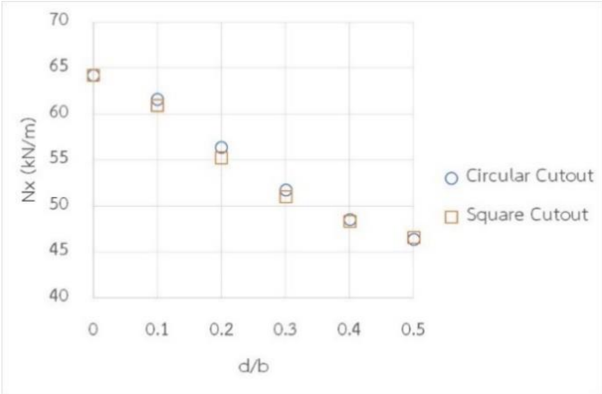


Figure 5: Buckling load of a plate with a central cutout subjected to uniaxial load

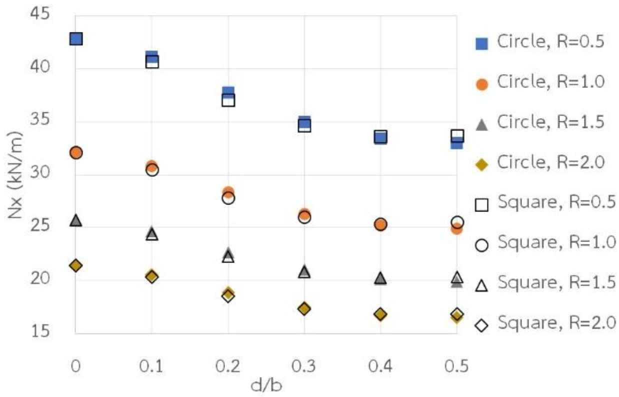


Figure 7: Buckling load of a plate with a central cutout subjected to biaxial compressive load

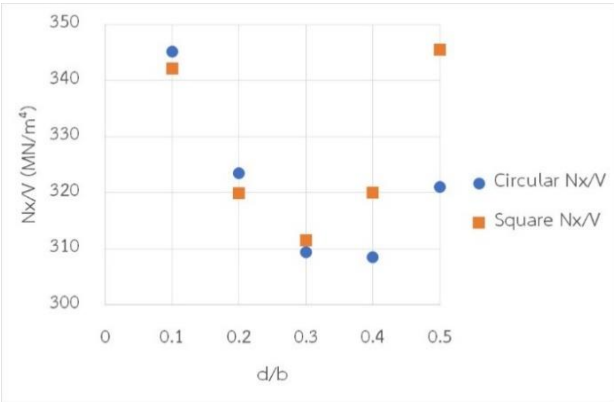


Figure 6: Buckling strength to volume ratio of a plate with a central cutout subjected to uniaxial load

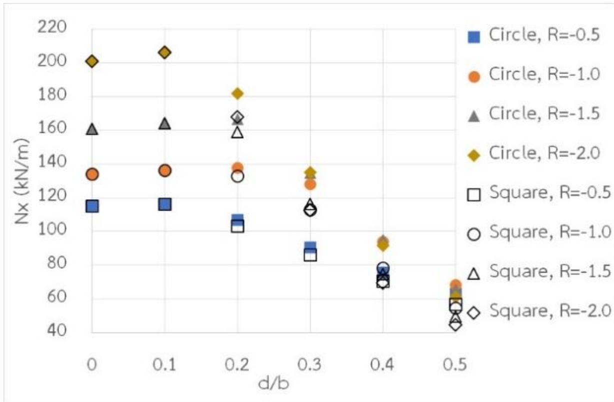


Figure 8: Buckling load of a plate with a central cutout subjected to biaxial tensile and compressive loads

B. Effect of Loadings

Comparing the results of three loading conditions, as shown in Figure 5–8, it can be seen that the biaxial compression case gives the critical buckling load less than that of the uniaxial loading case at the same cutout size. This is because the lateral compression in the y-axis has also weakened the plate in resisting the in-plane compressive force. It should be taken as a precaution in the design of plate-like structures where they may be subjected to biaxial compressive forces. In addition, the smaller the load ratio R, the higher the buckling load. It can be implied that by applying the lateral load the buckling load resistance of the plate will decrease as shown in Figure 7. The same results are achieved for both circular and square holes.

Considering the case of plates subjected to tensile and compressive forces, as shown in Figure 8, it is found that both circular and square cutouts cause higher buckling loads than that in the case of uniaxial and biaxial compression loadings for d/b = 0.1 to 0.4 while they result in close value for d/b = 0.5. It shows that applying the laterally tensile force to the plates will allow the plates to withstand more compressive force before the critical point is reached. However, when considering the load ratio R, it is found that the higher the load ratio, the higher the buckling load. In addition, in the case of a plate without a hole or with a small hole (d/b = 0.1), the buckling load at R = 2 is much greater than in the other cases. But as the hole size increased to the range of d/b = 0.3 - 0.5, the buckling loads with higher load ratios are significantly reduced to a close value with the lower load ratio. It can then be made a suggestion that for a large hole, i.e., d/b more than 0.3, it is not necessary to apply the lateral tensile load more than R = 1. Because the buckling strength that the plates achieved may not be worth it as the cost has much more increased.

C. Effect of the Cutout Shape

From Figure 5–8, when comparing circular holes and square holes with the same size of diameter and side, respectively, it is found that the plate with a square hole has a smaller area which, consequently, means that its volume and weight are also less than of the circular hole. But the buckling load is very close to each other. Accordingly, considering in terms of weight reduction to buckling strength reduction the plate with a square hole is better. It is evident from Figure 6, if the force-to-volume ratio is considered, that a small hole, e.g. d/b = 0.1 - 0.3, this ratio is quite similar. But when the holes are large enough as d/b = 0.4 or 0.5, the plate with a square hole has significantly better strength per volume.

V. CONCLUSION

This research examines the critical buckling load of square plates with circular and square cutouts by considering the effects of three loadings, hole size, load ratio, and the cutout shape. The problem is numerically analyzed by the finite element method. It can be concluded that the perforation reduces the strength of the plate against buckling. It will decrease as the hole size increases. It is the same for both circular and square holes.

In the case of loading types, if we introduce the case of the plate subjected to uniaxial loading as a benchmark, the biaxial compressive load will cause the reduction of the buckling strength of the perforated plate drastically. In practice, the use of plates in structural applications should avoid this type of load if the buckling strength is highly concerned. In contrast, when the lateral tensile force could be provided, the plate would be able to withstand much more buckling load. However, this advantage will fade as the hole grows larger.

Finally, if the hole is not large, both cutouts (circle and square) will not differ greatly in weight reduction and buckling strength. But if the cutout is large enough, a square hole has more benefit than a circular one in the view of strength to weight ratio.

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