SOLVE THE SERIES OF RLC CIRCUIT VIA THE TIME-DEPENDENT VOLTAGE SINE FUNCTION USING WRONSKIAN'S OF DIFFERENTIAL EQUATION

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Abstract

In this paper, we developed the mathematics and physics of the series of RLC circuits. The purpose of this study is to calculate for finding the time-dependent electric charge and the time-dependent energy static charge that because of time-dependent voltage which in sine function by using the second-order non-homogeneous linear ordinary differential equation and integration by part technique. We find that the charge corresponds to capacitance but inversely proportional to induction. The wave group of the time-dependent energy static charge depends on the ratio between resistance with double inductance and angular frequency parameter.

Keywords: Time-dependent energy static charge, Time-dependent electric charge, Time-dependent voltage

INTRODUCTION

One of the most familiar and important resonant systems is the time-dependent electrical system made up of the time-dependent voltage or the alternating current ($V(t)$), resistance (R), inductance (L), and capacitance (C) as shown in Figure 1. Mohazzabetal (2008) analyzes RLC response, which using the stage space method to model and that gives the effects of changing the resistances and capacitances are negligent systems and whereas the inductor output is changed, the parameter variation in the RLC includes the salient features of this algorithm. Sonam (2015) study the stability of the system using the bode plot method that is one of the convenient methods for stability calculation and also calculates phase margin, gain margin, phase crossover frequency, gain crossover at a time.

Dino (2019) study a systems theory perspective and very basic explanation explain the low–frequency hook which gives what has to happen phenomenological for this feature to appear. Ahammodullah et al. (2019) investigate the application of the RLC diagram in the catena of linear RLC closed series electric circuits. Using Kirchhoff's Voltage law solves the relevant second–order ordinary differential equations. So it gives the solution obtained was employed to procedure RLC diagram simulated by MATLAB and Mathematica 9.0.

The destination of this research, we can solve the series of RLC circuits in which the voltage is in sine function form and depends on time by using Wronskian's form of the differential equation. As the result gives us 2 cases of electric charge parameter and 2 cases of energy static charge parameter which the electric charge value corresponds to the capacitance but inversely proportional to induction, the energy static charge value corresponds to inductance but inversely proportional to the capacitance. Anywhere if the value of grammar (γ) is defined as the ratio between resistance with double inductance and angular frequency have slightly different values, the behavior of electric charge and energy static charge behave themselves like wave group. We have developed the series of RLC circuit of the time–dependent resonant system that can be used to effectively indicate new soft magnetic materials made possible the use of the time–dependent magnetic amplifier technology in styling competitive electric power supplies.

MATERIALS AND METHODS

The series of RLC circuits and the important resonant system is the time–dependent electrical system made up of the time–dependent voltage or the alternating current ($V(t)$), resistance (R), inductance (L), and capacitance (C) as shown in Figure 1.

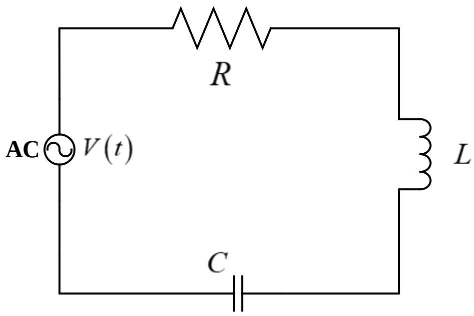


Figure 1 Illustration showing the series of RLC circuit with time–dependent voltage.

This is RLC circuit. The time-dependent voltage is in sine function form. To find the electric charge, we applied Kirchhoff's loop rule as shown in Equation (1).

$$V(t) = V_R + V_L + V_C \,, \tag{1}$$

where the voltage force $V(t)$ is $V_0 e^{-\lambda^2 t} \sin^2(\omega t)$, λ is a damping coefficient and ω is an angular frequency (Olutimo & Omoko, 2020). Substitute $V(t)$ into Equation (1), as shown in Equation (2) below.

$$V_0 e^{-\lambda^2 t} \sin^2(\omega t) = V_R + V_L + V_C \,. \tag{2}$$

From $V_R = IR$, $V_L = L(dI/dt)$ and $V_C = Q/C$, we may write the current $I = dQ/dt$, where Q is the electric charge in the loop, as shown in Equation (3). (Kishore, 2008)

$$\frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{1}{LC}Q = \frac{V_0}{L} e^{-\lambda^2 t} \sin^2(\omega t) \,. \tag{3}$$

We set 2γ is defined as R/L, ω^2 is defined as $1/LC$ and E_0 is defined as V_0/L, we may rewrite Equation (3) as

$$\frac{d^2Q}{dt^2} + 2\gamma\frac{dQ}{dt} + \omega^2 Q = E_0 e^{-\lambda^2 t} \sin^2(\omega t) \,. \tag{4}$$

This is a non-homogeneous, second-order, linear differential equation (Tikjha et al., 2018). The solution of Equation (4) can be found by summation of two parts of the time-dependent electric charge:

$$Q(t) = Q_C(t) + Q_P(t) \,, \tag{5}$$

where $Q_C(t)$ is the homogeneous equation's solution that gives

$$\frac{d^2Q}{dt^2} + 2\gamma\frac{dQ}{dt} + \omega^2 Q = 0 \tag{6}$$

Solve the solution by using the auxiliary equation, Equation (7) is auxiliary equation has the roots.

$$m^2 + 2\gamma m + \omega^2 = 0$$

$$m = -\gamma \pm \sqrt{\gamma^2 - \omega^2} \tag{7}$$

For this case, we considered the underdamped that in $\gamma < \omega$ case and it's convenient to make a substitution. We must have $m = -\gamma \pm i\omega_u$ Thus, the general solution of this homogeneous differential equation for Equation (6) with a and b is

$$Q_C(t) = e^{-\gamma t}(a\sin(\omega_u t) + b\cos(\omega_u t)) \,. \tag{8}$$

Then we find the constant number of a and b by using the initial conditions, $Q(0) = Q_0$ and $dQ(0)/dt = 0$. At first, we consider $Q(t)$ when t=0, so we get

$$Q_C(0) = b \tag{9}$$

Next, Consider derivative of charge depended on time term:

$$\frac{dQ_C(t)}{dt} = e^{-\gamma t}(a\omega_u \cos(\omega_u \mathrm{t}) - b\sin(\omega_u t)) - \gamma\,\mathrm{e}^{-\gamma t}(a\omega_u \sin(\omega_u \mathrm{t}) - b\cos(\omega_u t))$$

and $dQ(0)/dt = 0$ at $t = 0$. We must have

$$a = \frac{\gamma Q_0}{\omega_u} \,. \tag{10}$$

The solution of this homogeneous equation is

$$Q_C(t) = e^{-\gamma t}\left(\frac{\gamma Q_0}{\omega_u}\sin(\omega_u t) + Q_0\cos(\omega_u t)\right). \tag{11}$$

Then, suppose the particular solution $Q_P(t)$ of Equation (1) from the voltage by using Wronskian's method find the exponential and trigonometry solution by using this form.

$$Y_1(t) = \mathrm{e}^{-\gamma t}\sin(\omega_u t) \quad , \qquad Y_2(t) = \mathrm{e}^{-\gamma t}\cos(\omega_u t)$$

$$Y_1'(t) = \omega_u\,\mathrm{e}^{-\gamma t}\cos(\omega_u t) - \gamma\,\mathrm{e}^{-\gamma t}\sin(\omega_u t)\,,\; Y_2'(t) = -\omega_u\,\mathrm{e}^{-\gamma t}\sin(\omega_u t) - \gamma\,\mathrm{e}^{-\gamma t}\cos(\omega_u t)$$

and substitute in Wronskian's, we get

$$W = \begin{vmatrix} \mathrm{e}^{-\gamma t}\sin(\omega_u t) & \mathrm{e}^{-\gamma t}\cos(\omega_u t) \\ \omega_u\,\mathrm{e}^{-\gamma t}\cos(\omega_u t) - \gamma\,\mathrm{e}^{-\gamma t}\sin(\omega_u t) & -\omega_u\,\mathrm{e}^{-\gamma t}\sin(\omega_u \mathrm{t}) - \gamma\,\mathrm{e}^{-\gamma t}\cos(\omega_u t) \end{vmatrix}$$

$$W = -\omega_u\,\mathrm{e}^{-2\gamma t}\sin^2(\omega_u t) - \gamma\,\mathrm{e}^{-2\gamma t}\sin(\omega_u t)\cos(\omega_u t) - \omega_u\,\mathrm{e}^{-2\gamma t}\cos^2(\omega_u t) + \gamma\,\mathrm{e}^{-2\gamma t}\sin(\omega_u t)\cos(\omega_u t)$$

$$W = -\omega_u\,\mathrm{e}^{-2\gamma t} \tag{12}$$

Consider the first Wronskian's defined as W_1, we satisfy the solution as

$$W_1 = \begin{vmatrix} 0 & \mathrm{e}^{-\gamma t}\cos(\omega_u t) \\ \varepsilon_0 e^{-\lambda^2 t}\sin^2(\omega t) & -(\omega_u\,\mathrm{e}^{-\gamma t}\sin(\omega_u t) + \gamma\,\mathrm{e}^{-\gamma t}\cos(\omega_u t)) \end{vmatrix}$$

$$W_1 = -\varepsilon_0 e^{-(\lambda^2+\gamma)t}\sin^2(\omega t)\cos(\omega_u t) \tag{13}$$

Consider the second Wronskian's defined as W_2, we satisfy the solution as

$$W_2 = \begin{vmatrix} e^{-\gamma t}\sin(\omega_u t) & 0 \\ \omega_u\, e^{-\gamma t}\cos(\omega_u t) - \gamma\, e^{-\gamma t}\sin(\omega_u t) & \varepsilon_0 e^{-\lambda^2 t}\sin^2(\omega t) \end{vmatrix}$$

$$W_2 = \varepsilon_0 e^{-(\lambda^2+\gamma)t}\sin^2(\omega t)\sin(\omega_u t)\,. \tag{14}$$

We can satisfies the differential equation for Wronskian's as

$$\frac{du_1}{dt} = \frac{W_1}{W} = \frac{-\varepsilon_0 e^{-(\lambda^2+\gamma)t}\sin^2(\omega t)\cos(\omega_u t)}{-\omega_u\, e^{-2\gamma t}}\,.$$

So we set the parameter of $\delta = (\gamma - \lambda^2)$ as

$$\frac{du_1}{dt} = \frac{\varepsilon_0}{\omega_u} e^{\delta t}\sin^2(\omega t)\cos(\omega_u t)$$

$$\frac{du_1}{dt} = \frac{\varepsilon_0}{\omega_u} e^{\delta t}\left(\frac{1-\cos(2\omega t)}{2}\right)\cos(\omega_u t)\,. \tag{15}$$

We may write

$$u_1(\mathrm{t}) = \frac{\varepsilon_0}{2\omega_u}\left(\int e^{\delta t}\cos(\omega_u t)dt - \int e^{\delta t}\cos(2\omega t)\cos(\omega_u t)dt\right)\,. \tag{16}$$

The first integral function can be found by evaluation of the right-hand side of Equation (16) by using the integration by part technique

$$\int e^{\delta t}\cos(\omega_u t)dt = \frac{\omega_u e^{\delta t}\sin(\omega_u t) + \delta e^{\delta t}\cos(\omega_u t)}{(\omega_u{}^2 + \delta^2)}\,. \tag{17}$$

The second integral function can also be found by evaluation of the right-hand side of Equation (16) by using the integration by part technique. Therefore,

$$\int e^{\delta t}\cos(2\omega t)\cos(\omega_u t)dt = \frac{1}{2}\left[\frac{(2\omega+\omega_u)e^{\delta t}\sin((2\omega+\omega_u)t) + \delta e^{\delta t}\cos((2\omega+\omega_u)t)}{((2\omega+\omega_u)^2+\delta^2)}\right.$$
$$\left. + \frac{(2\omega-\omega_u)e^{\delta t}\sin((2\omega-\omega_u)t) + \delta e^{\delta t}\cos((2\omega-\omega_u)t)}{((2\omega-\omega_u)^2+\delta^2)}\right]. \tag{18}$$

From Equation (16), we will get

$$u_1(t) = \frac{\varepsilon_0}{2\omega_u}\left(\frac{\omega_u e^{\delta t}\sin(\omega_u t) + \delta e^{\delta t}\cos(\omega_u t)}{(\omega_u{}^2+\delta^2)} - \frac{(2\omega+\omega_u)e^{\delta t}\sin((2\omega+\omega_u)t) + \delta e^{\delta t}\cos((2\omega+\omega_u)t)}{((2\omega+\omega_u)^2+\delta^2)}\right.$$
$$\left. + \frac{(2\omega-\omega_u)e^{\delta t}\sin((2\omega-\omega_u)t) + \delta e^{\delta t}\cos((2\omega-\omega_u)t)}{((2\omega-\omega_u)^2+\delta^2)}\right) \tag{19}$$

The differential Equation for the second wronskian (Riley & Hobson, 2006) can be satisfied as

$$\frac{du_2}{dt}=\frac{W_2}{W}=\frac{-\varepsilon_0 e^{-(\lambda^2+\gamma)t}\sin^2(\omega t)\sin(\omega_u t)}{\omega_u\,\mathrm{e}^{-2\gamma t}}$$

or we may rewrite,

$$\frac{du_2}{dt}=\frac{-\varepsilon_0}{2\omega_u}e^{\delta t}(1-\cos(2\omega t))\sin(\omega_u t)\,.$$

Therefore,

$$u_2(t)=\frac{-\varepsilon_0}{\omega_u}\left(\int e^{\delta t}\sin(\omega_u t)dt-\int e^{\delta t}\cos(\omega t)\sin(\omega_u t)dt\right)\,. \qquad (20)$$

The first integral function of the right hand side in Equation (20) can be found by evaluation using integration by part technique.

$$\int e^{\delta t}\sin(\omega_u t)\,dt=\frac{\delta e^{\delta t}\sin(\omega_u t)-\omega_u e^{\delta t}\cos(\omega_u t)}{(\delta^2+\omega_u{}^2)}\,. \qquad (21)$$

Then, evaluate the second integral function of the right-hand side of Equation (20) by using the integration by part technique

$$\int e^{\delta t}\cos(2\omega t)\sin(\omega_u t)dt=\frac{\delta e^{\delta t}\sin(\alpha t)-\alpha e^{\delta t}\cos(\alpha t)}{2(\alpha^2+\delta^2)}-\frac{\delta e^{\delta t}\sin(\beta t)-\beta e^{\delta t}\cos(\beta t)}{2(\beta^2+\delta^2)} \qquad (22)$$

So we get the derivative of the second Wronskian's as

$$u_2(\mathrm{t})=\frac{-\varepsilon_0}{\omega_u}\left(\frac{\delta e^{\delta t}\sin(\omega_u t)-\omega_u e^{\delta t}\cos(\omega_u t)}{(\delta^2+\omega_u{}^2)}-\frac{\delta e^{\delta t}\sin(\alpha t)-\alpha e^{\delta t}\cos(\alpha t)}{2(\alpha^2+\delta^2)}+\frac{\delta e^{\delta t}\sin(\beta t)-\beta e^{\delta t}\cos(\beta t)}{2(\beta^2+\delta^2)}\right) \qquad (23)$$

The particular solution of Equation (4) with constant C is

$$\begin{aligned}Q_P(t)=\;&\frac{\varepsilon_0}{2\omega_u}e^{-\gamma t}\sin(\omega_u\,\mathrm{t})\left(\frac{\omega_u e^{\delta t}\sin(\omega_u t)+\delta e^{\delta t}\cos(\omega_u t)}{(\omega_u{}^2+\delta^2)}-\left(\frac{(2\omega+\omega_u)e^{\delta t}\sin((2\omega+\omega_u)t)+\delta e^{\delta t}\cos((2\omega+\omega_u)t)}{2((2\omega+\omega_u)^2+\delta^2)}\right)\right.\\ &\left.-\left(\frac{(2\omega-\omega_u)e^{\delta t}\sin((2\omega-\omega_u)t)+\delta e^{\delta t}\cos((2\omega-\omega_u)t)}{2((2\omega-\omega_u)^2+\delta^2)}\right)\right)-\frac{\varepsilon_0}{2\omega_u}e^{-\gamma t}\cos(\omega_u\,\mathrm{t})\left(\frac{\delta e^{\delta t}\sin(\omega_u t)-\omega_u e^{\delta t}\cos(\omega_u t)}{(\omega_u{}^2+\delta^2)}\right.\\ &-\left(\frac{\delta e^{\delta t}\sin((2\omega+\omega_u)t)-(2\omega+\omega_u)e^{\delta t}\cos((2\omega+\omega_u)t)}{2((2\omega+\omega_u)^2+\delta^2)}\right)\\ &\left.+\left(\frac{\delta e^{\delta t}\sin((2\omega-\omega_u)t)-(2\omega-\omega_u)e^{\delta t}\cos((2\omega-\omega_u)t)}{2((2\omega-\omega_u)^2+\delta^2)}\right)\right)+C\end{aligned} \qquad (24)$$

Let us find C by using the initial condition $Q_p(0) = Q_0$:

$$Q_p(0) = -\frac{\varepsilon_0}{2\omega_u}\left(-\frac{\omega_u}{(\omega_u{}^2+\delta^2)} + \frac{(2\omega+\omega_u)}{2((2\omega+\omega_u)^2+\delta^2)} - \frac{(2\omega-\omega_u)}{2((2\omega-\omega_u)^2+\delta^2)}\right) + C$$

$$C = Q_0 + \frac{\varepsilon_0}{2\omega_u}\left(-\frac{\omega_u}{(\omega_u{}^2+\delta^2)} + \frac{(2\omega+\omega_u)}{2((2\omega+\omega_u)^2+\delta^2)} - \frac{(2\omega-\omega_u)}{2((2\omega-\omega_u)^2+\delta^2)}\right) \quad (25)$$

Thus, the particular solution of Equation (4) is

$$\begin{aligned} Q_P(\mathrm{t}) = {} & \frac{\varepsilon_0}{2\omega_u} e^{-\gamma t}\sin(\omega_u\,\mathrm{t})\left(\frac{\omega_u e^{\delta t}\sin(\omega_u t) + \delta e^{\delta t}\cos(\omega_u t)}{(\omega_u{}^2+\delta^2)} - \left(\frac{(2\omega+\omega_u)e^{\delta t}\sin((2\omega+\omega_u)t) + \delta e^{\delta t}\cos((2\omega+\omega_u)t)}{2((2\omega+\omega_u)^2+\delta^2)}\right)\right. \\ & \left. - \left(\frac{(2\omega-\omega_u)e^{\delta t}\sin((2\omega-\omega_u)t) + \delta e^{\delta t}\cos((2\omega-\omega_u)t)}{2((2\omega-\omega_u)^2+\delta^2)}\right)\right) - \frac{\varepsilon_0}{2\omega_u} e^{-\gamma t}\cos(\omega_u\,\mathrm{t})\left(\frac{\delta e^{\delta t}\sin(\omega_u t) - \omega_u e^{\delta t}\cos(\omega_u t)}{(\omega_u{}^2+\delta^2)}\right. \\ & \left. - \left(\frac{\delta e^{\delta t}\sin((2\omega+\omega_u)t) - (2\omega+\omega_u)e^{\delta t}\cos((2\omega+\omega_u)t)}{2((2\omega+\omega_u)^2+\delta^2)}\right) + \left(\frac{\delta e^{\delta t}\sin((2\omega-\omega_u)t) - (2\omega-\omega_u)e^{\delta t}\cos((2\omega-\omega_u)t)}{2((2\omega-\omega_u)^2+\delta^2)}\right)\right) \\ & + Q_0 + \frac{\varepsilon_0}{2\omega_u}\left(-\frac{\omega_u}{(\omega_u{}^2+\delta^2)} + \frac{(2\omega+\omega_u)}{2((2\omega+\omega_u)^2+\delta^2)} - \frac{(2\omega-\omega_u)}{2((2\omega-\omega_u)^2+\delta^2)}\right) \end{aligned} \quad (26)$$

Then, we will get the solution of the electric charge,

$$\begin{aligned} Q(\mathrm{t}) = {} & e^{-\gamma t}\left[\frac{\gamma Q_0}{\omega_u}\sin(\omega_u\,\mathrm{t}) + Q_0\cos(\omega_u\,\mathrm{t})\right] + \frac{\varepsilon_0 e^{-\gamma t}\sin(\omega_u t)}{2\omega_u}\left(\frac{\omega_u e^{\delta t}\sin(\omega_u t) + \delta e^{\delta t}\cos(\omega_u t)}{(\omega_u{}^2+\delta^2)}\right. \\ & - \left(\frac{(2\omega+\omega_u)e^{\delta t}\sin((2\omega+\omega_u)t) + \delta e^{\delta t}\cos((2\omega+\omega_u)t)}{2((2\omega+\omega_u)^2+\delta^2)}\right) \\ & \left. - \left(\frac{(2\omega-\omega_u)e^{\delta t}\sin((2\omega-\omega_u)t) + \delta e^{\delta t}\cos((2\omega-\omega_u)t)}{2((2\omega-\omega_u)^2+\delta^2)}\right)\right) - \frac{\varepsilon_0 e^{-\gamma t}\cos(\omega_u\,\mathrm{t})}{2\omega_u} \\ & \left(\frac{\delta e^{\delta t}\sin(\omega_u t) - \omega_u e^{\delta t}\cos(\omega_u t)}{(\omega_u{}^2+\delta^2)} - \left(\frac{\delta e^{\delta t}\sin((2\omega+\omega_u)t) - (2\omega+\omega_u)e^{\delta t}\cos((2\omega+\omega_u)t)}{2((2\omega+\omega_u)^2+\delta^2)}\right)\right. \\ & \left. + \left(\frac{\delta e^{\delta t}\sin((2\omega-\omega_u)t) - (2\omega-\omega_u)e^{\delta t}\cos((2\omega-\omega_u)t)}{2((2\omega-\omega_u)^2+\delta^2)}\right)\right) + Q_0 \\ & + \frac{\varepsilon_0}{2\omega_u}\left(\frac{(2\omega+\omega_u)}{2((2\omega+\omega_u)^2+\delta^2)} - \frac{(2\omega-\omega_u)}{2((2\omega-\omega_u)^2+\delta^2)} - \frac{\omega_u}{(\omega_u{}^2+\delta^2)}\right) \end{aligned} \quad (27)$$

After that, we will calculate to find the energy static charge, in general

$$dw = \Delta V\, dQ = \frac{Q}{C}dQ\,, \qquad w = \int_0^Q \frac{Q}{C}dQ$$

The energy static charge in capacitors was stored by holding apart pairs of the opposite charges. The charge's mutual attraction stores potential energy into the capacitor. Thus, let us evaluate the energy value stored in the capacitor. We set the energy static charge as

$$U_{CC} = \frac{Q^2}{2C} \tag{28}$$

From Equation (27), we will get the equation of energy as

$$\begin{aligned}
U_{CC} = \frac{1}{2C}\Bigg[& e^{-\gamma t}\left[\frac{\gamma Q_0}{\omega_u}\sin(\omega_u\ \mathrm{t}) + Q_0\cos(\omega_u\ \mathrm{t})\right] + \frac{\varepsilon_0 e^{-\gamma t}\sin(\omega_u t)}{2\omega_u}\Bigg(\frac{\omega_u e^{\delta t}\sin(\omega_u t) + \delta e^{\delta t}\cos(\omega_u t)}{(\omega_u{}^2 + \delta^2)} \\
& -\left(\frac{(2\omega+\omega_u)e^{\delta t}\sin((2\omega+\omega_u)t) + \delta e^{\delta t}\cos((2\omega+\omega_u)t)}{2((2\omega+\omega_u)^2+\delta^2)}\right) \\
& -\left(\frac{(2\omega-\omega_u)e^{\delta t}\sin((2\omega-\omega_u)t) + \delta e^{\delta t}\cos((2\omega-\omega_u)t)}{2((2\omega-\omega_u)^2+\delta^2)}\right)\Bigg) - \frac{\varepsilon_0 e^{-\gamma t}\cos(\omega_u\ \mathrm{t})}{2\omega_u} \\
& \left(\frac{\delta e^{\delta t}\sin(\omega_u t) - \omega_u e^{\delta t}\cos(\omega_u t)}{(\omega_u{}^2+\delta^2)} - \left(\frac{\delta e^{\delta t}\sin((2\omega+\omega_u)t) - (2\omega+\omega_u)e^{\delta t}\cos((2\omega+\omega_u)t)}{2((2\omega+\omega_u)^2+\delta^2)}\right)\right. \\
& \left. + \frac{\varepsilon_0}{2\omega_u}\left(\frac{(2\omega+\omega_u)}{2((2\omega+\omega_u)^2+\delta^2)} - \frac{(2\omega-\omega_u)}{2((2\omega-\omega_u)^2+\delta^2)} - \frac{\omega_u}{(\omega_u{}^2+\delta^2)}\right)\right)\Bigg]^2
\end{aligned} \tag{29}$$

From Equation (27) and Equation (29) are the time-dependent charge and time-dependent energy static charge in order shown in program. Putting this into program Mathematica for plotting graph.

Case 1: time-dependent charge. $(\gamma << \omega)$

The graph that shown relation between electric charge has 2 cases which the capacitance (C_1 =40 μF , C_2 =80 μF , C_3 =120 μF , C_4 =160 μF) and the Inductance (L_1 =310 henry, L_2 =350 henry, L_3 =390 henry, L_4 =430 henry) are the early variant in Figure (2a) and Figure (2b) in order. The parameter of voltage force, inductance, initial charge, resistance, ω , λ , V_0 is the control variant (Teoh & Rahifa, 2018).

Case 2: time-dependent energy static charge. $(\gamma << \omega)$

The graph that shown relation between energy static charge has 2 cases which the capacitance (C_1 =40 μF , C_2 =80 μF , C_3 =120 μF , C_4 =160 μF) and the Inductance (L_1 =310 henry, L_2 =350 henry, L_3 =390 henry, L_4 =430 henry) are the early variant in

Figure (3a) and Figure (3b) in order. The parameter of voltage force, inductance, initial charge, resistance, ω, λ, V_0 is the control variant.

Case 3: time-dependent charge where γ and ω have slightly different value. $(\gamma < \omega)$

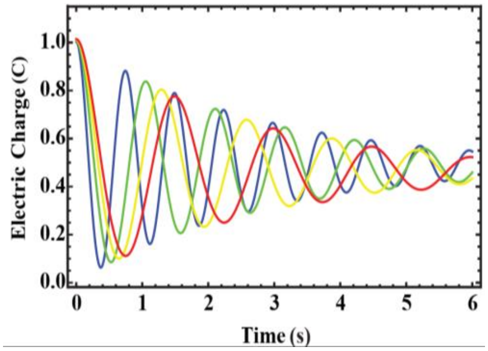
The graph that shown relation between energy static charge that behave themselves like wave group has 2 cases which the capacitance ((C_1 =420 μF , C_2 =422 μF , C_3 =424 μF , C_4 =426 μF) and the Inductance (L_1 = 1.102 henry, L_2 =1.104 henry, L_3 =1.108 henry, L_4 =1.012 henry)) are the early variant in Figure (4a) and Figure (4b) in order. The parameter of voltage force, inductance, initial charge, resistance, ω, λ, V_0 is the control variant.

Case 4: time-dependent energy static charge where γ and ω have slightly different value. $(\gamma < \omega)$

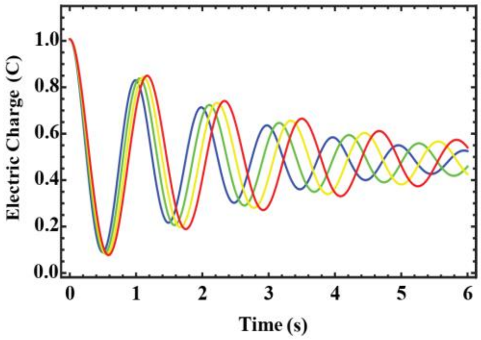
The graph that shown relation between energy static charge that behave themselves like wave group has 2 cases which the capacitance (C_1 =420 μF , C_2 =422 μF , C_3 =424 μF , C_4 =426 μF) and the Inductance (L_1 =1.102 henry, L_2 =1.104 henry, L_3 =1.108 henry, L_4 =1.012 henry)) are the early variant in Figure (5a) and Figure (5b) in order. The parameter of voltage force, inductance, initial charge, resistance, ω, λ, V_0 is the control variant.

RESULTS

We can describe of numerical and result of time-dependent electric charge as shown in Equation (27) which effects by sine voltage force as time-dependent charge are plotted in Figure 2. The time-dependent energy static charge of Equation (29) which effects by time-dependent voltage sine functions can be plotted in Figure 3. In this case, we can use the parameter value of $\gamma = R/2L$ is greater than the parameter value of ω $(\gamma << \omega)$.

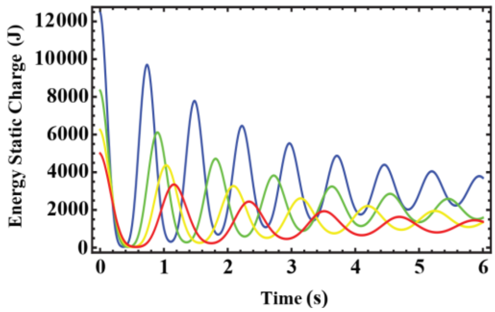


(a)

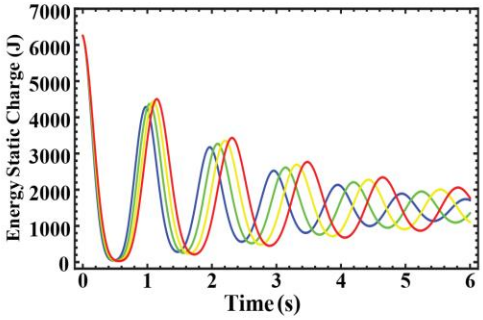


(b)

Figure 2 Illustration of the relative parameter between the time-dependent electric charge and time in Equation (27) versus the capacitor value (a) and the inductor value (b) $(\gamma << \omega)$



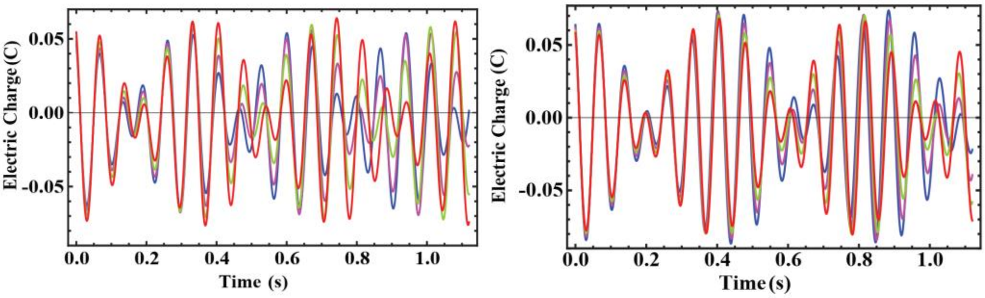
(a)



(b)

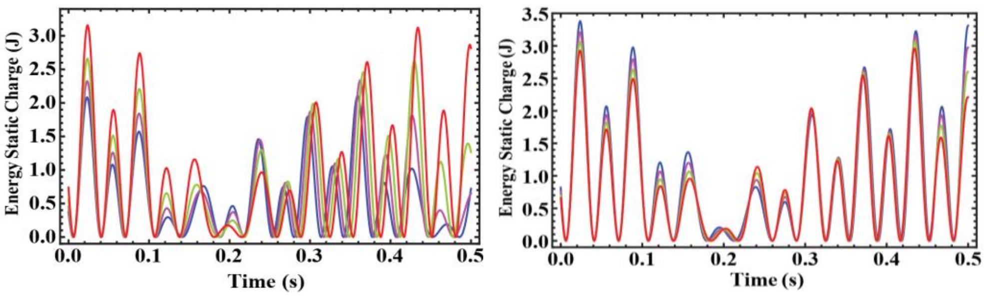
Figure 3 Plots of the time-dependent energy static charge in Equation (29) as a function of time showing versus the capacitor value (a) and the inductor value (b) $(\gamma << \omega)$

Finally, we can be represented plot of the graph relationship between of the time-dependent electric charge and time in the Figure 4. The time-dependent energy static charge and time can be plotted Figure 5. The parameter of the capacitor value and the parameter of the inductor value for studying the time-dependent electric charge was shown in Figure 4(a) and Figure 4(b), respectively. Figure 5(a) and Figure 5(b) shows the parameter of the capacitor value and the parameter of the inductor value for studying the time-dependent energy static charge, respectively.



(a) (b)

Figure 4 Plots of the time-dependent electric charge in Equation (27) as a function of time showing versus the capacitor value (a) and the inductor value (b) $(\gamma < \omega)$



(a) (b)

Figure 5 Illustration of plots of the graph time-dependent energy static charge in Equation (29) as a function of time versus the capacitor value (a) and the inductor value (b) $(\gamma < \omega)$

DISCUSSION

The Figure 2(a) is the underdamped model that shows the relation of parameter between the time-dependent electrics charge that γ and ω have more different value.We set the blue line as capacitor 40 μF, green line as capacitor 80 μF,yellow line as capacitor 120 μF and red line as capacitor 160 μF. You can see if we added the capacitor value, the electric wavelength increases but the amplitude would be decreased. From Figure 2(b) is the underdamped model that shows the relation between the time-dependent electric charge that γ and ω have more different value. We set the blue line as inductor 310 henry, green line as inductor 350 henry, yellow line as inductor 390 henry and red line as inductor 430 henry. You can see if we added the inductor value, the electric wavelength and amplitude would be increased. From Figure 3(b) is the underdamped model that shows the relation parameter between the time-dependent energy static charge that γ and ω have more different value. We set the blue line as capacitor 40 μF , green line as capacitor 80 μF ,yellow line as capacitor 120 μF and red line as capacitor 160 μF .You can see if we added the capacitor value, the electric wavelength would be increased but the amplitude would be decreased. From Figure 3(b) is the underdamped model that shows the relation parameter between energy the time-dependent static charge that γ and ω have more different value. We set the blue line as inductor 310 henry, green line as inductor 350 henry, yellow line as inductor 390 henry and red line as inductor 430 henry. You can see if we added the capacitor value, the amplitude and the electric wavelength would be increased.

From Figure 4(a), we can show the relation parameter between the time-dependent electrics charge that γ and ω have slightly different value. We set the blue line as capacitor 420 μF , green line as capacitor 422 μF ,yellow line as capacitor 424 μF and red line as capacitor 426 μF . You can see the charge has the behavior like wave group, if we added the capacitor value, the amplitude would be increased. From Figure 4(b), we can illustrate the relation parameter between the time-dependent electrics charge that γ and ω have slightly different value. We set the blue line as inductor 1.102 henry, green line as inductor 1.104 henry, yellow line as inductor 1.108 henry and red line as inductor 1.112 henry. You can see the charge hasthe behavior like wave group, if we added the inductor value, the amplitude would be decreased. From Figure 5(a), we can represent the relation parameter between energy the time-dependent static charge that γ and ω have slightly different value. We set the blue line as capacitor 420 μF , green line as capacitor 423 μF ,yellow line as capacitor 426 μF and red line as capacitor 429 μF . You can see the charge hasthe behavior like wave group, if we added the capacitor value, the amplitude would be increased.From Figure 5(b), we can illustrate the relation parameter between the time-dependent energy static charge that γ and ω have slightly different value. We set the blue line as inductor 1.102 henry, green line as inductor 1.104 henry, yellow line as inductor 1.108 henry and red line as inductor 1.112 henry. You can see the charge has the behavior like wave group, if we added the inductor value, the amplitude would decreased.We had developed an RLC circuit of resonant system that can be used to effectively demonstrate, in a very visual way, the phase relation between the voltages across reactive and resistive elements (Sokol et al., 2013). We have developed an RLC circuit of resonant system that can be used to effectively demonstrate new soft magnetic materials made possible the use of the magnetic amplifier technology in designing competitive electric power supplies (Nimaet al., 2019). You can see the electric charge has the behavior like wave group but the time-dependent electric charge in paper of AhammodullahHasan has not behavior like wave group, if we add the time, the amplitude slowly decreases.

CONCLUSIONS

We can be evaluated the time-dependent electric charge in the series of RLC circuit under the time-dependent voltage $V(t)=V_0 e^{-\lambda^2 t}\sin^2(\omega t)$. we may know the relation between electric charge depends on the time that the electric charge value corresponds to the capacitance but inversely proportional to induction and where γ and ω have slightly different value the electric charge behaves like a wave group that the electric charge value also corresponds to the capacitance. We can evaluate the time-dependent energy static charge in the RLC circuit loop under the time-dependent voltage $V(t)=V_0 e^{-\lambda^2 t}\sin^2(\omega t)$. we may know the relation between energy static charge depends on the time that the energy static charge value corresponds to inductance but inversely proportional to capacitance and where γ and ω have slightly different value the energy static charge behavior like a wave group that the amplitude of energy static charge value corresponds to the capacitance but inversely proportional to inductance.

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