

Case Studies on Response of Laterally Loaded Nonlinear Piles

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ABSTRACT: Closed-form solutions for laterally loaded free and fixed-head piles in elastic-plastic media have been developed and implemented into a spreadsheet program called GASLFP. Underpinned by a generic limiting force profile (LFP), the solutions offer an expeditious and sufficiently accurate method for predicting response of lateral piles. They also allow parameters to be deduced using measured pile response, as is evident in the study to date on ~70 test (elastic) piles. The solutions also well capture the impact of structure nonlinearity of pile body by employing reduced pile bending stiffness ($E_p I_p$). The law of the reduction in $E_p I_p$, however, needs to be verified. In this paper, the solutions are utilised to examine the response of four test-piles exhibiting structural nonlinearity. For each nonlinear concrete pile, the shear modulus of soil and the three parameters (of N_g , α_o and n) for constructing the LFP were deduced first using elastic pile assumption; and subsequently the variations of flexural rigidity with bending moment, the cracking moment M_{cr} and the ultimate moment M_{ult} were back-figured against measured pile deflection. The study provides parameters for modelling nonlinear piles in sand and clay, and justification on using pertinent expressions.

1. INTRODUCTION

A free-head pile is subjected to a lateral load P_t at an eccentricity e (thus a moment, $M_t = P_t e$) above ground level (GL), as schematically shown in Figure 1(a). The pile-head is free to rotate and translate with no constraint imposed at the pile head and along the effective pile length except for soil resistance. A number of approaches based on p-y concept were developed to capture the behavior of elastic pile (McClelland and Focht 1958). Guo (2001) proposed to model the pile-soil interaction at each depth x (below the GL) by a spring in series with a slider characterized by an idealized elastic-plastic p-y curve (p = net resistance per unit length, y = deflection along the pile axis) [see Figure 1(b)]. The spring has a subgrade modulus, k (i.e. the slope of the p-y curve), and the slider provides a limiting force per unit length p_u for the interaction [see Figure 1(c)]. The resistance p normally reaches the p_u from the GL and gradually extends to a depth called slip depth, x_p with increasing loading. Above the depth, the pile-soil interaction is in plastic state; whereas below the slip depth, it is in elastic state. The elastic zone of the pile is modelled by the independent springs, and a fictitious membrane (with a constant fictitious tension, N_p) to incorporate the coupled effect among springs (Guo and Lee 2001). The plastic zone is captured by the p_u , as the tension is negligible ($N_p = 0$) (i.e. an uncoupled load transfer model).

The subgrade modulus k and the fictitious tension, N_p are theoretically correlated with Young's modulus of an equivalent solid pile, E_p [$=E_p I_p / (\pi d^4 / 64)$] and shear modulus of soil, G_s , in which $E_p I_p$

and d = flexural rigidity, and outside diameter of the pile, respectively (Guo and Lee 2001). The limiting force per unit length p_u is well captured by (Guo 2001) (later referred to as Guo LFP).

$$p_u = A_L (\alpha_o + x)^n \quad (1)$$

where p_u = limiting force per unit length [FL^{-1}]; $A_L = s_u N_g d^{1-n}$ (clay), and $\gamma_s N_g d^{2-n}$ (sand), gradient of the LFP [FL^{-1-n}]; α_o = a constant to include the force at the GL [L]; x = depth below the GL [L]; n = a power to the sum of α_o and x ; N_g = limiting force factor; S_u = undrained shear strength of cohesive soil [FL^{-2}]; and γ_s = effective unit weight of the soil [FL^{-3}]. Guo (2006) shows (1) $G_s = (0.25 \sim 0.62)N$ (MPa) ($\tilde{G}_s = 0.5N$ MPa, note the bar denotes average; and N = blow counts of SPT); and $\alpha_o = 0$, $n = 1.7$ and $N_g = (0.4 \sim 2.5)K_p^2$ with respect to a uniform sand profile; in which $K_p = \tan^2 (45^\circ + \phi_s/2)$; and ϕ_s = effective friction angle of the soil (2) $G_s = (25 \sim 315)s_u$ ($\tilde{G}_s = 92.3s_u$); and $n = 0.7$, $\alpha_o = 0.05 \sim 0.2$ m ($\tilde{\alpha}_o = 0.11$ m), and $N_g = 0.6 \sim 3.2$ ($\tilde{N}_g = 1.6$) for a uniform clay profile. And (3) Use of an average G_s over a depth of $10d$, and the construction of an average LFP for a layered soil profile.

Other typical LFPs are defined previously (Guo 2006), such as Broms LFP (Broms 1964), Reese LFP for cohesionless soil (Reese et al. 1974), Matlock LFP for cohesive soil (Matlock 1970), and Hansen LFP for subsoil using cohesion c and ϕ_s' (Brinch Hansen 1961), which will be used directly herein.

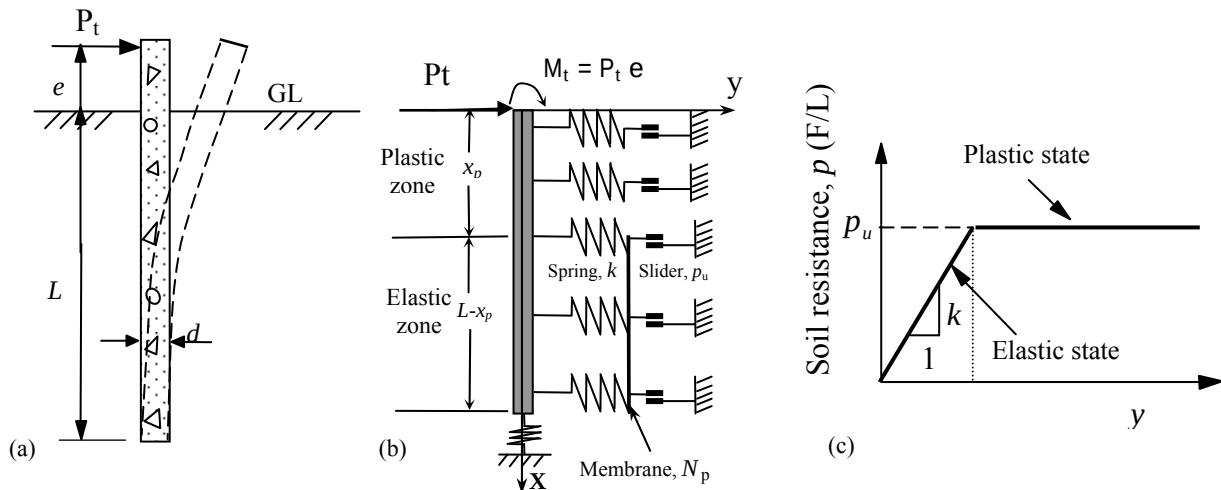


Figure 1 Coupled load transfer analysis for a free-head pile (Guo, 2001a, 2006): (a) The problem addressed, (b) Coupled load transfer model, and (c) Load transfer (p-y) curve

In light of the 'Guo LFP', elastic-plastic, closed-form (CF) solutions were developed for laterally loaded piles (Guo 2006). They encompass explicit expressions for estimating pile deflection, slope, bending moment, shear force and soil resistance, as functions of reciprocal of characteristic length of the pile $\lambda = \{[k/(4E_p I_p)]^{0.25}\}$ with $k/G_s = 2.4 \sim 3.92$, the slip depth x_p and the parameters A_L (or N_g), α_o , and n (for the LFP). They are readily estimated using a spreadsheet program called GASLFP, and are provided in simple closed-form expressions for $\alpha_o = 0$ and $N_p = 0$. For instance, ignoring ground level resistance (i.e. $\alpha_o = 0$), and using an uncoupled model ($N_p = 0$), the pile deflection at GL y_o is given by

$$\begin{aligned} \frac{y_o k \lambda^n}{A_L} = & \frac{2}{3} \bar{x}_p^{n+3} \frac{2\bar{x}_p^2 + (2n+10)\bar{x}_p + n^2 + 9n + 20}{(\bar{x}_p + 1 + \bar{e})(n+2)(n+4)} \\ & + \frac{(2\bar{x}_p^2 + 2\bar{x}_p + 1)\bar{x}_p^n}{\bar{x}_p + 1 + \bar{e}} + \frac{4}{3} \bar{P}_t \bar{e}^3 \\ & + \frac{2\bar{x}_p^4 + (n+4)(\bar{x}_p + 1)[2\bar{x}_p^2 + (n+1)(\bar{x}_p + 1)]}{(\bar{x}_p + 1 + \bar{e})(n+1)(n+4)} \bar{e} \bar{x}_p^n \end{aligned} \quad (2)$$

where $\bar{x}_p = \lambda x_p$, $\bar{e} = \lambda e$, and $\bar{P}_t = P_t \lambda^{1+n}/A_L$. The solutions and the program GASLFP well capture the nonlinear response of laterally loaded (elastic) piles.

Conversely, the program also allows the values of A_L (or N_g), α_o , n , and G_s to be deduced by matching the predicted with measured pile response, such as maximum bending moment (M_{max}), pile deflection y_o , and so forth under a spectrum of load P_t . The solutions are developed for a pile with a length $L > L_{cr}$ [$= 1.05d(E_p/G_s)^{0.25}$] in elastic zone; otherwise the solutions for short rigid piles (Guo 2008) should be adopted. Note the G_s may be taken as the average G_s over a depth of $10d$. GASLFP is used to conduct current investigation. The solutions (and programs) also allow response of nonlinear piles to be captured through reducing pile bending stiffness after crack (normally seen in concrete piles at a large deflection), as outlined subsequently.

2. MODELLING STRUCTURE NONLINEARITY

Tensile crack may be developed at a large deflection of a laterally loaded concrete pile, once the tensile stress at the extreme fibre reaching the modulus of rupture f_r . Section analysis of a beam allows the modulus f_r to be correlated with cracking moment, M_{cr} (upon which cracks is initiated) by:

$$M_{cr} = f_r I_g / y_r = k_r \sqrt{f'_c} I_g / y_r \quad (3)$$

where $k_r = 19.7 \sim 31.5$ for a normal weight concrete beam (ACI. 1993), and $16.7 \sim 62.7$ for lateral piles (revealed later); y_r = distance from centroidal axis of gross section to extreme fibres in tension; I_g = moment of inertia of gross section about centroidal axis neglecting reinforcement; f'_c = characteristic value of the compressive strength of concrete, in kPa. The f'_c and E_c may be empirically correlated by $E_c = 151,000(f'_c)^{0.5}$ (kPa) (ACI. 1993), in absence of experimental data.

The crack occurs in a pile at the depth of M_{max} once $M_{max} > M_{cr}$, while other regions may still be intact. This will render reduction in the flexural rigidity of the pile from the elastic $EI (= E_c I_g)$ to a new effective bending rigidity $E_p I_p (= E_c I_e)$ until a final cracked rigidity $(EI)_{cr} (= E_c I_{cr})$. Note I_{cr} = moment of inertia of cracked sections, at which fictitious hinges (thus ultimate bending moment, M_{ult}) takes place. The EI (or $E_p I_p$) represents the product of the modulus of elasticity of concrete, E_c and an effective moment of inertia, I_g (or I_e). The reduction in I_e with increase in M_{max} is currently based on the correlation proposed for beams (ACI. 1993), which may be written as

$$E_p I_p = \left(\frac{M_{cr}}{M_{max}} \right)^3 EI + \left[1 - \left(\frac{M_{cr}}{M_{max}} \right)^3 \right] (EI)_{cr} \quad (4)$$

Equation (4) is used to deduce Young's modulus of a pile, E_p [$= E_c I_e / (\pi d^4 / 64)$, or $E_c I_e / (bh^3 / 12)$], for entire pile body, regardless of the position and scale of cracks. Replacing E_p in the elastic-pile solutions with this new E_p allows (structurally) nonlinear pile response to be captured using the same solutions for elastic piles (Guo 2006). The M_{max} may be gained using elastic-pile (EI) analysis (Guo and Zhu 2011) (but for piles with an extremely high EI), as it is only slightly affected by development of crack. The new $E_p I_p$ for each M_{max} (or load) may be calculated using equation (4), together with M_{cr} from equation (3), and $(EI)_{cr}$ from rectangular block stress method (RBS) (Guo and Zhu 2011).

The solutions of elastic pile [e.g. equation (1)] well capture the impact of pile structure nonlinearity (Guo & Zhu 2011) via reducing the bending stiffness $E_p I_p$. The law of the reduction is examined herein against measured responses of structurally nonlinear piles (Nakai and Kishida 1982; Reese 1997; Huang et al. 2001; Ng et al. 2001; Zhang 2003) using GASLFP.

3. ANALYSIS OF FOUR NONLINEAR PILES

In this study, the M_{cr} (thus k_r) and $E_p I_p$ are deduced using GASLFP and measured pile response, as are parameters G_s and LFP for linear piles. Six piles were studied with input and deduced parameters being summarized in Tables 1, 2 and 3. The measured M_{cr} defines the start of deviation between the predicted $P_t - y_o$ curve using EI and the measured data. Thereafter, the $E_p I_p$ is reduced to fit the predicted with the measured $P_t - y_o$ curve until the final cracked rigidity $(EI)_{cr}$. The back-figured k_r is used to validate equation (3). The deduced values of $E_p I_p$ at various levels of M_{max} along with M_{cr} and $(EI)_{cr}$ are adopted to justify equation (4).

3.1 Case SN1 in Fine Sand

A bored pile B7 (termed as Case SN1) was instrumented with strain gauges and inclinometers, and tested under a lateral load applied near the GL ($e = 0$) (Huang et al. 2001). The soil profile at the site consisted of fine sand (SM) or silt (ML), and occasional silty clay. The ground water is located 1 m below the GL. The soil (to a depth of 15 m below the GL) had an average SPT blow count, $\tilde{N}$ of 16.9, a friction angle ϕ_s of 32.6°, and an effective unit weight, γ_s of 10 kN/m³. Thereafter, the SPT gradually increases from 25 (at a depth of 15 m) to 75 (at 80 m). The pile was 34.9 m in length, 1.5 m in diameter, and reinforced with 52 rebars (each had $d = 32$ mm and a yield strength f_y of 471 MPa). It had a concrete cover t of 50 mm thick with a compressive strength f'_c of 27.5 MPa. The EI and I_g were estimated as 6.86 GN-m², and 0.2485 m⁴ respectively, and thus $E_p = 27.6$ GPa. The cracking moment M_{cr} was estimated as 1.08~1.73 MN-m using equation (3), $k_r = 19.7 \sim 31.5$ and $y_r = d/2 = 0.75$ m. A ultimate moment M_{ult} of 8.77 MN-m, and $(EI)_{cr}$ of 1.021 GN-m² [i.e. $(EI)_{cr}/EI = 0.149$] were estimated using the RSB method.

Assuming an elastic pile, the response (with EI) was predicted using: (1) $n = 1.7$, $\alpha_o = 0$ and $N_g = 10.02$ ($= 0.9K_p^2$), deduced from the DMT tests (Huang et al., 2001), which gives the Guo LFP in Figure 2(a); and (2) $G_s = 10.8$ MPa ($= 0.64N$) (Guo 2006). The predicted load P_t - deflection y_o curve (denoted by EI) is shown in Figure 2(b), along with measured data, which indicates a cracking load P_t of 1.25 MN; and an ultimate capacity of 2.943 MN (indicating by fast increase in deflection). Thereby it follows: (1) A measured moment M_{cr} of 3.45 MN-m (at 1.25 MN) and $k_r = 62.7$ (i.e. 2~3 times that suggested by ACI). And (2) A measured M_{ult} of 10.5 MN-m (at 2943 kN). The $E_p I_p$ deduced by fitting the measured $P_t - y_o$ curve gives $(EI)_{cr}/EI = 0.55$, and $(EI)_{cr} = 3.773$ GN-m².

Table 1 Summary of information about piles investigated

Case	Reference	Pile details						
		d (m)	L (m)	EI (GN-m ²)	e (m)	f'_c (MPa)	f_y (MPa)	t (mm)
SN1	Huang et al. (2001)	1.5	34.9	6.86	0	27.5	471	50
SN2 ^c	Huang et al. (2001)	0.8	34.0	0.79	0	78.5 (20.6) ^a	1226 (471)	30
SN3	Ng et al. (2001)	1.5	28	10	0.75	49	460	
SN4 ^b	Zhang (2003)	0.86	51.1	47.67	0	43.4	460	75
CN1 ^c	Nakai & Kishida (1977)	1.548	30	16.68	0.5	153.7		
CN2		1.2	9.5	2.54	0.35	27.5		

^a: 78.5 is for the outer prestressed pipe pile and 20.6 for the infill;^b: 0.86×2.8 (m²) for a rectangular pile; ^c: Guo and Zhu (2012)

Table 2 Nonlinear properties of piles investigated

Cases	References	M_{cr} from ACI mtd (MN-m) (1)	M_{cr} deduced (MN-m) (2)	k_r derived from (2)	M_{ult} (MN-m)	$(EI)_{cr}/EI$ (%)	L_c/d
SN1	Huang et al. (2001)	1.08~1.73	3.45	62.7	8.77	14.8	8.9
SN2 ^a	Huang et al. (2001)	0.28~0.44	0.465	33.0	1.89	14.6	9.8
SN3	Ng et al. (2001)	1.44~2.31	3.34	45.5	4.0	40	9.9
SN4	Zhang (2003)	4.61~7.38	7.42	31.7	10.13	1.9	27.7
CN1 ^a	Nakai & Kishida (1977)	2.81~4.50	2.38	16.7		30	13.3
CN2		0.55~0.89	0.63	22.3		18	7.1

^a: Guo and Zhu (2012)

Table 3 Sand properties and derived parameters for piles in sand

Case	Soil type	Soil properties				Input parameters		
		γ'_s (kN/m ³)	ϕ_s (°)	N	D_r (%)	N_g/K_p^2	G/N (MPa)	n
SN1	Submerged silty sand	10	32.6	16.9		0.9	0.64	$\alpha_0 = 0$ $n = 1.7$
SN2 ^a	Submerged silty sand	10	32.6	16.9		1.0	0.64	
SN3	Submerged sand	11.9	35.3	17.1	44	1.2	0.64	
SN4	Silty sand with occasional gravel	13.3	49	32.5	61	0.55	0.4	

^a: Guo and Zhu (2012)

The deduced values of M_{ult} and $(EI)_{cr}$ are 1.2 and 3.7 times those estimated using RSB method, respectively. Using the calculated $(EI)_{cr}$ of 1.021 GN-m² (RSB method), $M_{cr} = 3.45$ MN-m, and $M_{ult} = 10.5$ MN-m, the $E_p I_p$ was determined using equation (4) for each M_{max} (obtained using EI) and the resulted $(EI)_{cr}/EI$ versus M_{max} is plotted in Figure 3(a). This $E_p I_p$ allows y_0 , M_{max} and x_p to be predicted for each specified load. The predicted $P-y_0$ curve is plotted in Figure 2(b) (triangular dots), and the M_{max} in Figure 2(c). A comparison between the predictions using elastic EI and nonlinear $E_p I_p$ indicates that: (1) The M_{max} predicted using $E_p I_p$ is ~ 3.2% less than that gained using EI ; and (2) The slip depth x_p at $P_t = 2571$ kN increases to 3.08d (using $E_p I_p$) from 2.49d (using EI); and the deflection y_0 exceeds the measured by 115% [indicating the impact of using an accurate $(EI)_{cr}$].

The large difference between the back-estimated and the calculated values of $(EI)_{cr}$ implies a partial development of crack in this particular pile.

3.2 Cases SN3 and SN4 in fill & Clay/Sand

3.2.1 Case SN3

Lateral loading test was conducted on a bored single pile embedded in very soft fill, followed by sandy estuarine deposit and clayey alluvium in Hong Kong (Ng et al. 2001). The ground water was located at 1.0 m below the GL. The sandy soil extending into a depth of 15m had $\tilde{N} = 17.1$, $\phi_s = 35.3^\circ$ (Guo and Zhu 2011), and $\gamma'_s = 11.0$ kN/m³. Lateral loads were applied at the middle of the 1.5 m (thickness) pile-cap. The pile, 28 m long and 1.5 m in diameter, had the properties of $f_y = 460$ MPa, $f'_c = 49$ MPa, $E_c = 32.3$ GPa, $EI = 10$ GN-m², $(EI)_{cr} = 4$ GN-m², $I_g = 0.2485$ m⁴, and $E_p = 40.2$ GPa. The M_{cr} was estimated as 1.44~2.31 MN-m using $k_r = 19.7\sim31.5$, $y_r = 0.75$ m and equation (3). The M_{ult} was not estimated owing to lack of the reinforcement detail.

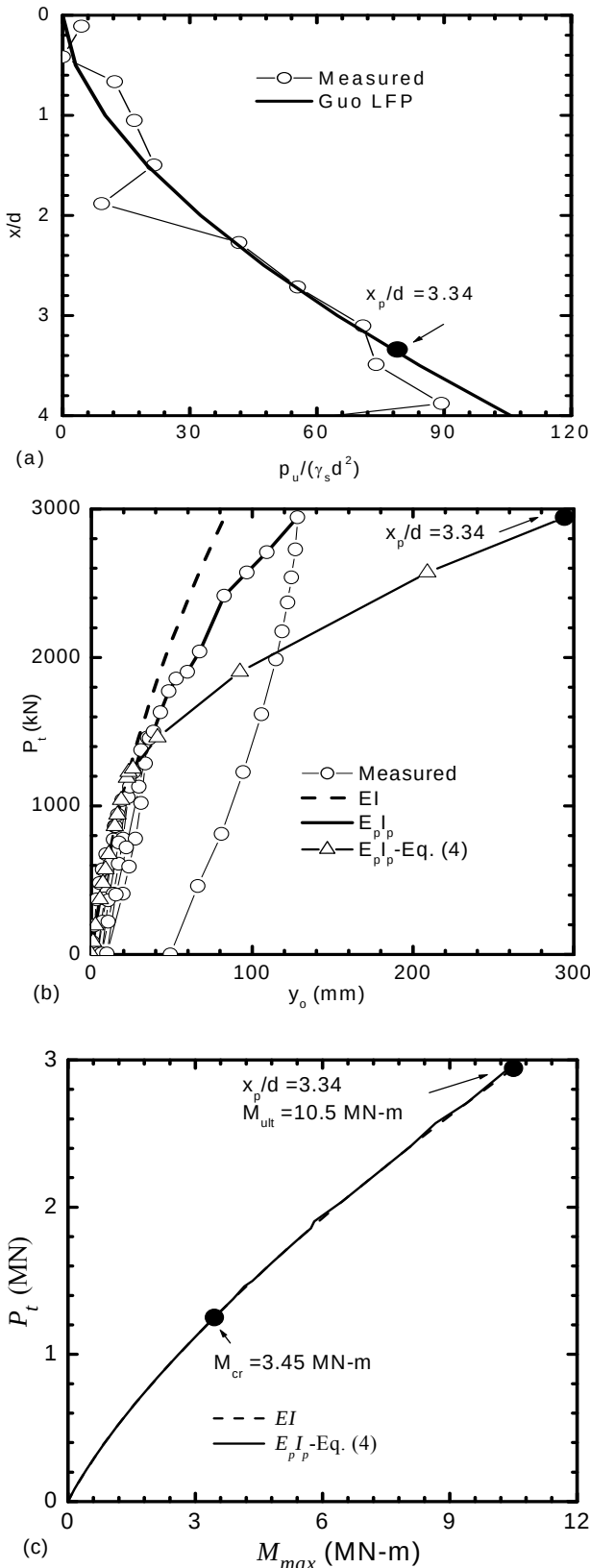


Figure 2 Comparison between measured (Huang et al., 2001) and predicted response of Pile B7: (a) LFPs, (b) P_t - y_o curves, and (c) P_t - M_{max} curves

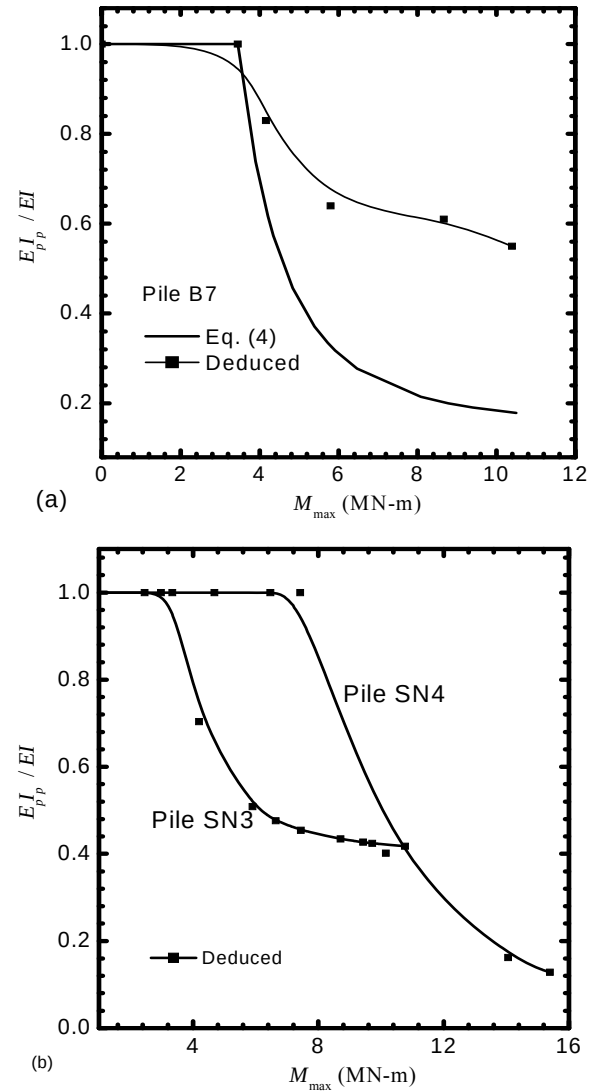


Figure 3 Variation of $E_p I_p / EI \sim M_{max}$.

The elastic pile analysis utilised $N_g = 1.2K_p^2 = 16.76$, $\alpha_0 = 0$ and $n = 1.7$ [see Figure 4(a)], and $G_s = 10.9$ MPa ($= 0.64N$). Note to a maximum slip depth x_p of $2.1d$, the p_u value, as shown in Figure 4(a), exceeds the predictions of Broms' LFP and Reese's LFP. The predicted deflection y_o , as shown in Figure 4(b) by 'EI', agrees with the measured data up to P_t of 1.1 MN, at which $M_{max} = 3.34$ MN-m. This value, taking as measured M_{cr} , offers $k_r = 45.5$, which again is about twice that of the ACI's suggestion. The $E_p I_p$ was deduced using the measured $P_t - y_o$ data, and is denoted as ' $E_p I_p$ ' in Figure 4(b). The associated $P_t \sim M_{max}$ and $E_p I_p / EI \sim M_{max}$ curves are plotted in Figures 4(c), and 3(b), respectively. In particular, it is noted that $(EI)_{cr} / EI = 0.4$, and at a load of 2.955 MN, $M_{max} = 10.77$ MN-m and $x_p = 2.1d$. Equation (4) was not checked without the M_{ult} .

Lateral load tests were conducted on two big barrettes (DB1 and DB2) embedded to a depth of 20 m in a reclaimed land in Hong Kong (Zhang 2003). The subsoil consisted either of sandy silty clay or loose to medium dense sand, and with cobbles in between the depths of 10.6 – 18.9 m below the GL. The ground water was located at a depth of 2.5 m.

The soil properties (to a depth of 15 m) were $\bar{N} = 32.5$, $\phi_s = 49^\circ$, and $\gamma'_s = 13.3 \text{ kN/m}^3$. The barrette DB1 (called SN4 herein) was investigated, which had a length of 51.1 m, and a rectangular cross-section of 2.8 m (height) $\times$ 0.86 m. Load was applied along the height direction at near the GL ($e = 0$). The strength and stiffness properties were $f_y = 460 \text{ MPa}$, $f'_c = 43.4 \text{ MPa}$ [$= f_{cu}/1.22$ (Beckett and Alexandrou 1997)], and a cubic concrete compressive strength f_{cu} of 53 MPa, $E_c = 30.3 \text{ GPa}$, $t = 75\text{--}100 \text{ mm}$, $I_g = 1.5732 \text{ m}^4$, $EI = 47.67 \text{ GN-m}^2$, and $E_p = 1.78 \times 10^9 \text{ kN-m}^2$. The M_{cr} was estimated as 4.61–7.38 MN-m using $k_r = 19.7\text{--}31.5$, and $y_r = 1.4 \text{ m}$; and M_{ult} as 10.13 MN-m and $(EI)_{cr}$ as 0.89 GN-m^2 for the upper 15 m, using the RSB method.

3.2.2 Case SN4

The elastic-pile analysis adopts $N_g = 28.15$ ($= 0.55K_p^2$), $\alpha_0 = 0$, $n = 1.7$, and $G_s = 13.0 \text{ MPa}$ ($= 0.4N$). In particular, as shown in Figure 5(a), up to a depth of $1.5d$, the current LFP lies in between Reese's LFP and Broms' LFP; and its average over a depth of $\sim 2.8d$ is close to that from Reese's LFP. The predicted deflection y_o , see Figure 5(b), agrees well with the measured data until the cracking load of 2.26 MN, which gives a measured M_{cr} of 7.42 MN-m and k_r of 31.7. Using the M_{cr} , and $(EI)_{cr}$ of 0.89 GN-m^2 , the $E_p I_p$, as shown in Figure 3(b), was deduced by matching the predicted and the measured deflection y_o [see Figure 5(b)], with the associated $P_t \sim M_{max}$ curve being depicted in Figure 5(c). These figures indicate: (1) the M_{ult} of 10.13 MN-m occurs at $P_t \approx 3.61 \text{ MN}$, with the measured y_o increasing sharply; (2) The initiation of crack renders increase of (i) the x_p at the maximum load P_t of 4.33 MN by 27.8%; and (ii) the M_{max} by 26.9%. The increase in M_{max} is not generally expected for piles with a normal relative stiffness E_p/G^* , but for the excessively high E_p/G^* of 1.1×10^5 [$G^* = (1+0.75\nu_s)G_s$, $\nu_s = \text{Poisson's ratio}$]. It is not difficult to use the increased M_{max} from nonlinear analysis in equation (4).

3.3 Pile Test (Case CN2) in Clay

Test Pile E was conducted at a clay site with SPT values (depth) of $N = 0$ (0–2 m), and $N = 4 \sim 6$ (2–10 m), and $S_u = 163.5 \text{ kPa}$ at $x = 3 \text{ m}$ below the GL. Lateral load was applied 0.35 m above the GL. The pile was characterised by $L = 9.5 \text{ m}$, $d = 1.2 \text{ m}$, $EI = 2.54 \text{ GN-m}^2$, and $f'_c \approx 27.5 \text{ MPa}$ [$= (E_c/151000)^2$]. The reported EI of 115.28 MN-m² (Nakai & Kishida 1982) was too low, as it gives an E_p of 1.13 GPa. Taking E_c (E_p) as the concrete modulus of 25 GPa, the M_{cr} was estimated as 554.2–886.7 kN-m (using $k_r = 19.7\text{--}31.5$, and $y_r = 0.6 \text{ m}$). This value is slightly higher than 527.6 kN-m by Nakai & Kishida (1977), implying the rationale of the E_c . The M_{ult} is not determined without the reinforcement detail. The model of elastic-pile utilises $n = 0.7$, $N_g = 2$, $\alpha_0 = 0.06$ [see Figure 6(a)], $G_s = 21.38 \text{ MPa}$ ($= 130.8S_u$) (Nakai and Kishida 1982), and $k = 3.77G_s$. Up to a depth of $5d$, Guo LFP is close to Hansen's LFP using $c = 8 \text{ kPa}$ and $\phi_s = 10^\circ$, but it is less than Matlock's LFP. The predicted pile deflection agrees well with the observed data up to P_t of 470 kN [see Figure 6(b)]. The measured M_{cr} (M_{max} at $P_{cr} = 470 \text{ kN}$) was computed as 628.4 kN-m and k_r as 22.3. The $E_p I_p$ was deduced using the measured deflection y_o [see Figure 6(b)]. This in particular, offers $E_p I_p / EI = 0.6$ and 0.18, respectively, for $P_t = 588.6 \text{ kN}$ and 735.8 kN, which in turn offer $(EI)_{cr} / EI$ of 0.35 and 0.005, respectively, as per equation (4). The drastic drop in $(EI)_{cr}$ implies pile failure prior to 735.8 kN (Nakai and Kishida 1982). The predicted bending moment profiles basically agree with the scattered, measured data as shown in Figure 6(c), in light of the difficulty in obtaining reliable measured profiles after crack.

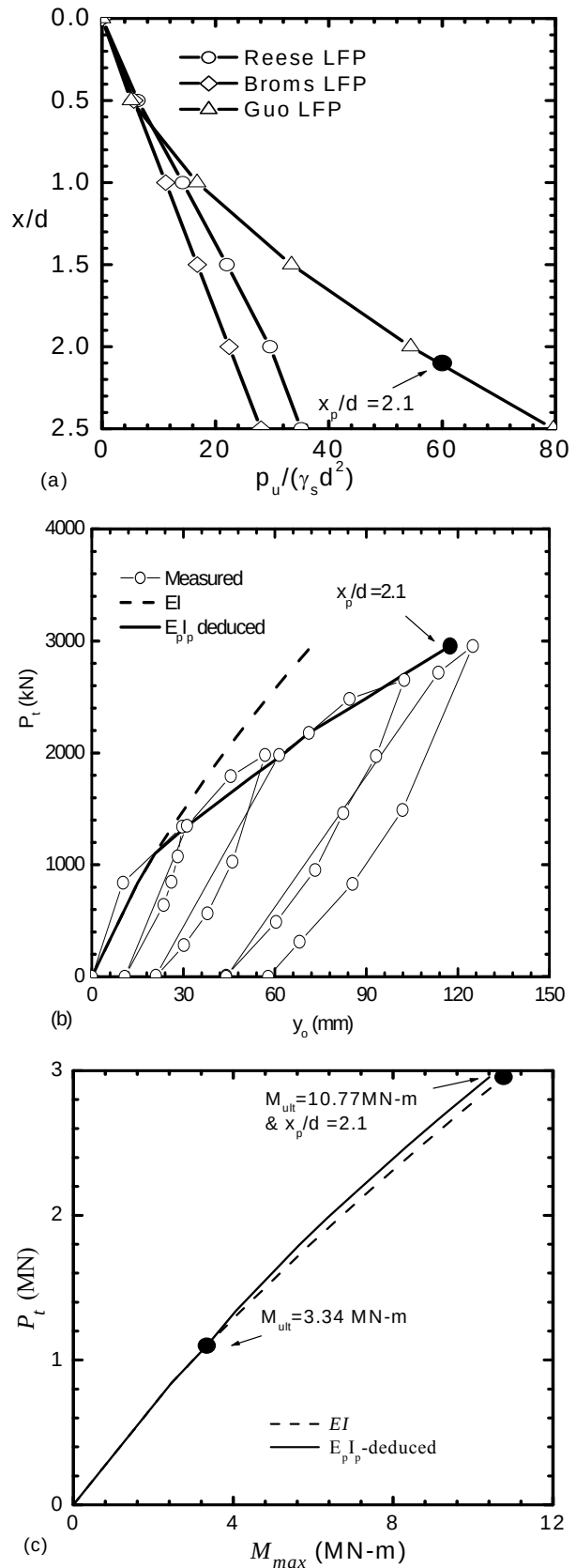


Figure 4 Comparison between measured (Ng, et al. 2001) and predicted response of Hong Kong pile: (a) LFPs, (b) P_t - y_o curves, and (c) P_t - M_{max} curves.

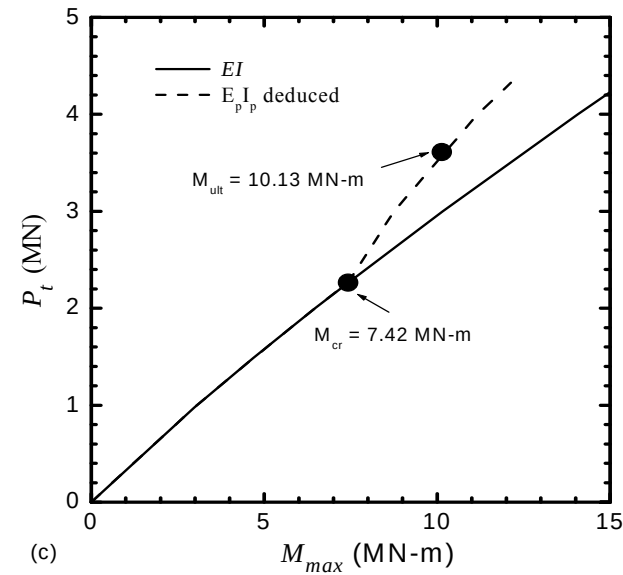
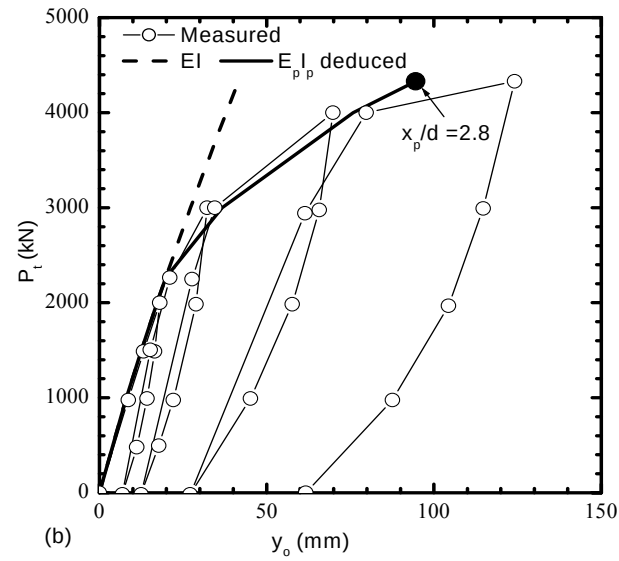
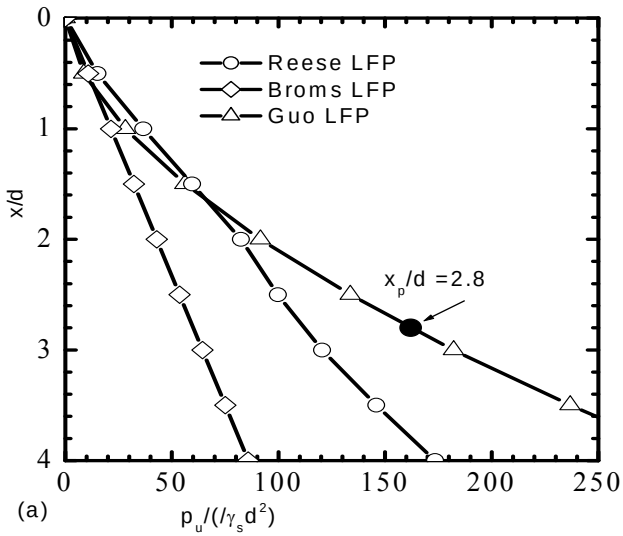


Figure 5 Comparison between measured (Zhang, 2003) and predicted response of DB1 pile: (a) LFPs, (b) P_t - y_o curves, and (c) P_t - M_{max} curve.

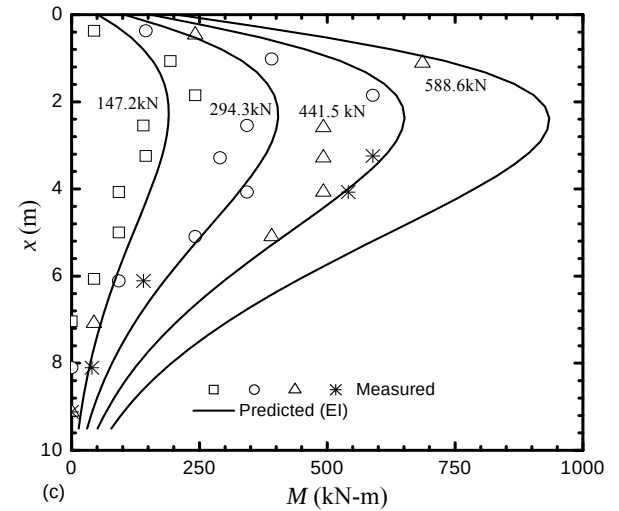
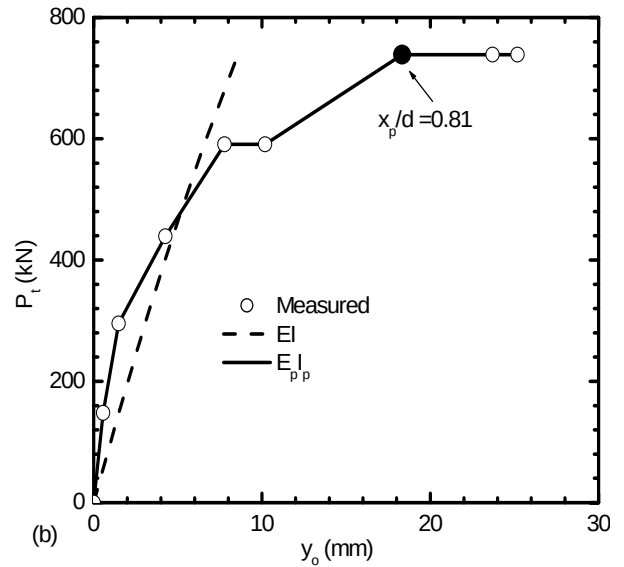
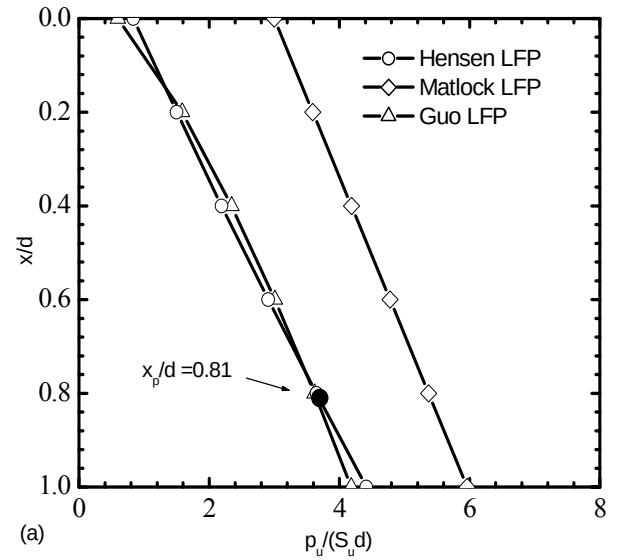


Figure 6 Comparison between calculated and measured (Nakai & Kishida, 1982) response of Pile E: (a) LFPs, (b) P_t - y_o curves, and (c) M profiles for $P_t = 147.2$ kN, 294.3 kN, 441.5 kN, and 588.6 kN

The deduced variations of the normalised critical bending stiffness $E_p I_p / EI$ with normalised bending moment M_{cr} / M_{max} for typical piles are plotted in Figure 7(a) together with those from three rock-socket piles (Guo 2012). The figure indicates a good agreement between each group of deduced dots and Equation (4) using the corresponding ratios of $(EI)_{cr} / EI$ (deduced). It is also interesting to see a linear correlation between the deduced values of k_r and $(EI)_{cr} / EI$ for all cases [see Figure 7(b)].

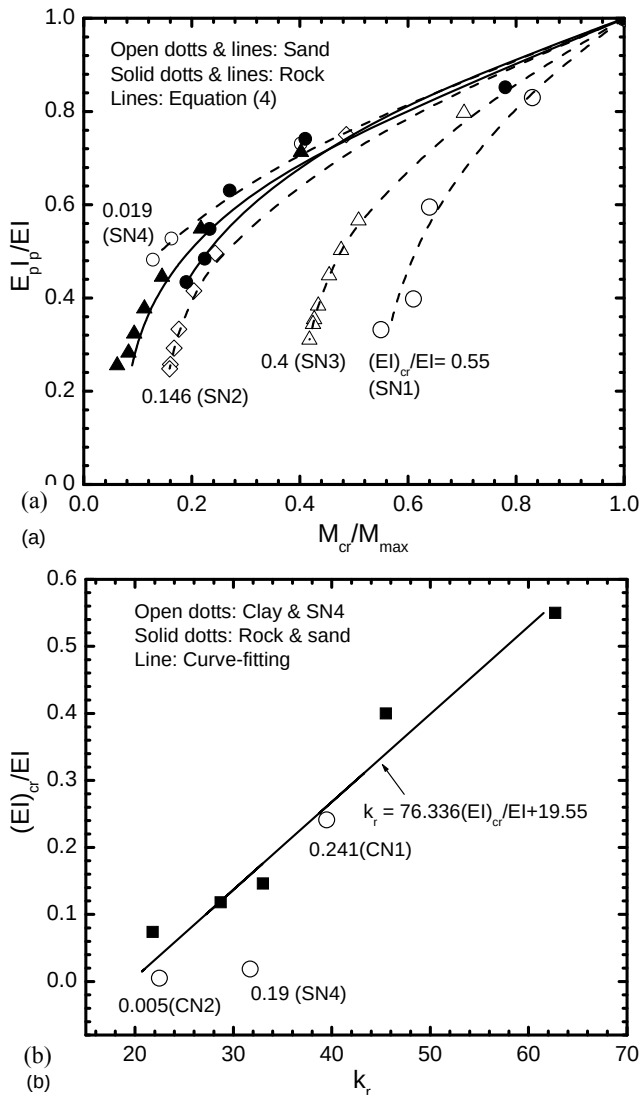


Figure 7 Normalised $(EI)_{cr} / EI$ versus (a) Normalised moment M_{cr} / M_{max} , and (b) the factor k_r

4. CONCLUSION

Study on six laterally loaded (structurally nonlinear) piles to date indicates similar ranges of soil parameters to elastic piles in clay, sand or layered soil, respectively, and the following features:

- The ultimate bending moment M_{ult} , flexural rigidity of cracked cross section $E_p I_p$ may generally be evaluated using the method recommended by ACI (1993) [thus equation (4)], although this would not always offer good prediction of measured pile response such as cases SN1 and SN4.
- Using equation (3) to predict the cracking moment, the k_r should be taken as 16.7~22.3 (clay) and 31.7~62.7 (sand). The k_r for piles in sand/ rock may be 2~3 times that for piles in clay and structural beams, and is linearly dependent of $(EI)_{cr} / EI$.

The current approach is underpinned by a reduced bending stiffness for entire piles. It well captures the overall response of a

crack pile, but its accuracy to predicting profiles of bending moment and deflection is yet to be confirmed by reliable measured data to be gained.

Dr. Bitang Zhu assisted the preliminary calculation.

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6. APPENDIX I. – Notation

The following symbols have been adopted in this paper.

A_L	gradient of the LFP [$\text{FL}^{-1-\eta}$];	M ($M_{\max}$)	bending moment in the pile (maximum M);
b	width of a rectangular cross section of a pile;	M_t	moment applied at the pile at the GL;
d	pile diameter;	n	power to the sum of α_0 and x ;
e	eccentricity of loading above groundline (GL);	N	blow count of Standard Penetration Tests (SPT) test;
E_c	modulus of elasticity of concrete;	N_g	a gradient correlated compressive strength with the limiting pile – soil pressure at ground level;
$E_c I_g$	initial flexural rigidity of the pile;	N_p	fictitious tension of a fictitious membrane linking the springs around the pile;
E_p	equivalent Young's modulus of pile;	P_t	lateral load applied at pile head;
f'_c	characteristic value of compressive strength of the concrete;	p_u	limiting force per unit length [FL^{-1}];
f_c''	design value of concrete compressive strength;	r	pile radius;
f_y	yield strength of reinforcement;	s_u	undrained shear strength [FL^{-2}];
G_s	shear modulus of soil;	x	depth below ground level [L];
h	depth of a rectangular cross section of a pile;	x_p	slip depth;
I_g	moment of inertia of gross section of the pile;	y (y_0)	pile deflection (y at ground level);
I_e	effective moment of inertia of the pile after cracking;	α_0	a constant to include the force at ground level [L];
I_{cr}	moment of inertia of cracked section;	ϕ_s	angle of friction of soil;
k	subgrade modulus of a spring between pile and soil;	γ	load transfer factor;
k_r	a constant for concrete rupture;	γ_s (γ_s')	unit weight of the soil (effective γ_s);
L_{cr}	critical length of a pile;	θ	the curvature.
M_{cr}	cracking moment;	λ	reciprocal of characteristic length of the pile;
		ν_s	Poisson's ratio of soil;