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The Design and Implement of Chaotic Jerk Oscillator with Application to Secure Communication Systems Based on Chaotic Masking

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Abstract— **Chaotic signals have recently been of much interest as a new promising carrier frequency or an unpredictable masking signal for applications in secure wireless communication systems. Existing chaotic circuits have extensively been constructed based on threedimensional ordinary differential equations. It has recently been reported that jerk chaotic systems can generate chaotic signals through a single dynamical equation, leading to an algebraically simple mathematical model as well as a cost-effective circuit implementation. This paper presents a very simple autonomous RC chaotic jerk oscillator with nine electronic components. The nonlinearity required for chaos is implemented through the use of a well-known diode equation. Basic dynamical properties are described including equilibria, eigenvalues of Jacobian matrix, chaotic attractors, time-domain waveforms, power spectrum, and bifurcations. Potential application of such a simple autonomous RC chaotic jerk oscillator is presented in message-masking and synchronization for secure digital communications. The results show that the chaotically masked message is fully synchronized at the receiver through the use of very simple circuit. Consequently, the proposed new paradigm on secure communication schemes offers not only a simple mathematical system, but also very cost-effective circuit and system implementations.**

Keywords-component; **Chaotic Jerk Oscillator, Secure Communications, Chaotic Masking**

I. INTRODUCTION

In 1963, Edward Lorenz encountered sensitive dependence of initial conditions of an atmospheric convection model, leading to the discovery of the Lorenz system with seventerms in three-dimensional ordinary differential equations (ODEs) and two quadratic nonlinearities. In 1976, Rössler proposed another chaotic system with seven terms in ODEs and a single quadratic nonlinearity, which is algebraically simpler than the Lorenz system. A single folded-band attractor of Rössler system is also topologically simpler than a twoscroll Lorenz attractor. The Rössler system has therefore been utilized as a basic system in searching for new chaotic systems and employed in various applications such as in secure communications or in control systems. The well-known Chua's circuit [1, 2] is a chaotic oscillator based on three firstorder ordinary differential equations (ODEs). Several chaotic oscillators such as [2], [3], [4], [5], [6], [7] are alternatively based on a single third-order ODE of a jerk form. The term 'jerk' comes from the fact that successive time derivatives of displacement are velocity, acceleration, and jerk. Most chaotic jerk oscillators have employed nonlinearity only in the x term of the jerk function. The simplest dissipative chaotic flow has, however, employed a quadratic nonlinearity in the term of the jerk function. Recently, [5] have shown minimal jerk flow of (1) with nonlinearity in the $\dot{X}$ term of the form

$$\dddot{x} + \ddot{x} + x = -\alpha e^{\dot{x}} \tag{1}$$

in which chaos occurs for $\alpha = 0.27$. Such a nonlinear function is of particular interest as it resembles diode charactreristics. Sprott [6] has subsequently implemented (1) in the form

$$\dddot{x} + \ddot{x} + x = -10^{-9}\left\{e^{\left(\frac{\dot{x}}{0.026}\right)} - 1\right\} \tag{2}$$

The oscillator (2), however, requires large counts of 14 electronic components including 4 op-amps. Although the oscillator (1) has been implemented by a minimal chaotic jerk equation of (1), the required number of electronic components does not seem to be minimal. It is natural to wonder in the opposite direction whether or not a slightly more complicated chaotic jerk equation may greatly reduce the large counts of electronic components for the chaotic oscillator.

This paper presents a very simple autonomous RC chaotic jerk oscillator with nine electronic components. The nonlinearity required for chaos is implemented through the use of a well-known diode equation. Basic dynamical properties are described including equilibria, eigenvalues of Jacobian matrix, chaotic attractors, time-domain waveforms, power spectrum, and bifurcations. Potential application of such a simple autonomous RC chaotic jerk oscillator is presented in message-masking for secure Communications.

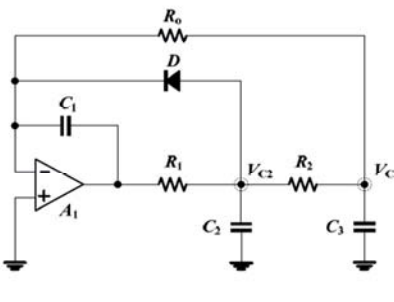


Figure 1. RC-Base chaotic oscillator

The results show that the chaotically masked message is fully synchronized at the receiver through the use of very simple circuit. Consequently, the proposed new paradigm on secure communication schemes offers not only a simple mathematical system, but also very cost-effective circuit and system implementations.

Circuit Realizations

In an attempt to reduce the number of electronic components, the minimal form (1) may be modified with a slightly more complicated jerk function and a simple diode equation expressed as

$$I_D = I_S\left\{exp\left(\frac{V_D}{nV_T}\right) - 1\right\} \quad (3)$$

in the following form

$$\dddot{x} = J\,(\ddot{x}, \dot{x}, x) + I_D R \quad (4)$$

where the voltage drop across the diode is $V_D = (\dot{x} + 2x)$, I_S is the reverse saturation current of diode, V_T is the thermal voltage at room temperature, n is the non-ideality factor of diode, R is a parameter and $J\,(\ddot{x},\,\dot{x},\,x)$ is a jerk function. Fig.1 illustrates an electronic circuit realization of the autonomous RC chaotic jerk oscillator, consisting of an amplifier, three resistors, three capacitors and a single diode. These components implement an integrator and a second-order RC passive filter in a feedback loop with a smaller nonlinear feedback loop containing a diode. Applying nodal analysis,

$$\begin{bmatrix}\dot{V}_{C3}\\ \dot{V}_{C2}\\ \dot{V}_{C1}\end{bmatrix} = \begin{bmatrix}-2/\tau_0 & 1/\tau_0 & 0\\ 1/\tau_0 & -2/\tau_0 & 1/\tau_0\\ -1/\tau_1 & 0 & 0\end{bmatrix}\begin{bmatrix}V_{C3}\\ V_{C2}\\ V_{C1}\end{bmatrix} + \begin{bmatrix}0\\ -I_D/C_2\\ -I_D/C_1\end{bmatrix} \quad (5)$$

Equation (5) can be expressed in terms of a normalized dynamical representation through the use of dimensionless variables and parameters as follows:

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{bmatrix} = \begin{bmatrix}-2 & 1 & 0\\ 1 & -2 & 1\\ -1/A & 0 & 0\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix} + \begin{bmatrix}0\\ -BI_D\\ -CI_D\end{bmatrix} \quad (6)$$

$$\begin{bmatrix}\dot{x} & x & A\\ \dot{y} & y & B\\ \dot{z} & z & C\end{bmatrix} = \begin{bmatrix}dX/d\tau & V_{C3} & \tau_0/\tau_1\\ dY/d\tau & V_{C2} & (\tau_0/\tau_2)/nV_T\\ dZ/d\tau & V_{C1} & (\tau_0/\tau_1)/nV_T\end{bmatrix} \quad (7)$$

where the time constants $\tau_0 = C_2R = C_3R, \tau_1 = C_1R$ and $\tau = t/\tau_0$ It is seen from (7) that $X = (\dot{X} + 2X)$ and consequently the diode equation I_D can be expressed as I_D=I_S\{exp[$V_{C2}/(nV_T)$]-1\} = I_S\{exp[Y]-1\}, resulting in

$$I_s\{exp[\dot{X} + 2X] - 1\} = i_D - I_s \quad (8)$$

Where $I_s exp(Y) = I_s exp(\dot{X} + 2X)$ Alternatively, the dynamical representation in (6) can also be written in a jerk representation as

$$\dddot{x} = -\ddot{X}\,(4 + Bi_D) - \dot{X}(3 + 2Bi_D) + AX - CI_D \quad (9)$$

It is obvious that a jerk model in (11) is described in the form shown in (4) as $\dddot{x} = J(\ddot{x}, \dot{x}, x) + I_D R$. In addition, the proposed circuit shown in Fig.1 has been designed with components R = 1 kΩ, C_1 = 0.1 μF, C_2 = C_3 = 10 μF, R_O is a potentiometer. The Diode model is 1N4148 where I_S=14.11x10^{-9}, n=1.984, and V_T=25.85x10^{-3}. The counting number of electronic components is only 8 and is therefore reduced by 43% compared to that of 14 counts in [6]. In particular, only a single op-amp is necessary for circuit implementation.

II. DYNAMICAL PROPERTY ANALYSIS AND NUMERICAL SIMULATIONS

Dynamic properties were mathematically analyzed using nonlinear theorems and numerically investigated using MATLAB. The initial condition was set at (0.1, 0, 0), which iles in the basic of attractor. The chaotic behaviors were simulated using the Fourth-order Runge-Kutta method with time step size of 5x10^{-6}. The system (7) is invariance under the transform $(x, y, z) \rightarrow (-x, -y, z)$, i.e. is symmetric around the z-axis and remains confined to the positive half-space with respect to the z state variable. The divergence of flow of the dynamic system is described as

$$\nabla . V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} = -\frac{4}{\tau_0} = -4 \times 10^3 \quad (10)$$

Therefore, the chaotic system (7) is a dissipative system with an exponential rate of contraction as

$$\frac{dV}{dt} = exp\left(-\frac{4}{\tau_0}\right) = exp(-4 \times 10^3) \quad (11)$$

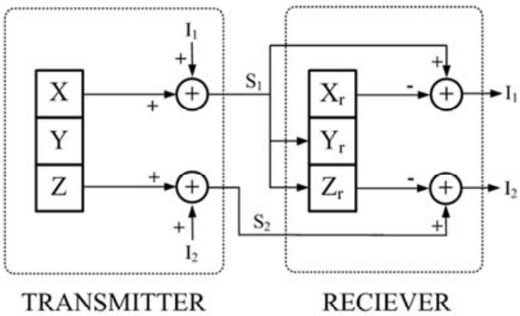


Figure2. Principle scheme of a general secure communication system with masking technique

In other words, a volume element $V0$ becomes smaller by the flow in time t into a volume element $V0exp$ (-t). Each volume containing the trajectories shrinks to zero as $t\rightarrow\infty$ at an exponential rate of -4x10^3. System orbits are ultimately confined into a specific limit set of zero volume, and the system asymptotic motion settles onto an attractor of the system. In order to investigate the linear stability, the system (5) was linearized, and a single fixed point was found at (0, 0, 0). The Jacobian matrix of partial derivatives is defined as

$$J = \begin{bmatrix}-2/\tau_0 & 1/\tau_0 & 0\\ 1/\tau_0 & -(2/\tau_0) - \beta_{C_2} & 1/\tau_0\\ -1/\tau_1 & -\beta_{C_1} & 0\end{bmatrix} \quad (12)$$

where the parameters $\beta_{C1} = \{I_s exp(V_{C2}/nV_T)\}/nV_T C_1$ and $\beta_{C2} = \{I_s exp(V_{C2}/nV_T)\}/nV_T C_2$. The resulting eigenvalues of the Jacobian matrix in (14) evaluated at the fixed point are consequently equal to

$$\lambda_1 = -6.1531$$
$$\lambda_2 = 1.0765 + 3.8852i$$
$$\lambda_3 = 1.0765 - 3.8852i \qquad (13)$$

It is evident from (13) that the fixed point is a saddle focus node as the eigenvalue λ_1 is negative real value and $\lambda_{2,3}$ are a pair of complex conjugate eigenvalues with positive real parts.

III. SECURE COMMUNICATION SYSTEMS BASE ON CHAOTIC MASKING

There are number of possible methods that have been developed for synchronization in chaotic communications. In the masking method, synchronization is achieved by simply if the conditional Lyapunov exponents for the systems are negative for the given operating parameters. Thus, one could simply recover the message signal from the received chaotic signal through by means of a subtraction at the receiver. This synchronization is robust against small perturbations of the carrier signal. In the chaotic modulation method the message signal becomes part of the dynamics, which is more robust because of the greater symmetry between chaotic oscillator and response. In the chaos shift keying technique the message information is encoded onto the attractor by means of modulating a parameter of the chaotic oscillator, typically in a

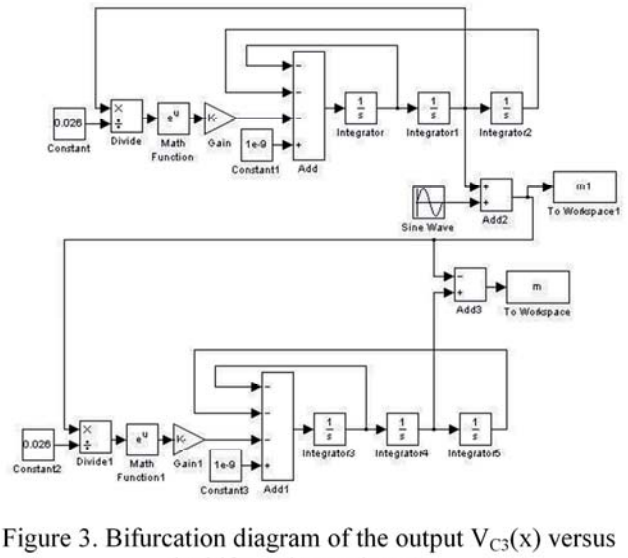


Figure 3. Bifurcation diagram of the output V_{C3}(x) versus the bifurcating resistor R_O.

binary manner. In all these three schemes synchronization is an obvious way of recovering the original information. Fig.2 shows the principle scheme of a general secure communication system with masking technique the transmitter can be used as a single drive system for a dual-channel transmitter independent of its response subsystem at the receiver.

IV. SIMULATIONS AND EXPERIMENTAL RESULTS

Fig.3 shows the simulation schematics in MATLAB. Electronic circuit simulation was performed in PSpice while the experiment was conducted on board using discrete components. The components value were set at R = 1 kΩ, C_1 = 0.1 μF, C_2 = C_3 = 10 μF, Diode is 1N4148 and an Op-amp is LM741, R_O is 1.2 kΩ. Fig. 5 shows the consistent chaotic attractors in x-y plane obtained from PSpice simulation and experiment. It is obvious that the proposed chaotic system circuit truly possesses chaotic behaviors with a single folded-band topology orbiting around the fixed point at (0, 0, 0). Fig.4 shows the bifurcation diagram of the peak of V_{C3} (X) versus the parameter R_O, exhibiting a route to chaos. I

t is seen that the bifurcating parameter R_O can be tuned for chaos in wide region of 0.8 kΩ to 3 kΩ. Upon setting R_O to a value of 1.2 kΩ, the chaotic attractors are displayed in Fig. 3 for a three-dimensional view, an *x–y* phase plane, an *x–z* phase plane, and a *y–z* phase plane. The attractor of three-dimensional view remains confined to the positive half-space of the z-axis. Fig.6 shows the experiment chaotic attractors in x–z plane. Fig.7 shows the synchronization results, (a) input and recovered output signal at the receiver, (b) synchronization errors. It is seen from Fig. 7 that the masked signal can be retrieved shortly with low errors. In addition, other types of signals such as rectangular or common human speech can also be applied to this method.

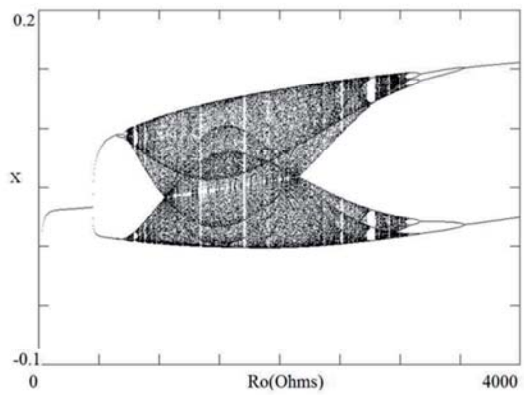


Figure 4. Bifurcation diagram of the output V_{C3} (X) versus the bifurcating resistor R_O.

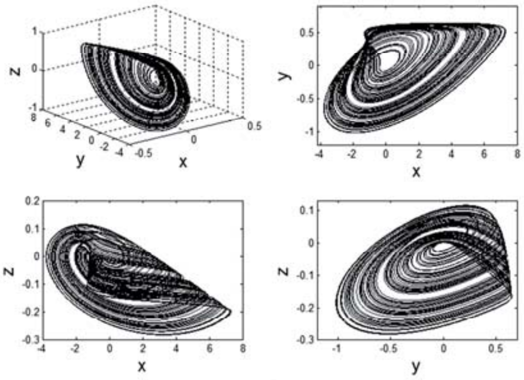


Figure 5. Chaotic attractors in three-dimensional view, an *x–y* plane, an *x–z* plane, and a *y–z* plane.

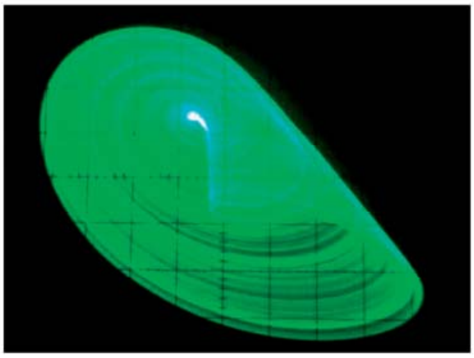
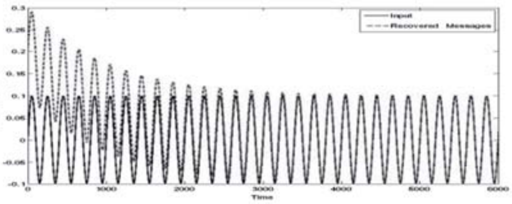


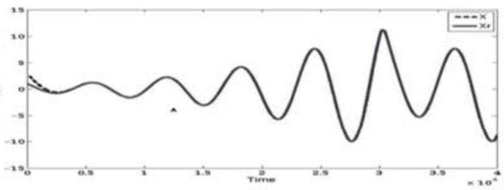
Figure 6. Experiment chaotic attractors in x–z plane

V. CONCLUSIONS

This paper has presented a very simple autonomous RC chaotic jerk oscillator with nine electronic components. The nonlinearity required for chaos is implemented through the use of a well-known diode equation. Basic dynamical properties are described including equilibria, eigenvalues of Jacobian matrix, chaotic attractors, time-domain waveforms and bifurcations. Potential application of such a simple



(a)



(b)

Figure 7. Syncronization results; (a) input and recovered output signal at the receiver, (b) synchronization errors.

autonomous RC chaotic jerk oscillator is presented in message-masking and synchronization for secure digital communications. The results show that the chaotically masked message is fully synchronized at the receiver through the use of very simple circuit. Consequently, the proposed new paradigm on secure communication schemes offers not only a simple mathematical system, but also very cost-effective circuit and system implementations.

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