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A New Exponentially-Controlled Logistic Chaotic Map

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***Abstract*— The typical logistic map has been utilized in a variety of applications such as in biological modeling and secure communications. Nonetheless, such a typical logistic map has only a single control parameter that sets all dynamic behaviors. This paper therefore introduces a new arbitrary power in the quadratic term in order to control stability of the system. The addition arbitrary power subsequently increases the degree of freedom of the logistic map and provides versatile responses as well as the flexibility of the system. Dynamic properties are described in terms of Cobweb plots, bifurcations, Lyapunov exponents, and chaotic waveforms in time domain. Experimental results utilize the Ardino microcontroller to generate chaotic waveforms with a relatively flat spectrum in frequency domain.**

Keywords—Generalization, Robust Chaos, Absolute Value nonlinearity, Bifurcation, Lyapunov exponent.

I. Introduction

Chaotic system is typically a dynamic system that possesses some significant properties, involving the sensitive dependence on initial conditions and system parameters, the density of the set of all periodic points, and topological transitivity. In particular, a chaotic map is the lowest one-dimensional evolution function in a discrete-time domain that can exhibits chaotic behaviors. The simplest chaotic map may be a logistic map which realizes quadratic polynomials as nonlinearity [1], i.e.

$$x_{n+1} = a - x_n^2 \tag{1}$$

where a is a control parameter. The well-known variant of (1) that has been employed in a variety of applications [2-4] is in the form

$$x_{n+1} = a x_n (1 - x_n) \tag{2}$$

In other words, it can be considered in (2) that the equation comprises two terms, i.e. linear and nonlinear terms as follows

$$x_{n+1} = a x_n - a x_n^2 \tag{3}$$

This equation normally utilizes for a classic example of nonlinear dynamic system behaviors. Originally, the Logistic Map is a discrete-time demographic model analogous to the first logistic equation created by Pierre François Verhulst. The variable x_n is a number between zero and one that represents the ratio of existing population to the maximum possible population. The coefficient a is a number means the sum of the rate of reproduction and malnutrition. This equation models the distribution of human populations and estimates the increasing in populations of other species under limited circumstances.

The common feature found in both (1) and (2) is a single changeable parameter a that sets overall dynamic behaviors of the overall system. Conventionally, the adaptive control of logistic map was therefore proposed using additional controller [5], i.e.

$$x_{n+1} = a x_n - b x_n^2 + u_k \tag{4}$$

where a and b are two system parameters and u_k is a contoller. Recently, a generalized Logistic Map has been proposed in the form

$$x_{n+1} = a x_n^{\alpha} (1 - x_n^{\beta}) \tag{5}$$

where α and β are arbritary power of the variable x_n. It can be considered from (4) and (5) that the equations are complicated in terms of atleast three parameters that contol both linear and nonlinar terms.

This paper therefore presents a new technique for controlling stability of the logistic map through the use of fractional power in the nonlinear term. The dynamic behaviors can be controlled effectively. The addition arbitrary power subsequently increases the degree of freedom of the logistic map and provides versatile responses as well as the flexibility of the system. Dynamic properties will be described in terms of Cobweb plots, bifurcations, Lyapunov exponents, and chaotic waveforms in time domain. Experimental results utilize the Arduino microcontroller to generate chaotic waveforms with a relatively flat spectrum in frequency domain.

II. Proposed Modified Logistic Map with Arbritary Power in Nonlinear Term

The proposed new technique for controlling stability of the logistic map through the use of fractional power in the nonlinear term is expressed as

$$x_{n+1} = ax_n(1 - x_n^{1-b}) \tag{6}$$

where a is a typical control parameter and b is a newly introduced parameter. Eq. (6) can be re-written as

$$x_{n+1} = ax_n - ax_n^{2-b} \tag{7}$$

It can be considered from (7) that the linear term (ax_n) is kept as original logistic map. However, the power of nonlinear term is reduced from 2 to frcational number. According to (8) the fixed point (x_p) can be found at

$$x_p = \left(\frac{1}{3-b}\right)^{\frac{1}{2-b}} \tag{8}$$

Generally, preliminary investigations of chaotic behaviors in chaotic maps can be achieved qualitatively and quantitatively through a bifurcation diagram and the Lyapunov Exponent (*LE*), respectively. The bifurcation diagram indicates possible long-term values, involving fixed points or periodic orbits, of a system as a function of a bifurcation parameter. The stable solution is represented by a straight line while the unstable solutions are generally represented by dotted lines, showing thick regions.

On the other hand, the *LE* is defined as a quantity that characterizes the rate of separation of infinitesimally close trajectories and is expressed as [5]

$$LE = \lim_{n\to\infty} \frac{1}{N} \sum_{n=1}^{N} \log_2 \left| \frac{dx_{n+1}}{dx_n} \right| \tag{9}$$

where N is the number of iterations. Typically, the positive *LE* indicates chaotic behaviors of dynamical systems and the larger value of *LE* results in higher degree of chaoticity. In this paper,

III. Simulation Results

A. Typical Logictic Map Characteristics

Dynamic behaviors of logistic map are investigated using MATLAB. Fig.1 shows an apparently chaotic waveform in time-domain where parameters were set at a = 3.99 and b = 0. It is seen from Fig.1 that the signal is random in both amplitude and frequency characteristics. Fig.2. shows the cobweb plot at a = 3.99 and b = 0.

The cobweb plot is typically used to investigate the qualitative behaviour of one-dimensional iterated functions. It can be seen in Fig.2 that a chaotic orbit shows a filled out area, indicating an infinite number of non-repeating values. Fig.3 show the period doubling bifurcation diagram in the region of [0,4] where chaos is induced from a gretaher than 3.5. Fig.5 shows the *LE* spectrum where chaos appears when *LE* is greater than zero.

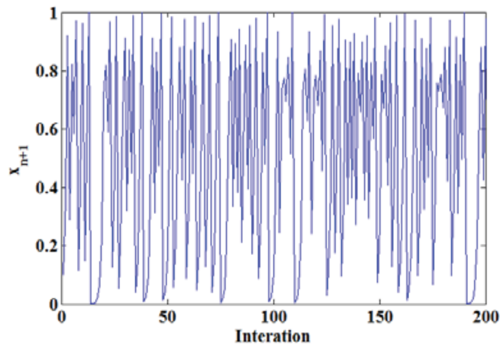

Fig.1. The pparently chaotic waveform in time-domain at a = 3.99 and b = 0.

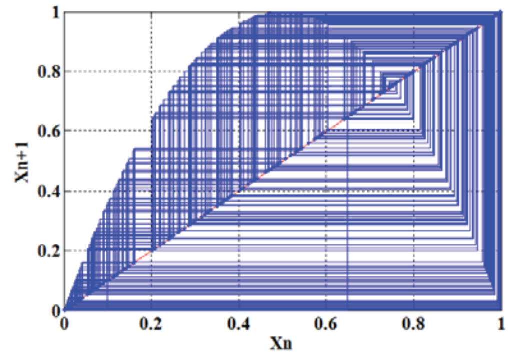

Fig.2. The cobweb plot at a = 3.99 and b = 0.

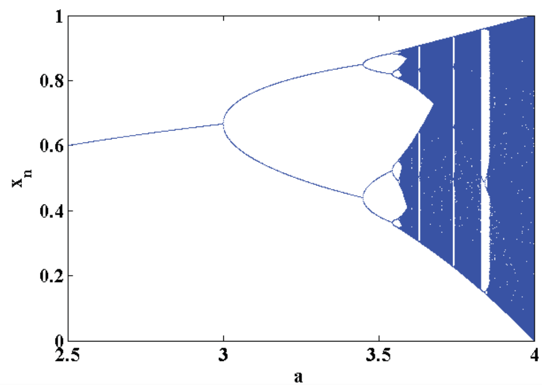

Fig.3. The period doubling bifurcation diagram in the region of [0, 4].

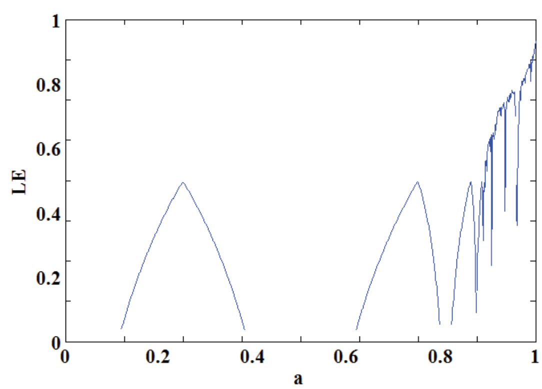

Fig.4 The *LE* spectrum where chaos appears when *LE* is greater than zero.

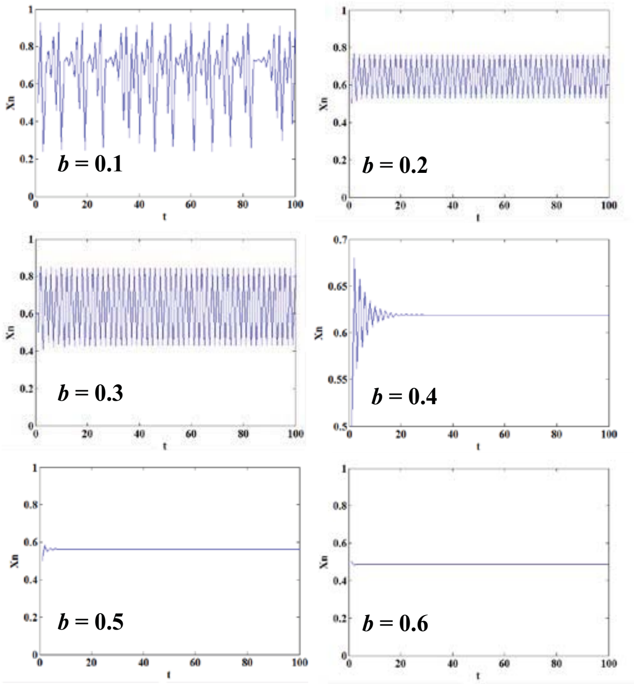

Fig.5. the time-domain waveforms of the modified logistic map at different value of the parameter *b*.

B. *Dynamic Behaviors of the proposed Modified Logistic Map*

Dynamic behaviors of the proposed modified logistic map can be investigated through the bifurcation of the parameter *b*. Fig.5.shows the period doubling bifurcation diagram of parameter *b* in the region of [0,1]. It can be seen that the bifurcation is inverse to the typical logistic map. In addition, the bifucation structure of parameter *a* versus *b* is shown in Fig.6. The system possesses relatively complex behaviors. In order to investigate the ability to control to behaviors, time domain waveforms are illustrated in Fig.7 It can be considered from Fig.7 that when the value of the parameter b is incresed, i.e. the power of the nonlinear term becomes fractional, the system become stable as indicated by the bifucation diagram shown in Fig.3 and the waveforms in Fig.7.

IV. Experimental Results

A. *Generation of Chaotic Signals Using Microcontroller*

The experimental results have been conducted using a cost effective Arduino with Atmel SAM3X8E ARM Cortex-M3 CPU as shown in Fig.6. The case where parameters $a = 3.99$ and b=0.1 was chosen as for demonstration. Fig. 7 shows the chaotic waveform generated from Arduino and Fig. 8. shows the Fast Fourier Transfrom (FFT) of the chaotic signal also showing a relatively flatspectrum over all frequency range. Other cases also exhibit the same characteristics in correspodance to the simulation results.

Fig.6 Arduino with Atmel SAM3X8E ARM Cortex-M3 CPU

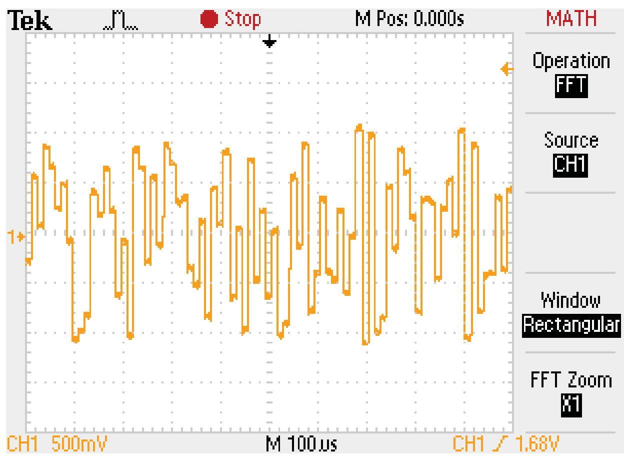

Fig.7. Chaotic waveform generated from Arduino.

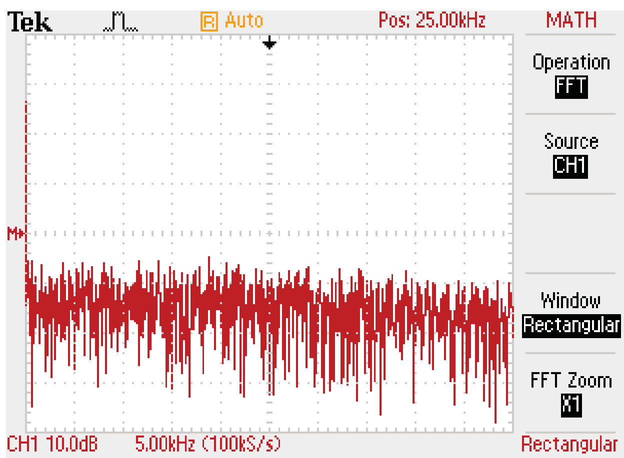

Fig.8 Fast Fourier Transfrom of the chaotic signal, showing a relatively flat spectrum over all frequency range .

V. Conclusion

This paper has presented a new arbitrary power in the quadratic term in order to control stability of the system. The additial arbitrary power subsequently increases the degree of freedom of the logistic map and provides versatile responses as well as the flexibility of the system. Dynamic properties are described in terms of Cobweb plots, bifurcations, Lyapunov exponents, and chaotic waveforms in time domain. Experimental results utilize the Ardino microcontroller to generate chaotic waveforms with a relatively flat spectrum in frequency domain. The application in random-bit generator that passes all NIST standard tests is also involved. The proposed chaotic map offer an alternative maps and random bit generator such as in cryptography or in communications

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