

# Engineering Protocols for the Assessment of the Net Moisture Flux at the Ground Surface

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**ABSTRACT:** The assessment of moisture flux boundary conditions at the ground surface has proved to be important for the analysis of “real world” geotechnical engineering problem. There are several components that must be quantified in order to determine the net moisture flux entering the soil at the ground surface including: precipitation, runoff, actual evaporation and transpiration. Preferred methodologies are becoming apparent for calculating each of the components that lead to the calculation of the net moisture flux at the ground surface. The purpose of this paper is to set out general engineering protocols for the assessment of the net moisture flux at the ground surface. Examples are presented to illustrate the applications of moisture flux at the ground surface for geotechnical engineering problem; the examples include: i.) movement of slabs built on grade or at shallow depths below ground surface, ii.) triggering of slope instability as a results of water infiltration, and iii.) design and performance of soil cover systems.

## 1. INTRODUCTION

There are two types of boundary conditions that are commonly associated with water seepage problems in soil mechanics; namely, the Dirichlet type boundary condition (i.e., the primary variable specified is hydraulic head), and the Neumann type boundary condition (i.e., the derivative of the primary variable or the moisture flux is specified). Prior to the advent of the digital computer, the Neumann boundary condition was generally restricted to the condition of zero moisture flux (i.e., an impervious boundary). However, geotechnical engineers are well aware that the earth’s surface is subjected to continuously changing randomly distributed flux boundary conditions. This paper will primarily focus on the assessment of net moisture flux at the ground surface.

The advent of the digital computer has brought about a renewed awareness that many geotechnical engineering problems can be addressed in a more refined and accurate manner when the net moisture flux at the ground surface is quantified and used for analysis purposes. A number of typical geotechnical engineering examples are briefly described later in this paper. In each case, it becomes clear that the ability to quantify the net moisture flux at the ground surface opens the way for a more rigorous and accurate assessment of questions commonly poised to the geotechnical engineer.

forecasting; however, it has largely been a resource that has not been fully utilized for geotechnical engineering purposes.

This paper illustrates some of the ways in which the assessment of ground surface moisture flux boundary conditions provides a tool for the analysis of “real world” geotechnical engineering problems. The solution of these “real world” engineering problems generally involves the numerical modeling of saturated-unsaturated soil conditions. There are several components that must be quantified in order to determine the net moisture flux entering the soil at the ground surface (e.g., precipitation, runoff, actual evaporation and transpiration) as shown in Fig. 1. Preferred methodologies are becoming apparent for calculating each of the components that lead to the calculation of the net moisture flux at the ground surface. This paper sets out general engineering protocols for the assessment of the net moisture flux at the ground surface.

The scope of this paper is restricted to illustrating the quantification of net moisture flux at the ground surface and its application to several engineering problems. The principles described are applicable to a wide range of near-ground-surface geotechnical engineering problems.

## 2. BOUNDARY VALUE CONTEXT FOR SOLVING SOIL MECHANICS PROBLEMS

The analysis of most geotechnical engineering problems involving saturated-unsaturated soil systems can be formulated within the context of a “boundary value” problem (Fig. 2). A “boundary-value” context suggests that there are common elements involved in the solution of a wide variety of engineering problems. The “boundary value” approach suggests that geotechnical engineering problems can be solved provided appropriate measured and estimated information is input to the computer. The ground surface and the stratigraphic interfaces form the geometric boundaries for the problem at-hand. Usually there are also two vertical boundaries outside the immediate problem area being analyzed as well as a lower limit boundary.

The “boundary value” approach suggests that targeted physical processes within the boundaries can be studied provided the processes can be mathematically described. The physical processes are generally described in the form of a partial differential equation (PDE), derived for a Representative Elemental Volume (REV) within the soil continuum. It is also necessary to input saturated-unsaturated soil properties for each of the materials involved. Most analyses involving unsaturated soils are highly nonlinear and as a result, it is necessary to provide starting or initial state conditions for the problem being analyzed.

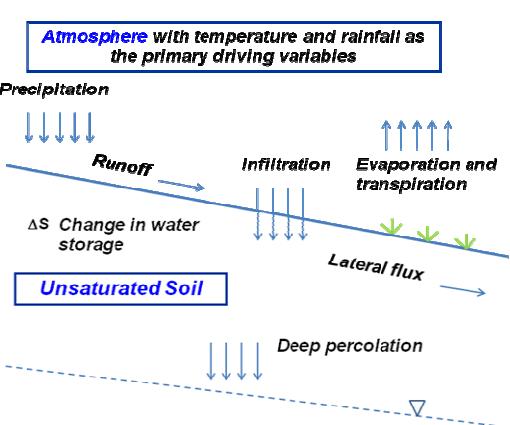


Figure 1. Primary components of a typical near-ground-surface geotechnical engineering problem

Thousands of weather stations around the world are collecting data relevant to energy and moisture transfer at the ground surface. The weather information provides the basic information necessary for the calculation of the net moisture flux at the ground surface. The weather data has become of great value for purposes of weather

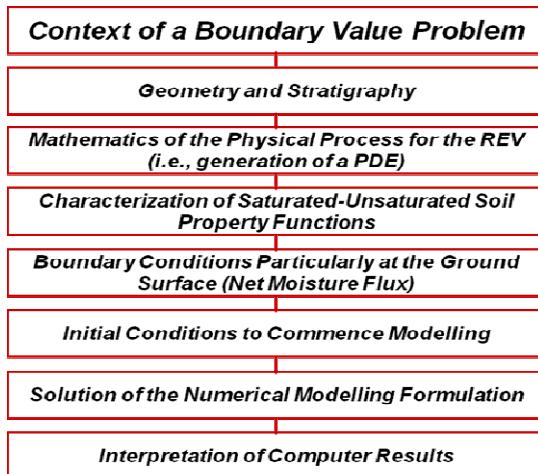


Figure 2. Steps associated with the solution of a “boundary value” problem

Most geotechnical engineering problems can be viewed as the solution of a partial differential equation. The solution of the PDE is performed using the finite element approach. The quantification of the net moisture flux at the ground surface constitutes information of primary importance. The climate parameters (e.g., precipitation, temperature, relative humidity, wind speed, and radiation) that contribute to the boundary condition at the ground surface are typically input as daily values. Assumptions must be made regarding the application of the climate data since the the variables being calculated are required on a finer time scale. For example, precipitation should be recorded on an hourly or sub-hourly basis in order to compute the separation between infiltration and runoff. Some present software packages can accept hourly (and sub-hourly) time data, leading to increased accuracy in performing the numerical simulations. Higher resolution data input leads to reduced convergence issues and increased accuracy in the computed results. The simulation time steps that are part of the numerical modeling can be fractions of a minute and the total time period may be in excess of 10 years. Consequently, each computer simulation may take considerable time to run.

The characterization of the unsaturated soil properties forms another important piece of input information. Research into the behavior of unsaturated soils over the past few decades has produced numerous procedures whereby saturated soil properties can be extended to embrace unsaturated soil behavior. This extension is generally accomplished through use of the relationship between the water content in the soil and soil suction known as the soil-water characteristic curve, SWCC. The unsaturated soil properties are generally estimated from the SWCC and take the form of nonlinear soil property functions. Nonlinearity of the soil properties, in turn, gives rise to nonlinearity in the numerical modeling process. Solving nonlinear PDEs brings along challenges associated with “convergence” of the solution.

### 3. GEOMETRY AND STRATIGRAPHY

The ground surface may be a natural terrain or a man-made ground surface. Most commonly, the ground surface has been controlled by the activities of humans as is the case for soil cover systems associated with mining activities and solid waste disposal. For example, the ground surface of mine waste materials is usually controlled by the waste disposal methodology. Tailings wastes may have a relatively flat surface whereas a waste rock pile often has steep side slopes. Steep side slopes make the problem more difficult to analyze. Both waste rock piles and tailings deposits are three-dimensional structures; however, design considerations are often limited to a one-dimensional analysis corresponding to a relatively flat surface. Two- and three-dimensional analyses are more realistic and may sometimes be used for the simulation of side-slope regions; however, the analyses may become extremely time-consuming.

The one-dimensional modeling of a cover system was originally solved using the SoilCover computer code (University of Saskatchewan, 2000) based on the Soil-Atmosphere formulation proposed by Wilson (1990). Two-dimensional analyses of covers on a sloping surface were later performed by Bussière and Aubertien (2003). And more recently, a quasi three-dimensional, net radiation approach has been developed by Weeks and Wilson (2005). Many geotechnical engineering problems can be solved using a one-dimensional analysis; however, there are situations where two- and three-dimensional analyses should be taken into consideration. The SVFlux software developed and maintained by SoilVision Systems Ltd., has the capability of solving one-, two-, and three-dimensional coupled heat and water mass flow problems. The recent studies by Weeks (2006) have shown that the computations of net evaporative flux from the soil surface can differ significantly depending upon the angle of the sun's rays and the orientation of the surface of the ground. Quasi three-dimensional analysis were performed by combining a large number of one-dimensional analyses into a network with only the net radiation being varied from one location to another.

The geotechnical engineer also needs to be aware that the ground surface may not be level and that there is a potential for runoff and “ponding”. The unevenness of the ground surface can also result from differential settlement of the underlying materials. The ground surface conditions might vary significantly from one location to another with the result that it is difficult to perform realistic moisture movement simulations by using a simple one-dimensional analysis.

### 4. THE PHYSICS OF SATURATED-UNSATURATED WATER FLOW FOR THE REPRESENTATIVE ELEMENTAL VOLUME

A Representative Elemental Volume (REV) must be selected within each of the continuum soil layers. It is necessary to mathematically describe the physics of saturated-unsaturated water flow through the REV while satisfying the conservation of mass requirement. The substitution of the constitutive behaviour for water flow and water storage, into the conservation of mass equation results in the derivation of a partial differential equation, PDE, for saturated-unsaturated seepage. The saturated-unsaturated water flow PDE can be written for the two-dimensional case as follows:

$$k_x^w \frac{\partial^2 h}{\partial x^2} + \frac{\partial k_x^w}{\partial x} \frac{\partial h}{\partial x} + k_y^w \frac{\partial^2 h}{\partial y^2} + \frac{\partial k_y^w}{\partial y} \frac{\partial h}{\partial y} = -m_2^w \gamma_w \frac{\partial h}{\partial t} \quad (1)$$

where  $h$  is the hydraulic head;  $k_x^w$  and  $k_y^w$  are the coefficients of permeability of the soil in  $x$ - and  $y$ - direction, respectively;  $m_2^w$  is the water storage;  $\gamma_w$  is unit weight of water; and  $t$  is time. Equation (1) is referred to as a “head based” formulation of the unsaturated seepage partial differential equation. The “mixed” formulation which designates water storage in terms of volumetric water content has been found to provide greater accuracy in terms of the calculations for water balance (Celia and Bouloutas, 1990).

The variable that must be determined from the PDE is the hydraulic head,  $h$ . In order to solve the PDE seepage equation it is necessary to have information on two soil properties; namely, the coefficient of permeability,  $k_w$ , of the soil and the water storage,  $m_2^w$ , of the soil. Unfortunately, both of the soil properties are nonlinear functions of the soil suction (Fredlund et al, 1994).

Equation (1) can be solved if the soil properties,  $k$  and  $m_2^w$  are known. However, these variables are a function of the pore-water pressure (or matric suction) in the soil. The pore-water pressure term of matric suction constitutes one component of hydraulic head,  $h$  (i.e.,  $h = u_w/\gamma_w + Y$  where  $u_w$  is the pore-water pressure, and  $Y$  is the elevation head). In other words, Eq. (1) has three unknowns and is nonlinear.

The nonlinearity requires that the soil properties first be estimated while the hydraulic heads are computed. Then the soil properties must be adjusted to obtain more reasonable values and the hydraulic heads are once again computed. This process is repeated until the equation has converged. Convergence means that reasonably accurate soil properties have been used when the hydraulic heads were computed.

The iterative process associated with solving the PDE may need to be repeated many times if the soil properties are highly nonlinear. It is also possible that the nonlinear PDE may never achieve convergence. It is also possible that even when the PDE has converged, the convergence may not correspond to the correct values for hydraulic head. Consequently, the solution of highly nonlinear PDEs has become an area of research in mathematics and computing science. Geotechnical engineers should be aware that the solution of highly nonlinear PDEs is a specialized area of study that is extremely relevant to solving unsaturated soils problems. Some software packages make use of PDE solvers that are specially designed for the solution of highly nonlinear PDEs. This constitutes an important feature when solving problems involving unsaturated soils.

The permeability and water storage functions are in reality more complex than what are shown in Fig. 3 since both functions exhibit hysteresis. There is actually one set of relationships corresponding to drying conditions and another set of conditions that apply for wetting conditions as shown in Fig. 4 (Pham et al, 2003). While hysteresis is known to exist in all soils, its effect is often not taken into account during computer simulations. This is just one of several approximations made in many design analyses associated with unsaturated soils.

The properties of the soil at the ground surface may change with time because of environmental influences. The soil may crack as a result of wetting/drying and freeze/thaw. Furthermore, the growth of vegetation creates a network of root holes, fissures and cracks. There may also be microbial contamination and other bio-intrusions that affect soil structure.

Changes in the soil structure can significantly change the soil-water characteristic curve (SWCC). Figure 5 illustrates the type of changes that might occur to the drying portion of a typical SWCC that contains clay. The SWCC may take on a bimodal character and the saturated hydraulic conductivity may increase by several orders of magnitude. Consequently, numerical modeling simulations based on the properties of originally intact materials can be considerably different from the soils that develop near the ground surface with time.

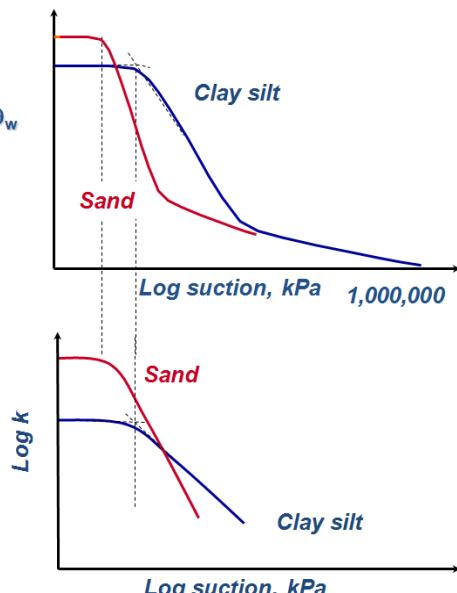


Figure 3. Typical soil-water characteristic curves (SWCCs) and permeability functions for two soils

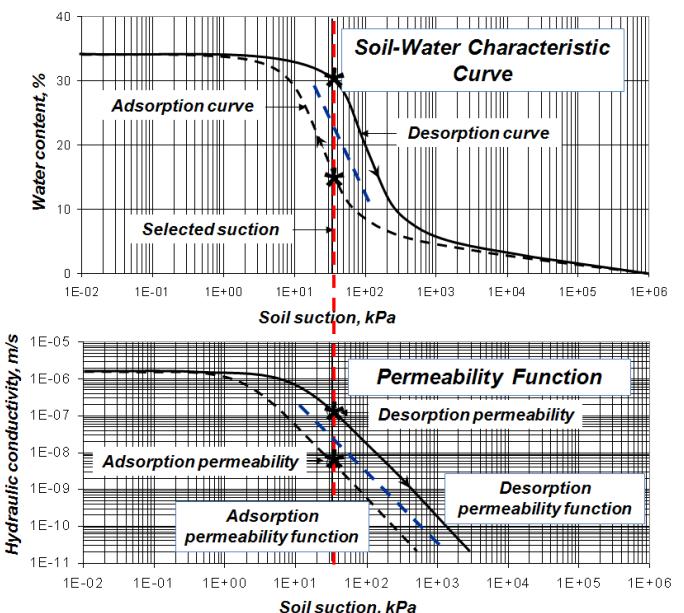


Figure 4. Effect of hysteresis upon drying and wetting of a soil

## 5. DETERMINATION OF THE SOIL-WATER CHARACTERISTIC CURVE

The SWCC can be defined as the relationship between the amount of water in a soil and the suction in the soil. There are two components to soil suction; namely, matric suction,  $(u_a - u_w)$ , where  $u_a$  is pore-air pressure and  $u_w$  is pore-water pressure, and osmotic suction,  $\pi$ . The sum of matric suction and osmotic suction is called total suction. There are two distinctive features of a SWCC; namely, the air entry value and the residual point. The air entry value designates the point at which the largest voids in the soil start to desaturate. The residual point is the point where it becomes extremely difficult to further extract water from the soil.

The SWCC is required for defining water storage and for the estimation of the permeability function for modelling water flow in a saturated/unsaturated soil system. The SWCC can either be estimated from soil classification properties or measured in the laboratory. The estimation of the SWCC is generally adequate for preliminary analysis, while the measured SWCC is required for detailed design of an engineering project. In general, only the drying curve (i.e., desorption curve) is measured or estimated. It is also possible to estimate a SWCC that is midway between the drying and wetting SWCCs.

There are three procedures that have been suggested for the estimation of an appropriate SWCC: i) through database mining of previously measured test results, ii) through estimation of the SWCC from grain-size distribution curves (Fredlund et al., 2002), and iii) from correlations with soil classification properties (Zapata et al., 2000).

There are a number of laboratory testing techniques that have been proposed and used for the measurement of the SWCC. The SWCC can be divided into two broad soil suction ranges; namely, the matric suction range with suctions less than 1500 kPa, and the total suction range with suctions greater than 1500 kPa. Consequently, the apparatuses used in the laboratory to measure the SWCC either apply matric suction or total suction and allow the soil to come to equilibrium with the applied suction value. The matric suction portion of the SWCC is measured by using pressure plate cells, while the total suction portion is usually measured using vacuum desiccators.

Figure 6 shows a disassembled Pressure Plate cell that was designed in the Golder office in Saskatoon, Canada (Golder, 2010). This new pressure plate cell is a significantly simplified device when compared to previously used Pressure Plate cells. The new cell has only two independent parts, and has a high design factor of

safety against breakage due to high air pressures. It is easy to operate and is less technician or operator dependent.

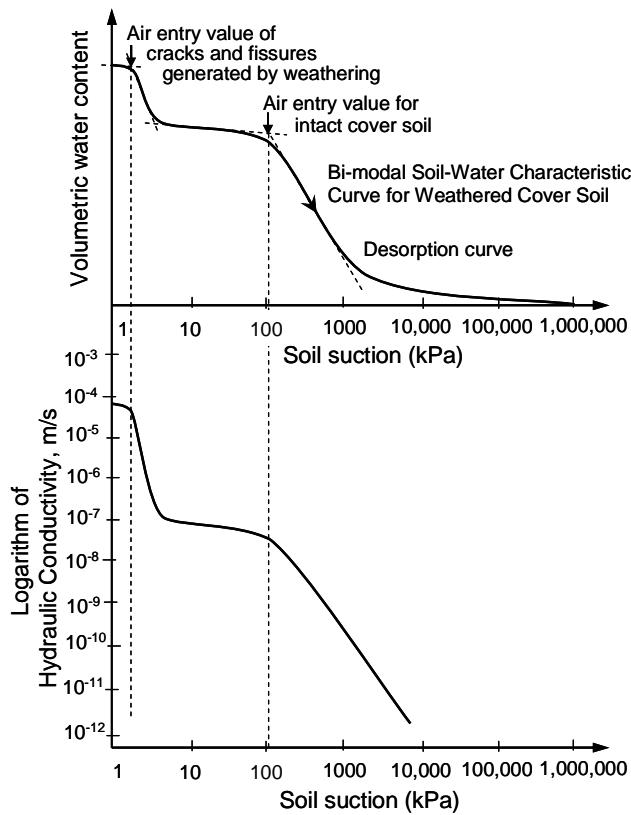


Figure 5. Effect of cracking that may occur as a result of weathering of near-surface soils



Figure 6. Pressure Plate Apparatus used at Golder Associates (Saskatoon) for the measurement of the Soil-Water Characteristic Curve, SWCC

Example laboratory results of measured SWCCs are shown in Fig. 7 for clay, silt and sand. These SWCCs were measured using the new Pressure Plate cell shown in Fig. 6. The measured data were best-fit using the Fredlund and Xing (1994) equation for the SWCC. The results show that the tested clay had an air entry value of about 150 kPa and a residual suction of about 20,000 kPa. The silt proved to be quite similar to the clay soil. The sand had an air entry value of 3.5 kPa and a residual suction of about 10 kPa. It should be noted that a ceramic disk of 500 kPa air entry value was used for the testing program. The SWCC portion in the high total suction range was not tested for these soils.

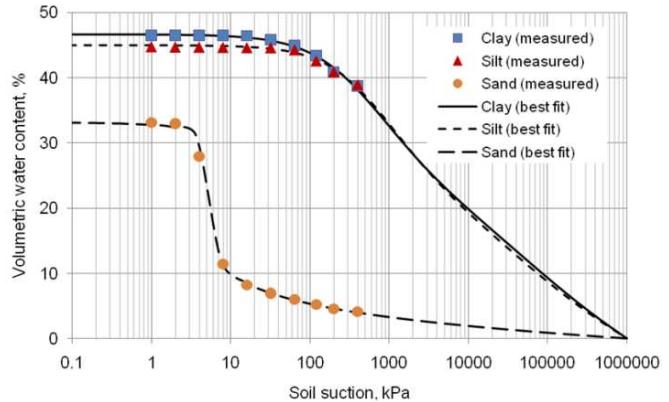


Figure 7. Typical drying SWCCs for three soils tested at Golder Associates (Saskatoon)

## 6. QUANTIFICATION OF THE GROUND SURFACE MOISTURE FLUX

Analyses to compute net moisture flux conditions at the ground surface were not part of historical soil mechanics. However, the calculation of ground surface moisture fluxes based on climatic data is now becoming an integral part of unsaturated soil mechanics developments. It should be noted that the calculation of net moisture flux at the ground surface involves numerous assumptions and approximations. Some of the inherent difficulties are mentioned in the following sections. Other factors such as the effects of freeze/thaw and wetting/drying are often not adequately taken into account during the analysis; however, their consideration is outside the scope of this paper.

The ground surface forms a flux boundary that interacts with the atmosphere. Water is either entering the ground surface boundary as a result of precipitation or it is leaving the ground surface through (actual) evaporation,  $AE$ , or transpiration,  $T$ . Water may also be shed across the ground surface through runoff,  $R$ , or intra-layer drainage. The components of moisture flux at the ground surface are described by the following equation.

$$\text{Net Infiltration } (I) = \text{Precipitation } (P) - \text{Actual Evaporation } (AE) - \text{Transpiration } (T) - \text{Runoff } (R) \quad (2)$$

Or in an abbreviated form, the net infiltration at ground surface can be written,

$$I = P - AE - T - R \quad (3)$$

The quantification of ground surface moisture flux conditions is a new analysis in soil mechanics. There has not been a long history of calculating ground surface moisture flux conditions because it is a complex problem and many assumptions must be made as part of the computational procedure. Considerable effort has been extended in trying to refine the calculations associated with determining actual evaporation,  $AE$ ; however, the runoff and the transpiration variables need to also be further studied.

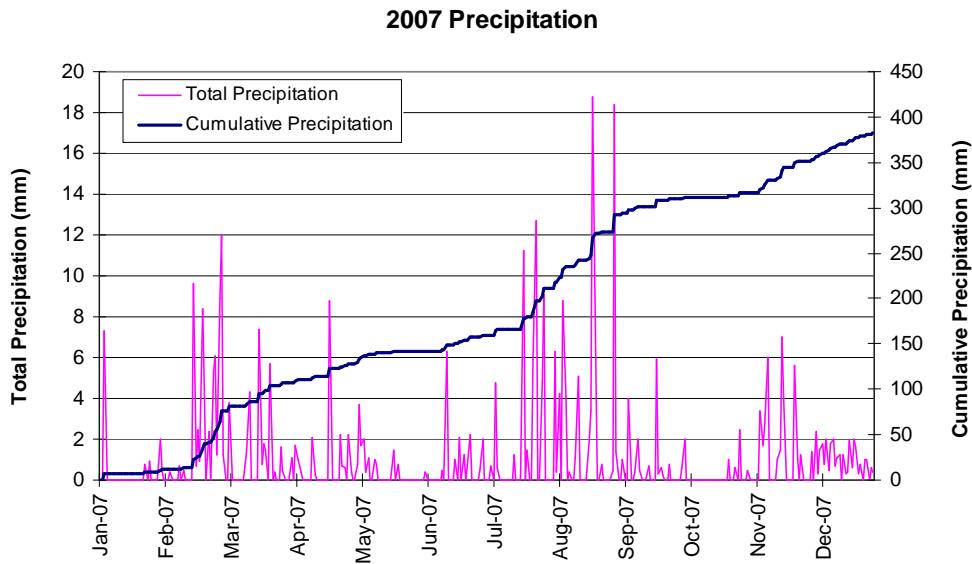


Figure 8. Typical weather station record showing the daily precipitation and the cumulative rainfall at a particular site in Canada

The physical processes associated with the determination of potential evaporation,  $PE$ , need to be fully understood prior to attempting to calculate actual evaporation,  $AE$ . Potential evaporation occurs from the ground surface when there is an ample supply of water while actual evaporation can be thought of as evaporation from a ground surface and transpiration from vegetation can be visualized as resisting or holding back evaporation. Consequently, actual evaporation requires that the effect of soil suction near ground surface be taken into account.

Each of the components contributing to net infiltration must be assessed in order to determine the moisture entering at ground surface. The components of net moisture flux are first discussed along with a brief description of the calculations and main assumptions required when performing calculations. The assessment of most variables related to net infiltration is made using average soil conditions and average imposed moisture flux loads. Temperature, relative humidity, wind speed and rainfall are usually the basic variables measured by an elementary weather station.

### 6.1 Precipitation

Precipitation can take the form of rainfall and snowfall. Its magnitude should be measured at or near the site under consideration. The daily measurements of precipitation may have been measured over a period of many years. Each year of data can be considered as an independent record and used as such for analysis purposes. An accumulated annual precipitation record can be plotted for each year (Fig. 8). The accumulated annual precipitation can take on a variety of shapes depending on the distribution of precipitation within the year as shown in Fig. 8. Even though the total precipitation in any two years might be the same, the response of the underlying soil may be quite different depending upon the distribution of precipitation throughout the year and the respective antecedent moisture conditions. Consequently, it is generally necessary to perform modelling simulations using several years of recorded climatic data.

An unsaturated soil can only accept water at a rate that is dependent mainly upon the hydraulic conductivity and water storage capabilities of the surface soil. It is possible for the surface soil to accept water at a rate in excess of the saturated hydraulic conductivity because of the effect of storage. However, it is likely that the intensity of rainfall during a storm may exceed the ability of the soil to accept water. When the intensity of rainfall exceeds the infiltration capacity at the ground surface, the remainder of the water becomes runoff or "ponding" on the ground surface.

The conventional collection of precipitation data often does not allow for the moisture flux variation during a storm event to be quantified. In other words, rainfall gauges are often set to measure

the precipitation once per day resulting in a situation where it is impossible to determine whether a storm was 10 minutes long or 10 hours long. A daily rainfall record will show all precipitation events as being spread out over most of the day and as a result the precipitation will appear to infiltrate the soil. The desire to reduce the data collection schedule to a daily resolution is most likely due to weaknesses in database systems used to manage weather station data. It should be noted, however, that it is possible to program some weather stations such that an hourly (or sub-hourly) record of rainfall intensity can be measured. Even if hourly records are kept for one year, these results provide valuable information for the quantification of potential runoff.

### 6.2 Runoff

Runoff can be calculated as the water that cannot gain entrance into the soil when it falls to the ground. The amount of moisture leaving the ground surface by actual evaporation must also be taken into account. As well, the slope of the ground surface must be taken into consideration when distributing the (vertical) rainfall onto a sloping surface. Figure 9 shows a simulation of infiltration and runoff performed using SVFlux software (SoilVision, 2005) (Gitirana Jr. et al, 2005).

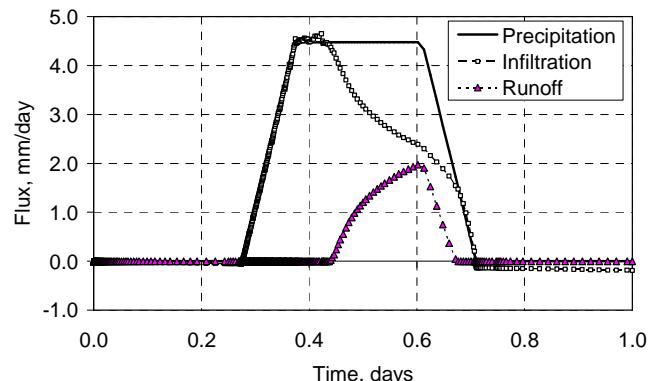


Figure 9. Illustration of the ability to simulate infiltration and runoff conditions

### 6.3 Potential Evaporation

The quantification of potential evaporation from the ground surface can be estimated using equations describing the effects of net radiation and "mixing". Numerous studies have been conducted since the 1920s with the intent of predicting "potential evaporation" from the ground surface. It is; however, the "actual evaporation" and

“evapo-transpiration” that are of primary interest in geotechnical and geo-environmental engineering.

Potential evaporation is the amount of water removed by the atmosphere through evaporation if water is freely available at the ground surface. In general, about 80% of the energy required for evaporation comes from the sun (in the form of net radiation) while wind (in the form of a mixing term) and the vapor deficit of the air forms a second important component contributing to evaporation. “Pan Evaporation” measurements (i.e., an open water surface) can also be used to measure “potential evaporation”.

Researchers have attempted to develop empirical, mathematical equations that embrace the primary variables controlling the rate of evaporation from a free water surface (i.e., potential evaporation). Each proposed “potential evaporation” equation uses specific weather-recorded data. The calculation of “potential evaporation” can be presented in units of mm/day. While the Thornthwaite (1948) equation is generally used to assess climatic conditions of aridity and humidity, it is the Penman (1948) equation that is generally used in geotechnical engineering for estimating potential evaporation.

Penman (1948) incorporated a number of variables commonly collected at weather stations (e.g., relative humidity, air temperature, wind speed, and net radiation) into the prediction of potential evaporation.

$$PE = \frac{\Gamma Q_n + \eta E_a}{\Gamma + \eta} \quad (4)$$

where:  $PE$  = potential evaporation in mm/day,  $\Gamma$  = slope of saturation vapour pressure vs. temperature curve, kPa/°C,  $Q_n$  = net radiation at the water (or saturated ground) surface, mm/day,  $\eta$  = psychrometric constant, kPa/°C,  $E_a$  =  $2.625(1 + 0.146W_w)$  ( $u_{v0}^{air} - u_v^{air}$ ), mm/day,  $W_w$  = wind speed, km/hr,  $u_v^{air}$  = vapour pressure in the air above the water (or saturated ground) surface, kPa, and  $u_{v0}^{air}$  = saturated vapour pressure at the mean air temperature, kPa.

The Penman equation shows that the vapour pressure gradient between the water surface and the air above the water becomes the primary driving mechanism for evaporation. There are two terms in the numerator of Eq. (4). The first term involving net radiation characterizes the power of the sun to evaporate water. Net radiation quantifies the net effect of short and long wave radiation from the sun, surface reflectance (albedo) and surface temperature. Net radiation values are not as commonly measured as other weather parameters and therefore, it is sometimes necessary to estimate net radiation values based on the latitude of the site under consideration as well as other variables. The second term involves “mixing” of the air above the water or the drying power of the air.

The vapour pressure in the air above the water and the saturated vapour pressure at the water surface are the dominant variables driving evaporation. The saturated vapour pressure is a function of temperature while the actual vapour pressure in the air is related to the relative humidity. The two variables on the bottom of the Penman equation are also related to vapour pressure.

When solving the Penman (1948) equation, it is necessary to know the minimum and maximum values for variables such as temperature and relative humidity for each day. An assumption can then be applied with regard to the variation of these variables throughout a 24 hour period.

The potential evaporation,  $PE$ , calculation provides the engineer with an understanding of the maximum evaporation that could occur from a water saturated surface. In the case of a soil at ground surface, the soil may be holding onto the water while the sun and wind are attempting to pull the water upward. The “struggle” between the climate and the soil gives rise to a lower actual evaporation,  $AE$ , from the ground surface.

#### 6.4 Actual Evaporation

The actual evaporation,  $AE$ , from a soil surface might be considerably less than the potential evaporation,  $PE$ . The

geotechnical engineer is most interested in calculating actual evaporation,  $AE$ , in order to compute the water-balances (or net infiltration) at the ground surface. Two equations are presented that can be used for calculating actual evaporation,  $AE$ , from a soil surface under varying soil suction conditions. Both equations are the outcome of research by Wilson (1990) who used evaporation from thin soil layers and sand column drying tests to verify the fundamental physical relationships used to extend the Penman (1948) equation for the calculation of actual evaporation,  $AE$ .

Wilson's (1990) first proposed equation takes the form of a modified Penman equation. The modification takes into consideration the reduced relative humidity (i.e., vapour pressure in the soil at ground surface), in the denominator of the Penman-Wilson equation (Wilson et al, 1994, 1997).

$$AE = \frac{\Gamma Q_n + \eta E_a}{\Gamma + \eta A} \quad (5)$$

where:  $AE$  = actual evaporation in mm/day,  $E_a = 0.35(1 + 0.15W_w)u_v^{air} \left( \frac{u_{v0}^{air}}{u_v^{air}} - \frac{u_{v0}^{air}}{u_v^{soil}} \right)$ , mm/day,  $u_v^{air}$  = water

vapour pressure in the air above ground surface, mm Hg,  $u_{v0}^{air}$  = saturated vapour pressure at the mean air temperature, mm Hg,  $u_v^{soil}$  = vapour pressure in the soil at ground surface, mm Hg,  $\Gamma$  = slope of saturation vapour pressure versus temperature curve, mm Hg/°C,  $Q_n$  = net radiation at the water surface, mm/day,  $\eta$  = psychrometric constant, mm Hg/°C,  $W_w$  = wind speed, km/hr. The relative humidity in the soil at ground surface,  $h_r$  is equal to  $u_v^{soil} / u_{v0}^{air}$  and the relative humidity of the air,  $h_{air}$  is equal to  $u_v^{air} / u_{v0}^{air}$ .

The ratio of actual evaporation to potential evaporation,  $AE/PE$ , can be understood using the thermodynamic equilibrium relationship between relative humidity and negative pore-water pressure (or total suction) (Edlefsen and Anderson, 1943).

$$\frac{u_v^{soil}}{u_v^{air}} = -\exp\left(\frac{u_w v_{wo} \omega_v}{\rho_w g R T}\right) \quad (6)$$

where:  $h_r$  = relative humidity in the unsaturated soil voids,  $u_{v0}^{air}$  = saturated air vapour pressure, kPa,  $u_v^{soil}$  = vapour pressure in the soil at ground surface, kPa,  $u_w$  = pore-water pressure, kPa,  $\omega_v$  = molecular weight of water, 0.018 kg/mol,  $v_{wo}$  = specific volume of water,  $g$  = gravity acceleration, m/s,  $T$  = temperature, °K. Equation [6] can be re-arranged and used to compute the vapour pressure in the soil at ground surface.

$$u_v^{soil} = u_v^{air} e^{\left(\frac{u_w v_{wo} \omega_v}{\rho_w g R T}\right)} \quad (7)$$

Another equation was proposed by Wilson et al (1994, 1997) for calculating actual evaporation,  $AE$ . The equation takes the form of a “limiting function” between zero and potential evaporation depending on the vapor pressure in the soil at ground surface. The  $AE$  is scaled in accordance with Lord Kelvin's equation. The “limiting function” equation is written as follows.

$$AE = PE \frac{u_v - u_v^{air}}{u_{v0} - u_v^{air}} \quad (8)$$

where:  $AE$  = actual evaporation in mm/day,  $PE$  = potential evaporation in mm/day,  $u_v$  = actual vapour pressure at the soil surface,  $u_{v0}$  = saturated vapour pressure at the soil surface temperature, and  $u_v^{air}$  = vapour pressure in the air above the soil

surface. Assuming that the air, water and soil temperatures are approximately equal allows temperature to cancel and Eq. (8) to be written in terms of the relative vapour pressure (i.e., relative humidity) of the air above the evaporating soil and water surfaces and Lord Kelvin's total potential equation.

$$AE = PE \frac{-\exp\left(\frac{u_w V_{wo} \omega_v}{\rho_w g R T}\right) - \frac{u_v^{air}}{u_{vo}^{air}}}{1 - \frac{u_v^{air}}{u_{vo}^{air}}} \quad (9)$$

where:  $u_v^{air} / u_{vo}^{air}$  = relative vapour pressure (or relative humidity,  $h_{air}$ ) of the air. Soil suction at the ground surface is obtained by combining either Eq. (8) or (9) with the partial differential equation that models liquid and vapour flow in the soil. The combined solution is referred to as a "soil-atmospheric model".

Wilson (1990) developed a Soil-Atmospheric model that combines heat and mass transport in the soil near to the ground surface and Lord Kelvin's equation relating vapour pressure to total suction. The water flow partial differential equation (i.e., liquid and vapour flow) predicts the total soil suctions at the ground surface. The total suction predictions then make use of Lord Kelvin's equation to yield the relative humidity (i.e., soil vapour pressure) at the ground surface. The vapour pressure in the soil provides an indication of the tenacity with which the soil is holding onto the water. Actual evaporation,  $AE$ , from the ground surface starts to be noticeably reduced from potential evaporation,  $PE$  when the soil suction in the soil at ground surface becomes greater than about 3000 kPa.

Wilson (1990) showed that it is the soil suction at the ground surface that primarily controls the actual rate of evaporation. Consequently, the soil type at ground surface is not a controlling factor when assessing the actual rate of evaporation (Fig. 10).

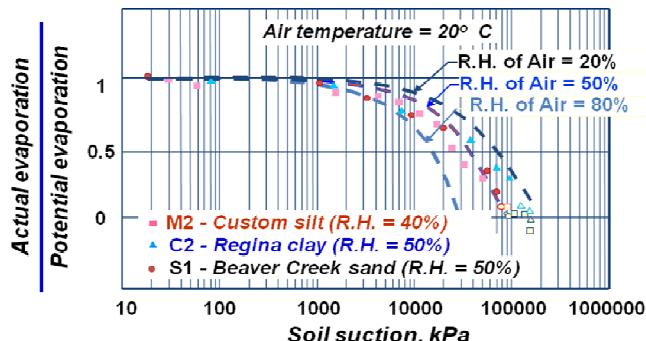


Figure 10. Actual evaporation rate from sand, silt or clay soil surfaces as a function of the soil suction at ground surface

## 6.5 Transpiration

Plants can be viewed as small pumps that remove water more efficiently from the soil than can be done through evaporation from the soil surface (Tratch et al, 1995) (Figure 11). Transpiration from plants can be considerably higher than actual evaporation. Therefore, it is important to take the ground surface vegetation into consideration. However, experientially the effect of vegetation has proven to be quite difficult to determine. Evapotranspiration is primarily a function of the root uptake zone and the leaf area index, LAI, of the plants. The growing season for the vegetation must be assumed and nutrients must be available in the soil to sustain plant growth. The long-term sustainability of plant growth has also proven to be a problem in some situations.

Numerical modelers are called upon to make numerous assumptions with regard to vegetation effects and these assumptions can have a significant effect on the outcome of the analysis. It is fair to say that more research is necessary with regard to the characterization of the effects of vegetation and how the results

should be incorporated into a vegetation moisture flux model (Tratch et al, 1995).

## Principles of Transpiration

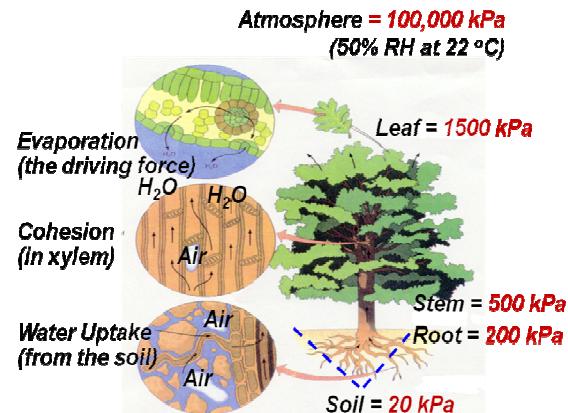


Figure 11. Concepts associated with transpiration from vegetation

## 7. NET MOISTURE FLUX AT THE GROUND SURFACE

Each of the components that influence the net moisture flux at the ground surface has been described. Once the information related to the net moisture flux at the ground surface boundary is complete, then it is possible to proceed with the calculations of infiltration of water into the soil. However, it needs to be understood that the above-mentioned calculations for moisture flux are not independent of modeling soil infiltration. The actual evaporation,  $AE$ , is dependent upon knowing the total suction at ground surface. Actual evaporation is computed in the infiltration model and as a result there is a "coupling effect" between the infiltration model and the calculation of the moisture flux boundary conditions. Stated another way, the calculations combine the climatic ground surface moisture flux conditions with the solution of the nonlinear partial differential equation of unsaturated soil seepage. The combination of the unsaturated soil moisture flow and the climatic boundary conditions is called a "soil-atmospheric model".

The soil-atmospheric model will need to be solved on an elapsed time scale that might be in the order of a few minutes. Each day is modelled and the time scale is continued for the entire year. However, one year may not be sufficient for design of the cover system. Rather, it may be necessary to perform these calculations for as much as 10 years or more. Needless to say, near-ground-surface simulations of moisture flow are computationally demanding. As well, the high nonlinearity of the partial differential moisture flow equation makes convergence of the solution a challenge.

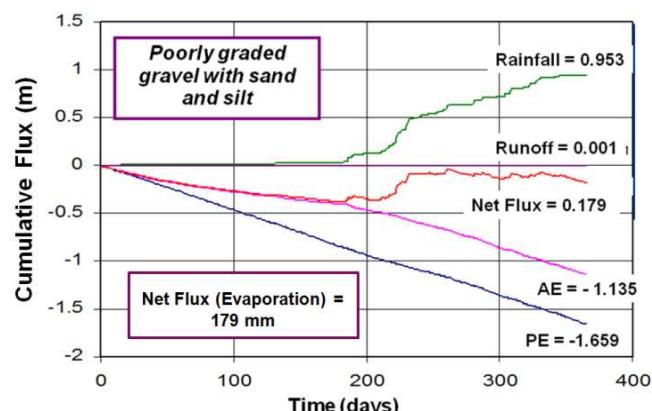


Figure 12. Net infiltration computed after the ground surface moisture flux has been applied to the soil-atmospheric model

Figure 12 shows the cumulative effects of precipitation, actual evaporation, and runoff for a portion of one year. The net effect is

called "net flux" or "net infiltration" at the soil surface. The magnitude of "net infiltration" provides an indication of the amount of water that is likely to pass below ground surface into the underlying materials. There are many assumptions and calculations that have gone into the calculation of infiltration.

## 8. EXAMPLES OF NEAR-GROUND-SURFACE SOIL MECHANICS PROBLEMS

The primary factor influencing the long-term performance of engineered structures is changes in soil suction of near-ground-surface soils. Soils change volume and shear strength as a result of changes in the net infiltration or the net moisture flux at the ground surface. There are a wide range of applications that can be considered; however, mention will only be made of a few examples such as: i.) the movement of slabs built on grade or at shallow depths below ground surface, ii.) the triggering of slope instability as a result of moisture infiltration, and iii.) the design and performance of soil cover systems.

### 8.1 Example No. 1 Slab-on-Ground

The movement of slabs-on-ground results in enormous cost to households in many countries of the world. It is often a soil mechanics problem that consulting engineers desire to avoid because of the high risk. However, the analytical tools are now available to perform numerous computer simulations of conditions that could occur. Figure 13 shows a slab-on-ground that is subjected to continuously changing environmental conditions. The edge of the slab moves upward as the underlying soil swells and moves downward when the underlying soil shrinks or dries out. Consequently, the concrete slab is subjected to a bending moment that can produce cracking at some distance from the edge of the slab. The variables required for the design of the slab are: i.) the possible amount of upward and downward movement, and ii.) the distance from the edge of the slab where movement is likely to cease.

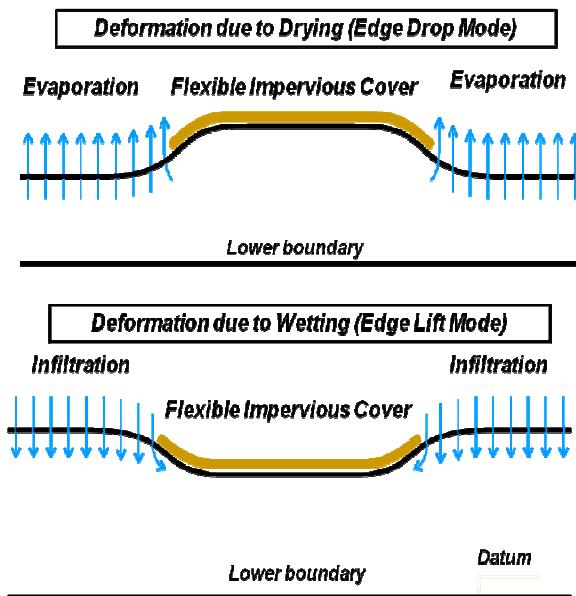


Figure 13. Illustration of the soil response to external loads and changes in matric suctions (after Post-tensioning Institute, 1996)

The engineering design solution involves the simulation of two physical processes; namely, i.) the movement of water in or out of the soil underlying the edge of the slab, and ii.) the stress-deformation modelling of soil movement as the stress state in the soil changes. Consequently, there are two partial differential equations that need to be solved in order to predict the movements that are likely to occur in the soils underlying the slab. The entire process is driven by the climatic and ground surface conditions surrounding the slab-on-ground.

Figure 14 presents a hypothetical case of a slab-on-grade used to illustrate the response of the system to the environmental changes (Fredlund and Vu, 2004).

Figure 15 shows a plot of matric suction profiles at the edge of the slab for various time when an upward moisture flux (i.e., the evaporation) of 10 mm/day was applied at the uncovered ground surface. Most of suction change took place near ground surface, and advanced deeper with time.

Vertical displacements versus depth are presented in Figure 16 for various times after the evaporation commences. Most of the settlements took place near ground surface where the change in matric suction is large. Figure 17 presents the vertical displacement at ground surface for various times after the evaporation commences. Differential settlement took place near the edge of the slab. This example problem illustrated that the predicted response of the soil moisture flux boundary conditions are consistent with those generally observed in the field.

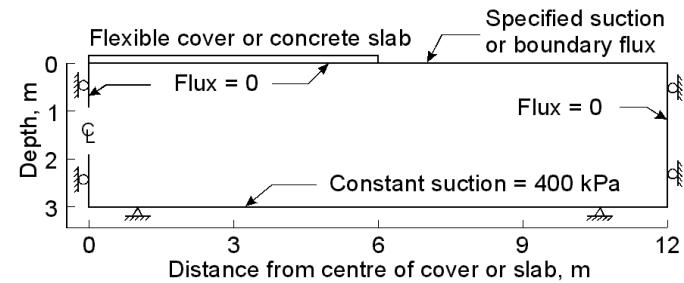


Figure 14. Illustration of the slab-on-ground example problem, boundary conditions for seepage and stress-deformation analysis

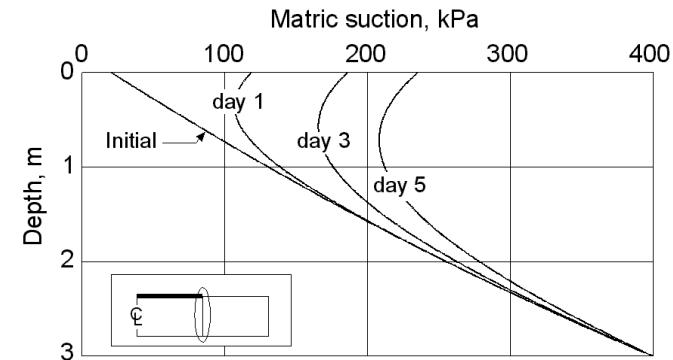


Figure 15. Matric suction profile at the edge of the slab for various elapsed times of evaporation

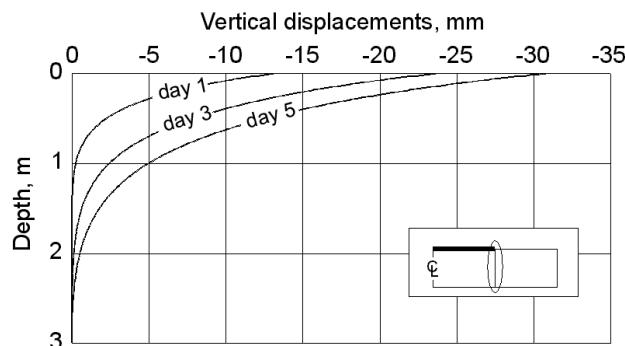


Figure 16. Vertical displacements (i.e., settlements) versus depth at the edge of the slab for various elapsed times of evaporation.

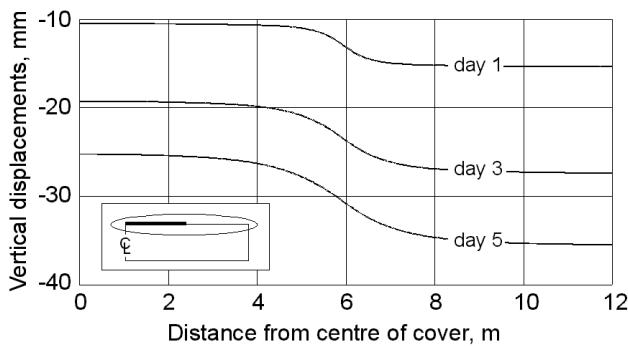


Figure 17. Vertical displacements at ground surface for various elapsed times of evaporation

## 8.2 Example No. 2 A Derailment Caused by a Prolonged Rainfall Event

This example problem shows the results of a post-incident numerical investigation associated with a train derailment that occurred after an extended period of rainfall at a site in Alberta, Canada. The trigger mechanism for the derailment was related to the net infiltration of water at ground surface (Vu et al., 2005).

The embankment of the derailment's location is about 3 m high and has 2:1 horizontal to vertical slope. The weather conditions leading up to the derailment were analysed. Figure 18 presents the 10 day variations of precipitation intensities at four weather stations near the derailment site. A comparison to climatic normal indicated above average rainfall and snowfall in the month of April for most stations. The results of climate evaluations at the site suggested that direct infiltration into the subgrade soil occurred prior to the derailment event.

The matric suction conditions and flow patterns within the railway embankment play an important role in the performance of the embankment. Increased water content and decreased matric suction, reduces shear strength and increase compressibility of the subgrade material. As a result, the railway embankment becomes less stable and the rail deflection becomes greater under train loading. Not all subgrade deflection is elastic and recoverable; a portion is plastic and leads to cumulative permanent settlement of the track. Different rail settlement is a factor that contributes to an uneven track.

Saturated/unsaturated flow modelling of the derailment section was undertaken to evaluate the potential impact that intensity and duration of the rainfall events could have on the subgrade suction conditions. Soil suction is an important variable in consideration of slope stability, bearing capacity and stress deformation conditions under train loading.

Figure 19 shows the changes in soil suction in the soil profile with time. It can be seen that the first two days of infiltration had little effect, but then the wetted front migrated relatively quickly into the subgrade, reaching a depth of 0.75 m into the subgrade on day 3 and 1.5 m on day 5. This 1.5 m depth is sufficient to accommodate development of a bearing capacity failure in the subgrade.

Figure 20 presents a summary of the bearing capacity analysis, stress deformation analysis, and slope stability analysis that were conducted for the site. As shown in this figure, the embankment failure was a result of reduced subgrade strength caused by the prolonged infiltration and repetitive dynamic train loading. Infiltration into the subgrade softened the embankment by reducing soil suction in the soils. The reduction of soil suction reduced the bearing capacity of the subgrade, reduced the overall slope factor of safety, and increased deformation of the rail-track system. Uneven track produced large deformation of the track system, resulting in a high impact factor under dynamic train loading.

Softening of the embankment subgrade would start approximately 2 days after the infiltration event commenced and would become pronounced 5 days after the infiltration started. The degree of subgrade softening would increase with time during the

precipitation process. Prolonged and heavy precipitation was the critical condition leading to instability of the track structure.

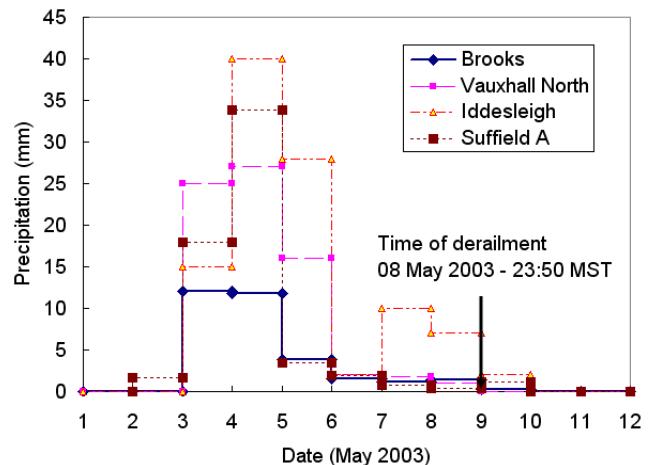


Figure 18. Variation of precipitation near the derailment site in May 2003

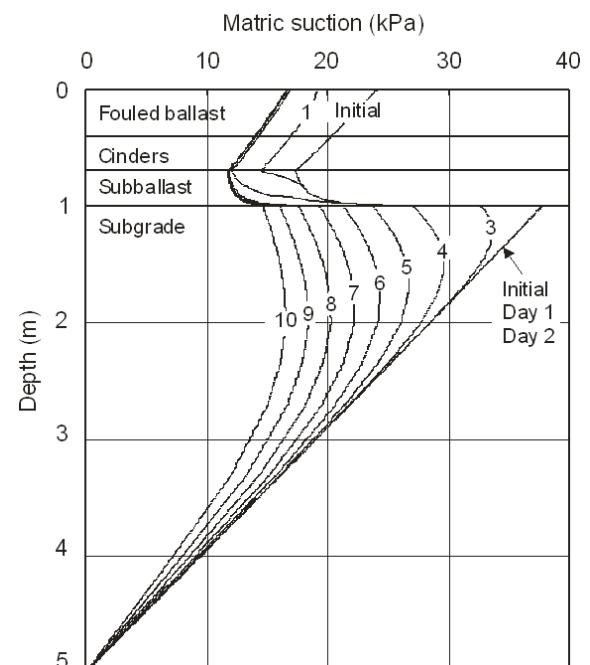


Figure 19. Distribution of suction versus depth with time after the commencement of infiltration

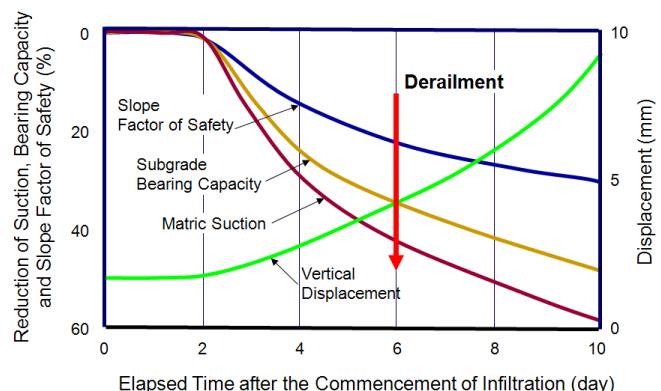


Figure 20. Illustration of the effect of infiltration to the soil suction, subgrade bearing capacity, vertical displacement (i.e., settlement) and slope factor of safety of the railway embankment

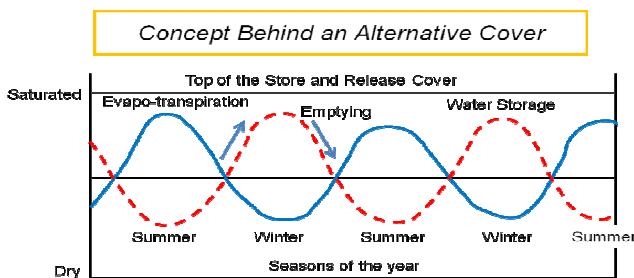


Figure 21. Concept of "Store and Release" used in the design of Alternative Cover systems (after Shackelford, 2005)

### 8.3 Example No. 3 Store and Release Soil Cover System

The engineering design of a "Store and Release" soil cover system involved the application of unsaturated soil mechanics principles (Fig. 21). A cover can change its degree of saturation with time and function in a manner that compensates for environmental fluctuations. Reductions in the degree of saturation reduce the coefficient of permeability of the cover system as long as the surface soil does not crack due to desiccation. The reduction in degree of saturation increases the storage capacity of the cover soil. The intent is for the cover to buffer the extreme climate forcing factors by storing water during wet periods and releasing it back to the atmosphere during dry periods.

The covers can consist of a variety of soil types and often make use of sand and silt soils. The covers are designed on the basis of water storage and water release (i.e., a water balance design). There must be sufficient capability for the annual precipitation to be removed from the cover on an annual basis. In other words, the cover must be in an area that tends towards being arid. However, an arid environment is not a sufficient criterion. The cover design must also take into consideration the distribution of precipitation throughout the year as well as the distribution of the thermal energy required to drive evapotranspiration. Stated another way, the cover material must be able to provide sufficient water storage capacity and water release capacity to accommodate the climatic weather patterns that are likely to be imposed on the cover at any time of any year.

It is necessary to test the functionality of the cover by subjecting the proposed design to several years of past climatic conditions. The cover may be subjected to 10 or more years of simulations through use of past climatic record data. The computer simulations may be reduced to time steps in the order of minutes and as a result the analysis becomes computationally intensive. There are also other factors that make the design analysis demanding and these will be later discussed. While the concept of "Storing" water and "Releasing water" throughout the year is simple, the analysis becomes dependent upon the assessment of many variables as well as several nonlinear unsaturated soil property functions.

## 9. SUMMARY

There are many assumptions that need to be made as part of the analysis and design procedure for engineered structures that are close to the ground surface. The soil conditions can change with time due to the effects of weathering with the result that the soil properties become far from the initially measured or assumed values. The changes can prove to differ by orders of magnitude from initial compacted or placement conditions. This does not make a realistic design impossible but simply shows that much greater care and detail must be given to the assessment of the unsaturated soil properties.

The climatic quantification that provides the "net moisture flux" at ground surface has utilized many broad assumptions. The tendency may be to focus the analysis on average conditions; however, the engineer needs to understand that it may be extreme weather conditions that may have the greatest effect over time. Extreme events may also lead to other processes such as erosion during significant water runoff.

The effect of cracks forming in soils near to the ground surface can significantly change the response of the surface soils to infiltration and exfiltration. Unsaturated soil properties are highly nonlinear and may even change to be bilinear in character. These extreme conditions need to be given more attention and may even turn out to constitute a controlling factor.

Modeling ground surface moisture flux conditions has proven to be one of the most challenging analytical procedures in soil mechanics. However, the benefits associated with analyzing ground surface moisture flux problems have proven to be of great value in geotechnical engineering practice. While great strides have been made in analyzing moisture flux problems there needs to be increased verification and monitoring studies in increase the engineer's confidence in the analyses being performed.

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