A Method for Determining Inventory Policy Parameters for a Single Product under Limited Space

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***Abstract*- This study involves a method for determining parameters for a (R, Q) inventory policy for a single product under limited storage space. Most previous research developed methods for finding optimal reorder point (R) and order quantity (Q) that minimizes total cost including holding cost, ordering cost and shortage cost. This study incorporates limitation of storage space by adding the cost of overstocking to the total cost. A method to find appropriate values of these parameters is proposed for the case where a single product is subject to uncertain demand and lead time limited storage space capacity. A numerical example based on real product data is used to demonstrate effectiveness of the method.**

***Keywords*- Inventory control; reorder point; overstock cost; order quantity; limited storage space**

I. INTRODUCTION

Effective inventory control that can sufficiently satisfy customer needs is one of the most important strategies of all businesses. To provide a competitive advantage and improved customer service levels, many enterprises have to keep excessive inventory and pay additional cost, even when capacity of storage space is limited. One of the commonly used inventory management policy is the Q, R policy, where the key decisions are the replenishment order quantity and reorder point. Under this policy, when the inventory position falls on or below the reorder point, a replenishment order will be placed. Optimal values of Q and R are the ones that minimize the total inventory management cost.

Numerous research has been conducted on inventory policies. However, one important aspect of the problem, which is the limitation of storage space, has not been thoroughly studied. Some research studies on the Q, R policy that also consider storage space capacity in inventory management problem [1] - [7].

Huang [4] developed an inventory policy to find retailer's optimal cycle time and optimal payment time under the supplier's trade credit policy and cash-discount policy with space restriction and perishable goods. Roy and Maiti [5] presented a fuzzy EOQ inventory model. In contrast, the study of Özer and Wei [6] shows an optimal policy under fixed cost, planning horizon with advance demand information. While Sana [7] developed an EOQ model and Hariga [2] concentrated on (R, Q) policy, both studies considered storage spaces with respect to two types of warehouses: owned warehouse (OW) and rented warehouse (RW).

However, in those studies, the total cost consists of the ordering, holding and shortage costs, without explicitly considering the overstock cost for over-capacity units. In terms of the total cost, Hariga [1] - [2], and Zhoa et al. [3] considered three cost components: production or ordering cost, holding cost, and shortage cost; while Maiti [5] and Özer and Wei [6] only considered the first two cost components, and Huang [4] and Sana [7] considered the last two components.

When over-capacity units occur, they can be returned to the supplier [1], kept at an external rented capacity [2], [4], [7], or they are simply not allowed in the system [3], [5], [6]. Regarding demand, [1] - [3], [6], [7] considered stochastic demand following some distributions, e.g. normal, exponential, uniform, lognormal, and Poisson, whereas Huang [4] assumed constant demand, and Roy and Maiti [5] used the demand-dependent unit price. Similarly, for lead time, Hariga [1] - [2] assumed that the lead time follows normal distribution, while some researchers assumed constant or no lead time [3] - [7].

This paper aims at filling research gap in previous research studies. The paper concentrates on the Q, R inventory policy for managing a single product that has varying demand and lead time under limited storage space capacity. The objective is to find appropriate values of Q and R that minimize the total cost, while explicitly consider the overstock cost as a component of the total cost in addition to the ordering, holding, and shortage cost.

The paper is organized as follows. The problem statement is described in Section 2. Notations and methodology is presented in Section 3. An illustrative numerical example is provided in Section 4. Finally, conclusion is made in Section 5.

II. PROBLEM STATEMENT

The inventory management problem of interest can be described as follows.

In this system, inventory position of a single item is periodically reviewed. An order of size Q is placed if the ending inventory position is below the reorder point. When the inventory on-hands cannot satisfy demand, shortage will occur in the form of lost sales. The storage space for the item has a limited capacity. When the inventory on-hands exceed the capacity of the storage space, an overstock cost will be incurred according to the amount of overstock. The system measure of performance is the total inventory management cost, consist of four components: (1) cost of placing orders, (2) cost of holding inventory, (3) cost of lost sales, and (4) cost of overstock. The objective is to determine appropriate order quantity (Q) and reorder level (R) that minimizes the total cost under limited storage space.

III. NOTATIONS AND METHODOLOGY

A. Notations

The method described in this section uses the following notation.

Q	replenishment order quantity
R	reorder level
EOQ	economic order quantity
D	the demand rate of the item, in units/time period
CP	fixed reordering cost
CH	unit inventory carrying cost per time period
CS	unit shortage cost
d	demand during the replenishment lead time
p(d)	probability that demand during lead time is equal to *d*
d	average demand during the replenishment lead time
$E\{s\}_R$	expected number of units short incurred when the reorder level is set to R
W	limited capacity of the storage space
TC	Total cost

B. Distribution of Demand During Varying Lead Time

The distribution of the demand during varying lead time forms the basis to evaluate the performance of a specified reorder point. Specifically, it is used to estimate the expected amount of shortage for a given reorder point. In addition, the distribution can lead to determining the reorder point for a given value of service level. Therefore, the pre- processing step in the methodology is to formulate this distribution.

Due to the uncertainties in demand and replenishment lead time, the process of determining the distribution of demand during varying lead time must account for both uncertainties. First, the empirical distribution of one-period demand is constructed from historical demand data. This distribution also represents the distribution of demand during lead time (DDLT) when the lead time is one period. Then, the two-period DDLT distribution is derived by tabulating the one-period DDLT distribution with itself. The possible values of the two-period DDLT distribution are the sum of all possible pairs of one-period DDLT values, and the associated probability of each value is the product of the probabilities of the one-period DDLT. The process repeats until the DDLT distribution of the maximum lead time is derived. The final step is then to combine the DDLT for all possible values of lead time to obtain the DDLT of varying lead time. A short example of deriving the DDLT is given next.

Suppose the distribution of one-period demand is as shown in Table 1. An empirical distribution of demand during two-day lead time can be developed by tabulating the distribution of one-day demand. The possible values are from adding all possible pairs of demand values, and the associated probabilities are from multiplying the probability of demand values. The resulting DDLT distribution, after projecting all possible demand values and combining probability of demand for each demand value, is as shown in Table 2.

TABLE I
PROBABILITY OF ONE-DAY DEMAND

One-day demand	1	2	3
Probability	0.2	0.5	0.3

TABLE II
DISTRIBUTION OF DDLT, L = 2

Demand in day 1	Demand in Day 2: 1	2	3	Demand in day 1	Demand in Day 2: 1	2	3
1	2	3	4	1	0.04	0.10	0.06
2	3	4	5	2	0.10	0.25	0.15
3	4	5	6	3	0.06	0.15	0.09

Two-day demand	2	3	4	5	6
Probability	0.04	0.20	0.37	0.30	0.09

The DDLT distribution for three-day lead time is then created by further tabulating the DDLT distributions of two- day lead time and of one-day lead time, as shown in Table 3.

TABLE III
DISTRIBUTION OF DDLT, L = 3

Three-day demand	3	4	5	6
Probability	0.008	0.060	0.186	0.305
Three-day demand	7	8	9	
Probability	0.279	0.135	0.027	

In addition, when the lead time is uncertain, the DDLT distribution can be derived by combining both the lead time demand distribution and the lead time distribution. Now, suppose the lead time distribution follows the empirical distribution is shown in Table 4. Then, the final lead time demand distribution is as shown in Table 5. For example, the 3-unit demand occurs with the probability of 0.3, 0.2, and 0.008, if the lead time is one day, two days, and three days, respectively. Therefore, the overall probability of 3- unit demand would occur with the total probability of (0.3×0.7 + 0.2×0.2 + 0.008×0.1) = 0.02508.

TABLE IV
PROBABILITY OF LEAD TIME

Lead time	1	2	3
Probability	0.7	0.2	0.1

TABLE V
DISTRIBUTION OF DEMAND DURING VARYING LEAD TIME

Demand	1	2	3	4	5
Probability	0.1400	0.3580	0.2508	0.0800	0.0786
Demand	6	7	8	9	
Probability	0.0485	0.0279	0.0135	0.0027	

C. Iterative Q, R procedure

The algorithm for determining Q and R is adopted from the iterative method, first introduced by Felter and Dalleck (1961). The method combines shortage cost to the EOQ [9]. The shortage cost is subject to the expected amount of shortage, which depends on R, while R is also a function of Q. Thus, the algorithm proceeds iteratively until Q and R converge.

The method incorporates the following five steps:

Step 1 : Compute EOQ as an initial solution for Q,

$$Q = \sqrt{\frac{2DC_P}{CH}}$$

Step 2: Compute the probability of shortage,

$P(d > R) = \frac{QC_H}{DCs}$. Then, find R from the DDLT distribution.

Step 3: Estimate the amount of shortage for a given R (from Step 2), $E\{s\}_R = \sum_{d=R+1}^{d\max} (d - R)\, p(d)$

Step 4: Update $Q = \sqrt{\frac{2D[C_P + C_S\, E\{s\}_R\,]}{C_H}}$

Step 5: Repeat Step 2 to Step 4 until the values of Q in Step 4 and R in Step 2 converge.

The algorithm begins with an initial solution of $Q = EOQ$ in Step 1. In Step 2, the probability of shortage is calculated. Then, the value of R that is associated with this probability can be found from the DDLT distribution. This is done by deriving the cumulative probability distribution from the DDLT. Then, look up the cumulative probability of the demand that matches the probability of shortage, the value of the demand at that point is the appropriate R.

Based on the value of R from Step 2, $E\{s\}_R$ can be easily computed in Step 3. Step 4 updates Q based on the total cost that includes ordering cost, carrying cost, and shortage cost. Finally, in Step 5 the algorithm repeats Steps 2-4 until the values of R and Q converge.

At this point, it can be noticed that the algorithm does not take into account one cost component, which is the overstock cost. The values of Q and R will be adjusted to take into account the storage space limitation later in Sub-section E.

D. Simulation to evaluate the performance of R, Q

To evaluate the performance of Q and R, simulation of the inventory management process is created. In the simulation, periodically (e.g. daily) demand is randomly generated from the empirical distribution of one-period demand. After the customer demand arrives, the system updates the amount of inventory on-hands, inventory position, the amount of lost sales (if inventory on-hands is insufficient to satisfy demand), and the system determines whether a replenishment order Q is needed. If so, then the system randomly generated a lead time. At the end of the update, the system checks if there is incoming shipment from previous order so as to update the inventory on-hands and position accordingly. At this point, if the inventory on- hands exceed the capacity of the storage space, then the overstock amount is updated. At the end of the day, the system updates the inventory carrying cost based on the ending inventory on-hands, and then checks if the simulation terminating condition is met. The simulation is set to run for T periods. At the end of the simulation, all four costs that have incurred are combined to be the total cost. Fig. 1 shows a flow chart for simulation model of inventory management.

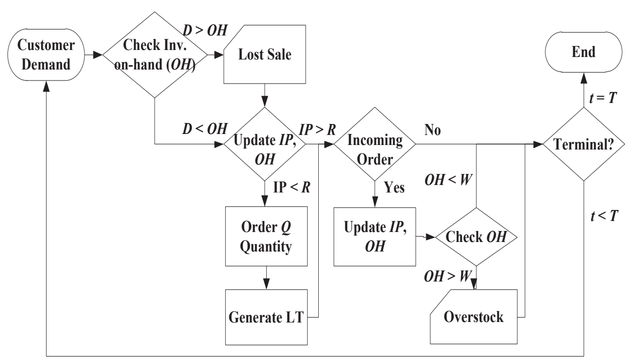

Fig. 1 Flow chart of inventory management

E. Adjusting the Inventory Parameters using the Cyclic Coordinate Method

The final step of the methodology is the adjustment of Q and R to take into account the limited space restriction. The method for adjustment is based on the multidimensional search without derivative, called cyclic coordinate method, for the unconstrained non-linear optimization [10]. The cyclic coordinate method uses the following notations.

l the allowable final length of the uncertainty, $l = 0$

$R_{L(k)}$ (or $Q_{L(k)}$) Lower bound of reorder point (or order quantity) at kth iteration in $[R_L; R_U]$ (or $[Q_L; Q_U]$).

$\lambda_{(k)}$ Eligible lower bound of reorder point (or order quantity) at kth iteration in $[R_L; R_U]$ (or $[Q_L; Q_U]$).

$\mu_{(k)}$ Eligible upper bound of reorder point (or order quantity) at kth iteration in $[R_L; R_U]$ (or $[Q_L; Q_U]$).

$R_{U(k)}$ ($Q_{U(k)}$) Upper bound of reorder point (or order quantity) at kth iteration in $[R_L; R_U]$ (or $[Q_L; Q_U]$).

$TC_{\lambda(k)}$ The total cost of the eligible lower bound of reorder point (or order quantity) at k^{th} iteration.

$TC_{\mu(k)}$ The total cost of the eligible upper bound of reorder point (or order quantity) at k^{th} iteration.

In the cyclic coordinate method performed at this step, Q and R are treated as the decision variables. The objective function is the total cost that now includes the fourth cost component, the overstock cost. To handle the uncertainty in the simulation result, the simulation model is run in multiple replications, and the average total cost is used as the objective function in the cyclic coordinate method.

This algorithm performs the search in the directions of the two decision variables, R and Q, with $d_R = (1, 0)$; $d_Q = (0, 1)$, one direction at a time in a cyclical manner. In the first iteration, starting from an initial solution (R_0, Q_0) from the iterative Q, R algorithm, the search is performed along the dR direction. The optimal step size is determined by using a one-dimensional search, called Golden Section Search method. After reaching a new solution, $(R_{(1)}, Q_0)$, the quality of the solution is evaluated using the simulation as described in Subsection D. Then, the search continues in the d_Q direction, and reach the new solution of $(R_{(1)}, Q_{(1)})$, which completes the first iteration. The algorithm repeats until the solution converges. The steps of cyclic coordinate are as shown in Fig. 2.

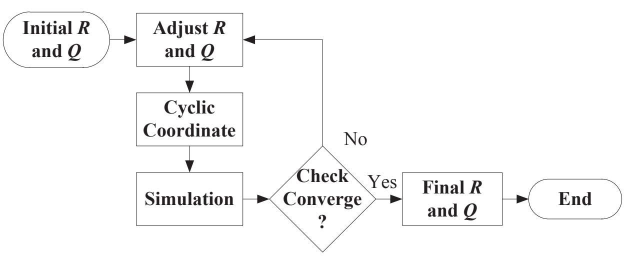

Fig. 2 Flow chart of R and Q adjustment

The golden section search follows these steps. First, arbitrarily select $l = 0.05$ to be used as stopping criterion for the length of the interval of uncertainty. Let $[1, R_0]$ (or $[1, Q_0]$) be the initial interval of uncertainty where the optimal R (or Q) lies. These intervals are chosen because R and Q must be non-negative and be smaller than the initial Q and R. Let $\lambda_{(1)} = 1 + (1\text{-}\alpha)(R_0 - 1)$, (or for finding the optimal Q, $\lambda_{(1)} = 1 + (1\text{-}\alpha)(Q_0 - 1)$) and $\mu_{(1)} = 1 + \alpha(R_0 - 1)$ (or $\mu_{(1)} = 1 + \alpha(Q_0 - 1)$), where $\alpha = 0.618$ (a golden ratio). Evaluate $TC(\lambda_{(1)})$ and $TC(\mu_{(1)})$, let $k = 1$, and go to next step.

Step 1 If $R_{U(1)}\text{-}R_{L(1)}$ (or $Q_{U(1)}\text{-}Q_{L(1)}$) < 0.05, then stop; the optimal solution is in the interval $[R_{L(1)}, R_{U(1)}]$ (or, $[Q_{L(1)}, Q_{U(1)}]$. Otherwise, if $TC(\lambda_{(1)}) > TC(\mu_{(1)})$, go to step 2; and if $TC(\lambda_{(1)}) <= TC(\mu_{(1)})$, go to Step 3.

Step 2 Let $R_{L(k+1)}$ (or $Q_{L(k+1)}$) $= \lambda_{(k)}$;
$R_{U(k+1)} = R_{U(k)}$ (or $Q_{U(k+1)} = Q_{U(k)}$)
$\lambda_{(k+1)} = \mu_{(k)}$;
$\mu(k+1) = R_{L(k+1)} + \alpha(R_{U(k+1)} - R_{L(k+1)})$
or $\mu_{(k+1)} = Q_{L(k+1)} + \alpha(Q_{U(k+1)} - Q_{L(k+1)})$)
And then evaluate $TC(\mu_{(k+1)})$ and go to step 4.

Step 3 Let $R_{L(k+1)} = R_{L(k)}$ (or $Q_{L(k+1)} = Q_{L(k)}$);
$R_{U(k+1)}$ (or $Q_{U(k+1)}$) $= \mu_{(k)}$;
$\lambda_{(k+1)} = R_{L(k+1)} + (1 - \alpha)(R_{U(k+1)} - R_{L(k+1)})$,
(or $\lambda_{(k+1)} = Q_{L(k+1)} + (1 - \alpha)(Q_{U(k+1)} - Q_{L(k+1)})$)
$\mu_{(k+1)} = \lambda_{(k)}$.

After that evaluate $TC(\lambda_{(k+1)})$ and go to step 4.

Step 4 Replace k by $k + 1$ and go to Step 1

The search will continue until $R_{U(1)}\text{-}R_{L(1)}$ (or $Q_{U(1)}\text{-}Q_{L(1)}$) < 0.05 at which point the solution convergence occurs (k is number of iterations). In the next section, one numerical example that illustrates the inventory system model in detail will be given.

IV. NUMERICAL EXAMPLE

Consider a fast moving item. All demands that occur in one day are aggregated to a daily demand. One year of daily demand data (294 working days) are used as input data. Cost parameters and demand data are obtained from industry. They are as shown in Table 6.

TABLE VI
COST COMPONENTS

Ordering cost (THB/order), C_P	9.6
Inventory holding cost (THB/unit/day), C_H	0.0118896
Shortage cost (THB/unit), C_S	8
Average daily demand (unit)	468
Standard deviation of demand (unit)	465

The demand data are used to fit an empirical distribution of one-day demand. Twenty replications of demand data are then generated from this distribution. These generated data are for solution evaluation purpose through simulation model. Replenishment lead time from the supplier is variable based on the probability of lead time as shown in Table 7.

TABLE VII
PROBABILITY OF LEAD TIME

Lead time (days)	Probability
1	0.7
2	0.2
3	0.1

The distribution of demand during varying lead time (DDLT) can be constructed as described in Section 3B. The iterative approach is applied to find initial R_0 and Q_0 values. Table 8 shows the iteration of the iterative approach until the two parameters converge.

TABLE VIII
ITERATIVE Q, R PROCEDURE

Iteration	Step 1: *EOQ*	Step 2: *P(d>R)*	*R*	Step 3: *E(s)*	Step 4: *Q*
1	869	0.00276	3,491	1.36274	1,271
2		0.00404	3,311	1.96371	1,412
3		0.00448	3,259	2.18522	1,460
4		0.00464	3,242	2.26277	1,477
5		0.00471	3,237	2.28610	1,482
6		0.00472	3,235	2.29551	1,488
7		0.00472	3,235	2.29551	1,488

The iterative procedure starts with computing initial Q from EOQ = 869 units. Secondly, compute $P(d>R)$ using Q from Step 1 and the cost parameters from Table 6 to obtain $P(d>R)$ = 0.00276. Then, find the value of R that is associated with $P(d>R)$ from the cumulative probability of demand during lead time distribution. After that, the expected number of shortage for a given value of R (from Step 2) is determined at 1.3627. Finally, Q is updated to be equal to 1,271 units. In the end of iterative Q, R procedure, R and Q parameters converge to R = 3,235 units and Q = 1,488 units.

After finding R_0 and Q_0, the quality of the initial solution is evaluated using simulation. The initial inventory is set on 6,470 units (two times of R). The process of simulation is that when inventory position drops to or below 3,235 units, then an order of 1,488 units will be placed. If the retailer cannot satisfy demand, it counts as lost sales.

To incorporate the storage space capacity, the maximum allowable of storage space is set to (average inventory level + L×SD of inventory level); for example, for L = 0.6, the capacity is 3,759 units. The overstock cost of 0.24 THB per unit will incur if the inventory level exceeds the space capacity. Table 9 shows the results from simulation evaluation.

TABLE IX
TOTAL COST OF ITERATIVE METHOD

Number of order	114
Number of shortage (units)	1,044
Average inventory level (units/day)	3,308
Ordering cost (THB)	1,094
Shortage cost (THB)	8,352
Inventory holding cost (THB)	14,356
Overstock cost (THB)	9,110
Total cost (THB)	**32,912**

After evaluating performance of R_0 and Q_0, the cyclic coordinate method is applied to adjust these parameters. The length of the interval of uncertainty, l, is set to 0.05. The initial boundary for the golden section search is [1, R_0] = [1, 3,235]. The iterations of the golden section in the R and Q directions are shown in Table 10 and Table 11, respectively.

From Table 10, it showed that the golden section search is terminated in the 21st iteration. Each TC is computed, while holding Q at Q_0 = 1,488. The first two new solutions are positioned at $\lambda_{(1)}$ = 1,236 units; $\mu_{(1)}$ = 1,999 units. $TC(\lambda_{(1)})$, including ordering cost, shortage cost, inventory on hand, and overstock cost combinations at $\lambda_{(1)}$ = 1,236 units, is 210,031.7 THB, while $TC(\mu_{(1)})$ is 72,378.8 THB. It can be easily seen that because $TC(\lambda_{(1)}) > TC(\mu_{(1)})$, in Step 2 $R_{L(2)} = \lambda_{(1)}$ = 1,236 units; $R_{U(2)} = R_{U(1)}$ = 3,235 units; $\lambda_{(2)} = \mu_{(1)}$ =1,999 units; and $\mu_{(2)} = R_{L(2)} + 0.618\ (R_{U(2)} -$ RL(2)) = 2,472 units. At the end of the golden section search, the interval of uncertainty is [2,982; 2,982], the optimal R1 is therefore 2,982 and the total cost is equal to 30,720.62 THB, which is a 7% cost reduction.

TABLE X
SUMMARY OF THE ITERATIONS ALONG DIRECTION
(1, 0) FOR CHANGING R

k	$R_{L(k)}$	$\lambda_{(k)}$	$\mu_{(k)}$	$R_{U(k)}$	$TC(\lambda_{(k)})$	$TC(\mu_{(k)})$
1	1.00	**1,236.4**	1,999.6	3,235.0	210,031.7	**72,378.8**
2	1,236.4	**1,999.6**	2,471.5	3,235.0	72,378.8	**42,291.9**
3	1,999.6	**2,471.5**	2,763.1	3,235.0	42,291.9	**33,437.2**
4	2,471.5	**2,763.1**	2,943.4	3,235.0	33,437.2	**31,017.9**
5	2,763.1	2,943.4	**3,054.7**	3,235.0	**31,017.9**	31,065.4
6	2,763.1	**2,874.5**	2,943.4	3,054.7	31,427.2	**31,017.9**
7	2,874.5	**2,943.4**	2,985.9	3,054.7	31,017.9	**30,775.3**
8	2,943.4	2,985.9	**3,012.2**	3,054.7	**30,775.3**	30,894.8
9	2,943.4	**2,969.6**	2,985.9	3,012.2	30,938.4	**30,775.3**
10	2,969.7	2,985.9	**2,995.9**	3,012.2	**30,775.3**	30,890.1
11	2,969.7	2,979.7	**2,985.9**	2,995.9	**30,768.2**	30,775.3
12	2,969.7	**2,975.8**	2,979.7	2,985.9	30,920.4	**30,768.2**
13	2,975.9	**2,979.7**	2,982.0	2,985.9	30,768.2	**30,720.6**
14	2,979.7	2,982.0	**2,983.5**	2,985.9	**30,720.6**	30,733.1
15	2,979.7	**2,981.2**	2,982.0	2,983.5	30,775.1	**30,720.6**
16	2,981.2	2,982.0	**2,982.6**	2,983.5	**30,720.6**	30,720.6
17	2,981.2	**2,981.7**	2,982.0	2,982.6	30,775.1	**30,720.6**
18	2,981.7	2,982.0	**2,982.3**	2,982.6	**30,720.6**	30,720.6
19	2,981.7	**2,981.9**	2,982.0	2,982.3	30,775.1	**30,720.6**
20	2,981.9	2,982.0	**2,982.1**	2,982.3	**30,720.6**	30,720.6
21	2,981.9	2,982.0	**2,982.0**	2,982.1	**30,720.6**	30,720.6

TABLE XI
SUMMARY OF THE ITERATIONS ALONG DIRECTION (1, 0) FOR CHANGING Q

k	$Q_{L(k)}$	$\lambda_{(k)}$	$\mu_{(k)}$	$Q_{U(k)}$	$TC(\lambda_{(k)})$	$TC(\mu_{(k)})$
1	1.0	**569.0**	920.0	1,488.0	73,249.0	**34,479.7**
2	569.0	**920.0**	1,137.0	1,488.0	34,479.7	**30,801.5**
3	920.0	**1,137.0**	1,271.0	1,488.0	30,801.5	**30,202.7**
4	1,137.0	**1,271.0**	1,353.9	1,488.0	30,202.7	**30,128.4**
5	1,271.0	1,353.9	**1,405.1**	1,488.0	**30,128.4**	30,737.3
6	1,271.0	**1,322.2**	1,353.9	1,405.1	30,304.8	**30,128.4**
7	1,322.2	1,353.9	**1,373.5**	1,405.1	**30,128.4**	30,644.2
8	1,322.2	**1,341.8**	1,353.9	1,373.5	30,314.9	**30,128.4**
9	1,341.8	1,353.9	**1,361.4**	1,373.5	**30,128.4**	30,330.1
10	1,341.8	1,349.3	**1,353.9**	1,361.4	**30,124.0**	30,128.4
11	1,341.8	**1,346.4**	1,349.3	1,353.9	30,320.7	**30,124.0**
12	1,346.4	1,349.3	**1,351.0**	1,353.9	**30,124.0**	30,509.6
13	1,346.4	**1,348.2**	1,349.3	1,351.0	30,190.3	**30,124.0**
14	1,348.2	1,349.3	**1,350.0**	1,351.0	**30,124.0**	30,188.8
15	1,348.2	1,348.9	**1,349.3**	1,350.0	**30,066.0**	30,124.0
16	1,348.2	1,348.6	**1,348.9**	1,349.3	**30,039.8**	30,066.0
17	1,348.2	**1,348.4**	1,348.6	1,348.9	30,173.7	**30,039.8**
18	1,348.4	1,348.6	**1,348.7**	1,348.9	**30,039.8**	30,061.9
19	1,348.4	**1,348.5**	1,348.6	1,348.7	30,068.7	**30,039.8**
20	1,348.5	1,348.6	**1,348.6**	1,348.7	**30,039.8**	30,089.3
21	1,348.5	**1,348.6**	1,348.6	1,348.6	30,058.7	**30,039.8**
22	1,348.6	1,348.6	**1,348.6**	1,348.6	**30,039.8**	30,051.8
23	1,348.6	**1,348.6**	1,348.6	1,348.6	30,045.3	**30,039.8**

After finding R, it is fixed at 2,982, and the cyclic coordinate turns to the Q direction. The results from the golden section search from Table 11 shows the optimal Q_1 = 1,349. The cyclic coordinate method keeps searching cyclically until R and Q converge. Iteration results from the cyclic coordinate method are shown in Table 12.

After seven iterations, the optimal solution converges at R = 3,191, a reduction of 1.4%, and Q = 1,105, a reduction of 26%. Note that both R and Q are rounded to the nearest integer. The performance measures of the final solution are shown in Table 13. Overall, the number of order and shortage increase, while the average inventory and overstock decrease. This lead to the final total cost of 28,331 THB, or a cost reduction of 13.92%. The average inventory on-hands reduced by 8.59%.

TABLE XII
SUMMARY OF THE CYCLIC COORDINATE METHOD

K	$R_{(k)}$	$Q_{(k)}$	j	$d_{(j)}$		$R_{(j+1)}$	$Q_{(j+1)}$	TC (THB)
1	3235	1488	1	1	0	2,982.0	1,488.0	30,720.6
			2	0	1	2,982.0	1,348.6	30,041.0
2	2982.0	1348.6	1	1	0	3,058.4	1,348.6	29,738.2
			2	0	1	3,058.4	1,254.3	29,189.6
3	3058.4	1254.3	1	1	0	3,119.5	1,254.3	28,949.4
			2	0	1	3,119.5	1,203.6	28,694.8
4	3119.5	1203.6	1	1	0	3,139.8	1,203.6	28,701.2
			2	0	1	3,139.8	1,104.2	28,211.9
5	3139.8	1104.2	1	1	0	3,190.2	1,104.2	28,103.3
			2	0	1	3,190.2	1,104.3	28,110.2
6	3190.2	1104.3	1	1	0	3,190.5	1,104.3	28,091.4
			2	0	1	3,190.5	1,104.3	28,091.4
7	3190.5	1104.3	1	1	0	3,190.5	1,104.3	28,091.4
			2	0	1	3,190.5	1,104.3	28,091.4

TABLE XIII
TOTAL COST OF CYCLIC COORDINATE METHOD

Number of order	154
Number of shortage (units)	1,437
Average inventory level (units/day)	3,034
Average number of overstock (units/day)	25
Ordering cost (THB)	1,478
Shortage cost (THB)	11,496
Inventory holding cost (THB)	13,167
Overstock cost (THB)	2,190
Total cost (THB)	**28,331**

V. CONCLUSIONS

This article considers the inventory management problem with the Q, R policy under storage space limitation. A two- stage method featuring iterative Q, R method to find initial solution without considering storage space limitation, and cyclic coordinate method to adjust the solution for space limitation is proposed. A numerical example that illustrates the effectiveness of the method is provided. For future work, further test of the method on other products with various characteristics will be conducted.

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