A Study on Analysis and Design of Geometric Tolerances of Machine Tools

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***Abstract*- The objective of the research is to establish a systematic design method applicable to analysis and design of kinematic motion deviations of five-axis machining centers based on the tolerances of the guide-ways. A systematic procedure is proposed to determine the tolerance values of the guide-ways theoretically under the constraints on the kinematic motion deviations of five-axis machining centers, by applying ISO tolerance definitions. The proposed method provides us with theoretical way to design the geometric tolerances of the guide-ways connecting the com- ponents of five-axis machining centers, based on allowance of the kinematic motion deviation of the tools against the workpieces.**

***Keywords*- Machine tools, Machining accuracy, Geometric accuracy, Motion deviations, Tolerance design**

I. INTRODUCTION

The kinematic motion deviations of five-axis machining centers are deeply influenced by the geometric deviations of the components, such as guide-ways and bearings. A systematic design method is, therefore, required for specifying suitable geometric tolerances of the guide-ways, in order to improve the kinematic motion deviations of five-axis machining centers.

Many researches have been carried out to model and to analyze the kinematic motion errors of the machine tools, such as modelling of shape generation motions of machine tools and machining error measurements [1-4]. These researches mainly focused on the analysis of the influences of the kinematic motion deviations of machine tool components on the kinematic motion deviations of the tools against the workpieces and the geometric deviations of the machined surfaces.

Many researches have also been carried out to deal with the dimensional tolerances and the geometric tolerances, aimed at realizing systematic analysis and design methodologies for the three dimensional machine products [5-6]. However, the relationships between the motion deviations and the geometric tolerances of the components have not yet been clarified.

The objectives of the present research are to establish the mathematical models representing the kinematic motion deviations of the machine tools, on the basis of the geometric tol- erances of the components, and to apply the models to the theoretical analysis of the kinematic motion deviations of the machine tools. The issues discussed in the paper are summa- rized in the followings.

(1) Modelling of kinematic motion deviations for three types of five-axis machining centers,
(2) Applications of proposed models to analysis of kinematic motion deviations of five-axis machining centers, and
(3) Applications of the proposed models to design of geomet- ric tolerance values of guide-ways.

II. GEOMETRIC TOLERANCES

The geometric tolerances of the machine components specify the allowable areas of the features named "tolerance zones," which constrain the position and orientation deviations of the associated features against the nominal features, as shown in Figure 1. The associated features and the nominal features mean the features of the manufactured products and the ideal features defined in the design phase, respectively. The geometric deviations of the associated features from the nominal features are represented by sets of parameters named "deviation parameters." For example, one position parameter i_1 and two rotational parameters i_2 and i_3 are required to represent the geometric deviations of the associated plane features against the nominal plane features, for the case where the tolerance zone is given by the area between a pair of parallel planes.

In the research, the followings are assumed for the ease of the modelling and the analysis of the geometric deviations.

(1) The deviation parameters δ_i representing the position and orientation deviations of the associated features follow the normal distribution N(μ_i, σ_i), and $\mu_i = 0$. Where, μ_i and σ_i are the mean values and the standard deviations, respectively.

(2) The manufacturing processes of the components are well controlled, and the proportion of the non-conforming components is as small as a value P_d called "percent defective." Here, the non-conforming components mean the components, in which the toleranced features exceed the tolerance zones.

(3) Equation (1) represents the relationships between the standard deviations ıi of the deviation parameters of the toleranced features and the sizes of the tolerance zones.

$$\sigma_i = \delta_{imaz} \ / \ C_{pd} \qquad (1)$$

where,

i : Indices of deviation parameters of the toleranced features.

δ_{imax} : Tolerance limits of the deviation parameters δ_i, if the other deviation parameters $\delta_j = 0$, $(i \neq j)$.

C_{pd} : A constant representing the ratio of the maximum values δ_{imax} and the standard deviations σ_i. It is assumed that C_{pd} has same value for all the deviation parameters of one tolerance feature.

III. KINEMATIC MOTION DEVIATIONS

(1) Linear Tables

The kinematic motion deviations of y-axis liner tables shown in Figure 2 are formulated in the following equations, under the assumption that the priority among the guide-ways is "(***a*** = ***c***) > (***b*** = ***d***)." Here, "(***a*** = ***c***)" means that there is not any priority between the guide-ways ***a*** and ***c*** to determine the positions and orientations of the tables against the bases considering the geometric deviations of the guide-ways. "***g*** > ***h***" means that the higher priority is given to the guide-way ***g*** against ***h*** to determine the positions and orientations of the tables against the bases.

$$\mathbf{A}_2(y) = \begin{pmatrix} 1 & -\frac{\delta_{\gamma b}+\delta_{\gamma d}}{2} & \frac{\delta_{\beta a}+\delta_{\beta c}}{2} & \frac{1}{2}\{(\delta_{ub}+\delta_{ud})-(\delta_{\gamma b}+\delta_{\gamma d})y\} \\ \frac{\delta_{\gamma b}+\delta_{\gamma d}}{2} & 1 & -\frac{\delta_{\alpha a}+\delta_{\alpha c}}{2} & y \\ -\frac{\delta_{\beta a}+\delta_{\beta c}}{2} & \frac{\delta_{\alpha a}+\delta_{\alpha c}}{2} & 1 & \frac{1}{2}\{(\delta_{wa}+\delta_{wc})+(l_{a1}\delta_{\beta a}+l_{c1}\delta_{\beta c})+(\delta_{\alpha a}+\delta_{\alpha c})y\} \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (2)$$

where,

y: y-direction travel.

δ_{pq}: Differences of the position and orientation deviations of the features in base side and table side consisting of the q-th guide-ways of in the p-th directions.

p (=$\alpha,\beta,\gamma,u,v,w$): Components of the position and orientation deviations.

q (= **a, b, c, d**): Guide-ways.

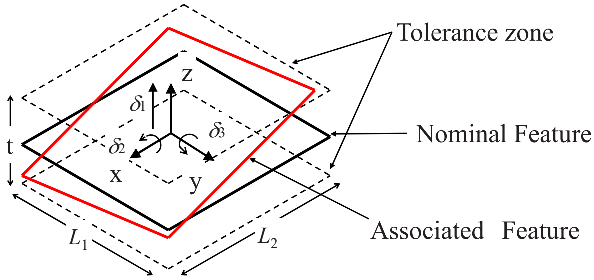


Fig. 1 Example of geometric tolerances

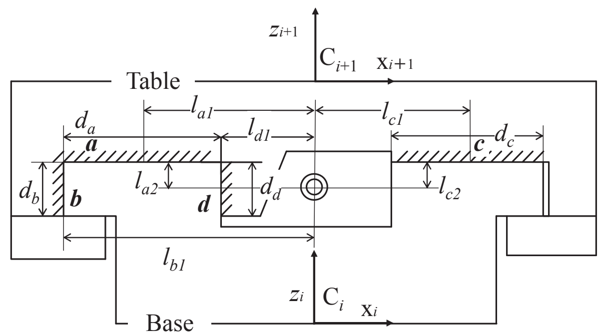


Fig. 2 Linear tables

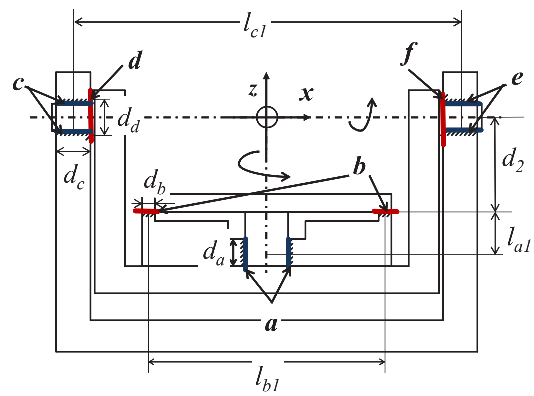


Fig. 3 Vertical and horizontal rotary tables

(2) Vertical Rotary Tables

The kinematic motion deviations of the vertical rotary tables shown in Figure 3 are formulated in Eq. (3), under the assumption that the priority among the guide-ways is "***a*** > ***b***."

$$\mathbf{A}_6(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta & \delta_{V\beta 1} & \delta_{Vx} \\ \sin\theta & \cos\theta & \delta_{V\alpha 1} & \delta_{Vy} \\ \delta_{V\beta 2} & \delta_{V\alpha 2} & 1 & \delta_{Vz} \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad (3)$$

$$\delta_{V\alpha 1} = -\beta_{b2}\sin\theta + \alpha_{b2}\cos\theta - \alpha_{b1}$$
$$\delta_{V\alpha 2} = \beta_{b1}\sin\theta + \alpha_{b1}\cos\theta - \alpha_{b2}$$
$$\delta_{V\beta 1} = -\beta_{b2}\cos\theta - \alpha_{b2}\sin\theta + \beta_{b1}$$
$$\delta_{V\beta 2} = -\beta_{b1}\cos\theta + \alpha_{b1}\sin\theta + \beta_{b2}$$
$$\delta_{Vx} = -l_1\beta_{b2}\cos\theta - l_1\alpha_{b2}\sin\theta + l_1\beta_{b1} - \delta_{xb2}\cos\theta + \delta_{y_{b2}}\sin\theta + \delta_{xb1}$$
$$\delta_{Vy} = -l_1\beta_{b2}\sin\theta + l_1\alpha_{b2}\cos\theta - l_1\alpha_{b1} - \delta_{xb2}\sin\theta - \delta_{y_{b2}}\cos\theta + \delta_{y_{b1}}$$
$$\delta_{Vz} = \delta_{za1} - \delta_{za2}$$

where,
θ : Rotation angle of table.
α_{ij}, β_{ij}, γ_{ij} : Orientation deviations of the j-th geometric feature of guide-way i around x, y and z-axis of the rotary tables.
δ_{xij}, δ_{yij}, δ_{zij} : Position deviations of the j-th geometric feature of guide-way i along x, y and z-axis of the rotary tables.

(3) Horizontal Rotary Tables

For the cases of the horizontal rotary tables shown in Fig- ure 3, the kinematic motion deviations are formulated in Eq. (4), under the assumption that the priority among the guide- ways is “(***c*** = ***e***) > (***d*** = ***f***).”

$$\boldsymbol{A}_4(\varphi)=\begin{pmatrix} 1 & \frac{1}{2}\delta_{H\gamma 1} & \frac{1}{2}\delta_{H\beta 1} & \frac{1}{2}\delta_{Hx} \\ \frac{1}{2}\delta_{H\gamma 2} & \cos\varphi & -\sin\varphi & \frac{1}{2}\delta_{Hy} \\ \frac{1}{2}\delta_{H\beta 2} & \sin\varphi & \cos\varphi & \frac{1}{2}\delta_{Hz} \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

$$\delta_{H\beta 1}=-\beta_{c2}-\beta_{e2}+(\gamma_{c1}+\gamma_{e1})\sin\varphi+(\beta_{c1}+\beta_{e1})\cos\varphi$$
$$\delta_{H\beta 2}=-\beta_{c1}-\beta_{e1}-(\gamma_{c2}+\gamma_{e2})\sin\varphi+(\beta_{c2}+\beta_{e2})\cos\varphi$$
$$\delta_{H\gamma 1}=\gamma_{c2}+\gamma_{e2}-(\gamma_{c1}+\gamma_{e1})\cos\varphi+(\beta_{c1}+\beta_{e1})\sin\varphi$$
$$\delta_{H\gamma 2}=\gamma_{c1}+\gamma_{e1}-(\gamma_{c2}+\gamma_{e2})\cos\varphi-(\beta_{c2}+\beta_{e2})\sin\varphi$$
$$\delta_{Hx}=\delta_{xd1}-\delta_{xd2}+\delta_{xf1}-\delta_{xf2}$$
$$\delta_{Hy}=\delta_{yc1}+\delta_{ye1}-(\delta_{yc2}+\delta_{ye2})\cos\varphi+(\delta_{zc2}+\delta_{ze2})\sin\varphi$$
$$-\frac{l_{c1}}{2}\{(\gamma_{e1}-\gamma_{c1})-(\gamma_{e2}-\gamma_{c2})\cos\varphi-(\beta_{e2}-\beta_{c2})\sin\varphi)\}$$
$$\delta_{Hz}=\delta_{zc1}+\delta_{ze1}-(\delta_{zc2}+\delta_{ze2})\cos\varphi-(\delta_{yc2}+\delta_{ye2})\sin\varphi$$
$$+\frac{l_{c1}}{2}\{(\beta_{e1}-\beta_{c1})-(\beta_{e2}-\beta_{c2})\cos\varphi+(\gamma_{e2}-\gamma_{c2})\sin\varphi)\}$$

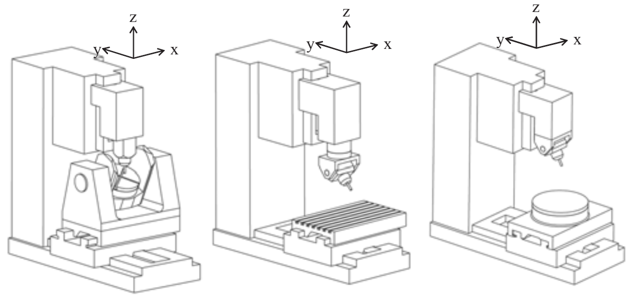
where,
M : Rotation angle of table.

IV. EFFECTS OF GEOMETRIC TOLERANCES

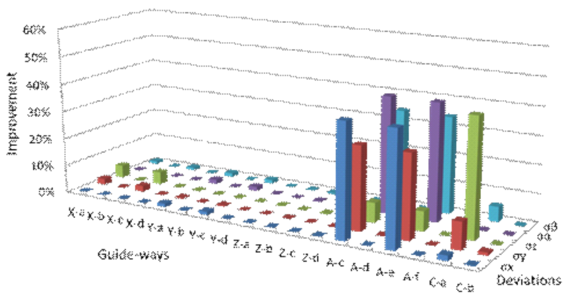
The five-axis machining centers are composed of two rota- ry tables and three liner tables shown in Figures 2 and 3. Three types of five–axis machining centers shown in Figure 4 are considered here for the analysis. The models of the shape generation motions of the machining centers are obtained by combining the formulas represented in Eqs. (2), (3) and (4). The detail of the analysis method is pre- sented in the previous paper [7].

The effects to the kinematic motion deviations are investi- gated by applying the various tolerance values to the guide-ways. Figure 5 summarizes the analysis results of the improvement ratio of the tolerance values to the kinematic motion deviations.

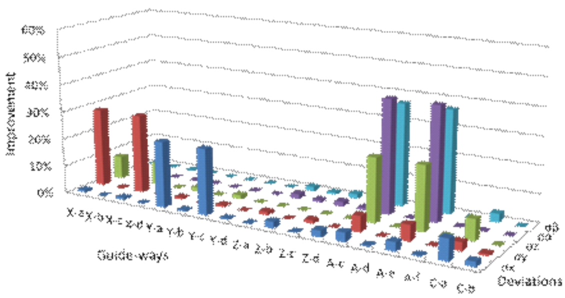
It is found that the improvement ratio of the A-axis and Caxis rotary tables is higher than ones of the linear tables for all cases of the analysis conditions. In particular, the guide-ways ***c*** and ***e*** of A-axis and the guide-way a of C-axis have larger effects than the other guide-ways. So, the tolerance values of the guide-ways of A-axis and C-axis are very important to improve the kinematic accuracy of the five-axis machine tools.



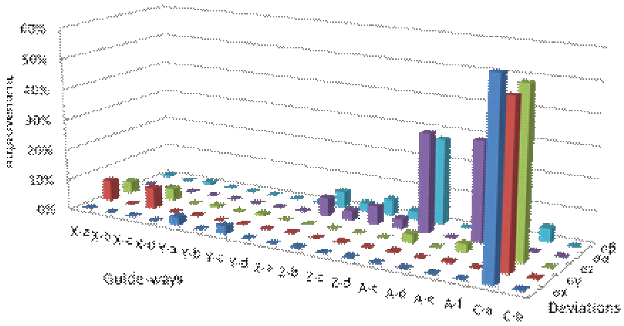
(a) Type-1 (b) Type-2 (c) Type-3
Fig. 4 Five-axis machining centers



(a) Type-1



(b) Type-2



(c) Type-3
Fig. 5 Effects of tolerances of guide-ways

V. TOLERANCES DESIGN

A systematic design system is proposed for planning a suitable set of the geometric tolerances of the guide-ways consid- ering the trade-off between the requirements on the kinematic motion deviations and the ease of the manufacturing processes of the guide-ways of the machine tools. The design variables to be determined here are the geometric tolerances t_i of individual guide-ways shown in Figure 5.

Objective functions are considered to represent the tradeoff between the requirements on the kinematic motion deviations and the ease of the manufacturing processes. Figure 6 shows the relationships between the sizes of the toleranced features and the tolerance values for various IT tolerances defined in ISO. As shown in the figure, the tolerance values depend on the feature sizes even for the same IT tolerances, such as IT6, 8 and 10. The following equation is proposed as the objective functions for the tolerance design, based on the relationships shown in Figure 6.

$$G_{obj}(\boldsymbol{t}) = \text{Minimize} \sum_{i=1}^{n} \frac{r_i^{0.34}}{t_i} \qquad (5)$$

where,
r_i : Size of toleranced features.
t_i : Tolerance values.

An optimization method is applied to determine the geometric tolerances t_i of the individual guide-ways on the basis of both the objective function given in Eq. (5) and the constraints on the kinematic motion deviations represented by Eqs. (2), (3) and (4). Figure 7 shows an example of the designed tolerance values for the individual guide-ways under the con- straints that the allowable kinematic motion deviations are less than 0.05 mm.

The designed tolerances of t_3, t_5, t_7, t_9, t_{11}, and t_{15} are smaller than the other tolerance values, as shown in Figure 7. This means that the tolerance values of those guide-ways are rather important to reduce the standard deviations of the kinematic motion deviations between the tools and the workpieces.

Figure 8 shows the values of objective function estimated by applying the designed tolerance values. The tolerances become smaller, the values of objective function become higher. This tendency coincides with the purpose to ease the manufacturing processes by making the tolerance values smaller.

VI. FUTURE RESEARCH PLANS

The tolerance analysis and tolerance design problems have been considered in the research project, based on the theoretical approach for the shape generation functions of the machine tools. The following two issues will be considered in the future research works.

(1) Investigation of the real machining centers, and
(2) Application of the proposed method to the other types of manufacturing devices.

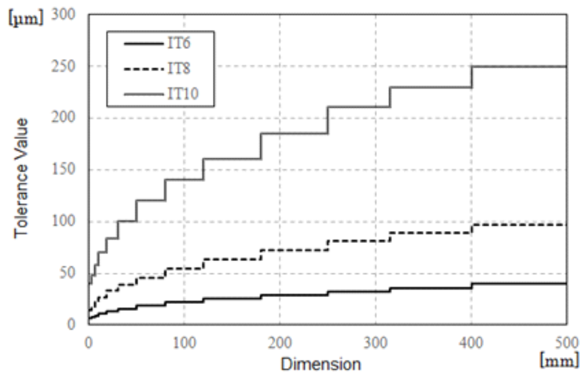


Fig. 6 Relationships between sizes and tolerance values

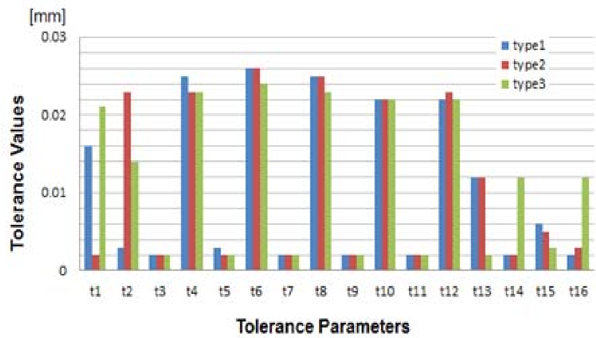


Fig. 7 Examples of designed tolerance values

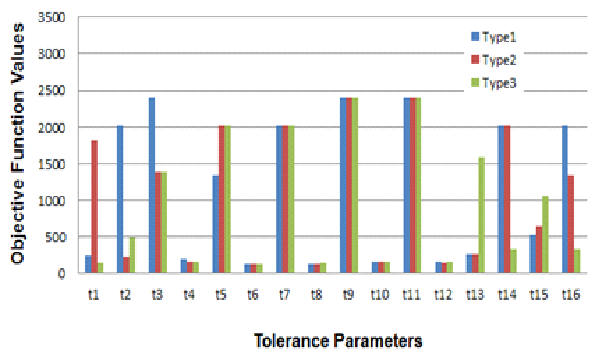


Fig. 8 Examples of objective function values

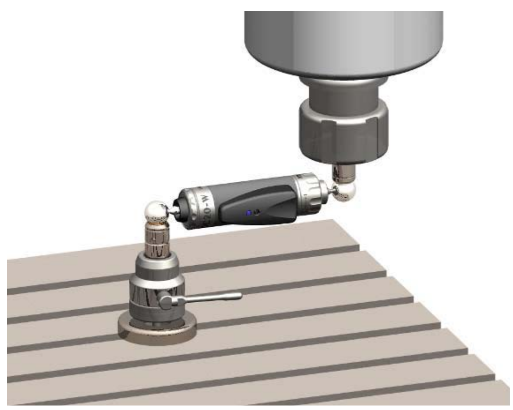


Fig. 9 DBB (Double Ball Bar) system
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Figure 9 shows an example of the systematic evaluation methods of the kinematic motion deviations of the tools against the worlpieces. A measuring device named DBB (Double Ball Bar) systems have been widely applied to evaluate and to improve the machining accuracy of the multi-axis

machine tools. The DBB systems have been applied to the evaluation of the alignment errors of the machining centers. The resolution of the DBB systems is smaller than 100 nm, and the kinematic deviation of the machining centers may be evaluated more accurately. A systematic method will be considered, in the future research, to evaluate the kinematic motion deviations of the individual linear and rotary tables of the multi-axis machining centers, based on the DBB measurement data.

Another future research plan is the extension of the appli- cation fields of the method proposed here. Figure 10 shows an example of the newly developed robots applied to the assembly processes for the small components. The robots have three link mechanisms to control the end effectors in 3-dimensional space, as shown in Figure 11. The position and the moving velocity of the end effectors are numerically controlled by three motors equipped within the link mechanisms, based on the mathematical models. The positions and the velocities have some errors due to the geometric tolerances of the indi- vidual links and the joints.

The tolerance design method proposed here will be applied to the design problems of the parallel link robots, aimed at improving the kinematic motion accuracy.

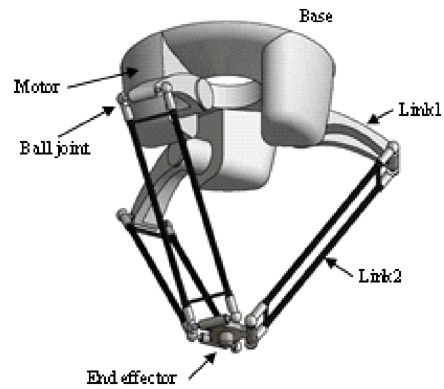

Fig. 7 Examples of designed tolerance values

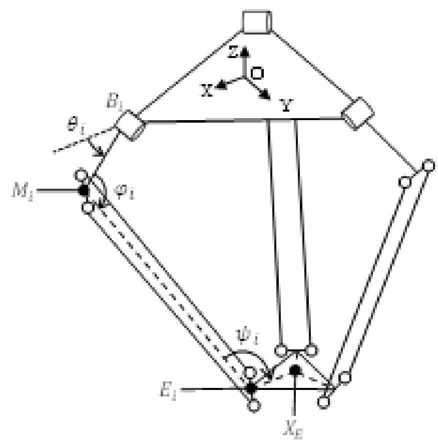

Fig. 7 Examples of designed tolerance values

VII. CONCLUSIONS

Following remarks are concluded.

(1) A mathematical model has been proposed to represent the kinematic motion deviations of the five-axis machining centers.

(2) The kinematic motion deviations of the five-axis machining centers are estimated by applying the proposed models. It is understood that the kinematic motion deviations are affected by the geometric tolerances of the guide-ways and that the geometric tolerances of the guide-ways of the rotary tables have higher effects on the kinematic motion devi- ations than the ones of the linear tables.

(3) Theoretical method is proposed to design the geometric tolerances of the guide-ways connecting the components of five-axis machining centers, based on allowance of the kinematic motion deviations of the tools against the work-pieces. This method is to design tolerances under the constraints on the standard deviations of shape generation functions of the tools against the workpieces.

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