

End-Bearing Granular Pile-Effect of Non-Linearity of Soil and Granular Pile

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ABSTRACT: The solutions for consideration of linear and non-linear non-homogeneity of floating granular pile in homogeneous soil conditions are available, still solutions are not available for non-homogeneous end-bearing granular pile in non-homogeneous soil conditions. The present paper deals with the mathematical solution, for calculating the top displacement, normalized shear stresses, normalized axial load and percentage load transferred to the base for the non-homogeneous end-bearing granular pile in non-homogeneous soil conditions based on the elastic continuum approach. The contemplation of non-homogeneity of the granular pile in the stiffness range of stone columns/granular piles in non-homogeneous soil mass is more realistic and could represent true in-situ behavior. The analysis concludes that with the increase of the degree of non-homogeneity of granular pile, the settlement influence factor decreases significantly for the shorter length of granular pile ($L/d \leq 20$) because of higher values of deformation modulus of a granular pile at all depths as compared to modulus of longer ones. With the increase of soil non-homogeneity, the shear stresses decrease in the top 15 % and bottom 10 % portions of the granular pile while they increase in the rest (middle part) of its length. A comparative study has been made for present analysis and average analysis and it was found that by average analysis the values of settlement influence factors are underestimated in comparison to exact analysis therefore the average analysis is not suitable.

KEYWORDS: End-bearing granular pile, Relative stiffness of bearing stratum, Relative stiffness of granular pile, Settlement influence factor.

1. INTRODUCTION

Construction of granular piles is done in stages with granular material placed in lifts in the hole and then compacted. Increase of in-situ confining stresses from the surrounding soil, with depth may lead to different degrees of compaction and unit weight with depth leading to non-homogeneity of granular pile in terms of its deformation modulus, although the energy input for compaction at each stage of construction of granular pile is constant.

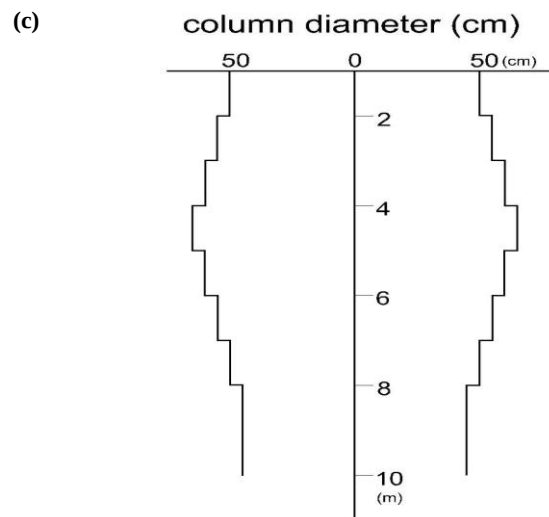
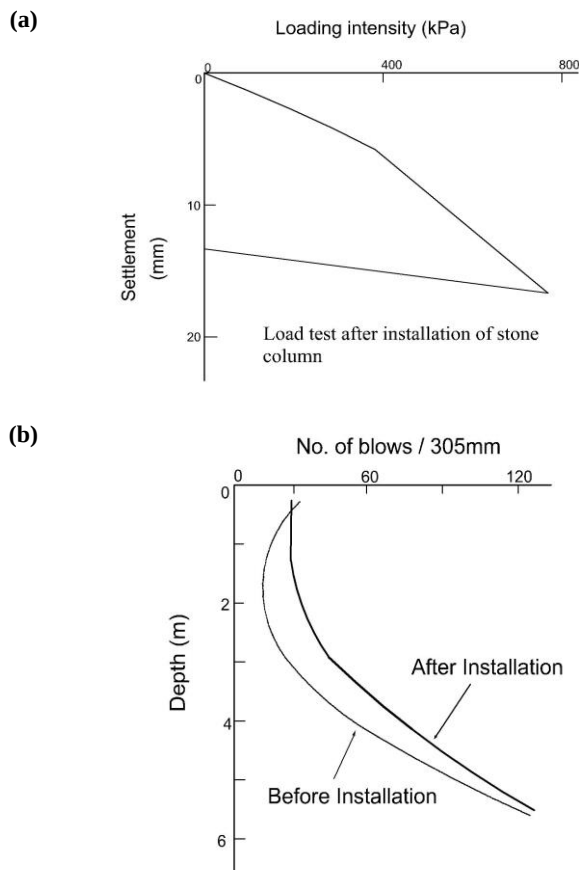


Figure 1 Installation Effects (a) Load Test, Madhav et al. (1988), (b) SPT (N) vs Depth, Loh (1982), (c) Shape of stone column De Cock et al. (1994)

Various studies regarding experimental and analytical cases for granular pile treated ground is listed below.

Balaam N. P. (1978)	Discussed the method of analysis of single stone column. The load displacement relationships of single granular pile for adhesive slip, frictional slip and no-slip conditions are evaluated.
Balaam N. P. and Booker J. R. (1981)	Presents an analytical solution to predict the settlement response of rigid foundation on soft clay stabilized by large number of fully penetrating stone columns assuming no-slip condition between column and surrounding soil.
Balaam N. P. et al. (1977)	Presented a finite element approach for the prediction of load settlement response considering the effect of relative stiffness of the granular pile.

Michael J.V. Baldinelli (1999)	The newly developed Statnamic load test and its basic theory are described. In this approach a one dimensional model was developed to represent the pile-soil system accounting for soil nonlinearity, slippage at the pile-soil interface and energy dissipation through wave propagation and different types of damping.
Banerjee, P. K. and Davies T. G. (1977)	Discussed the Analysis of pile groups embedded in Gibson soil by utilizing the concept of the interaction factors. These factors have been derived by using the Boundary layer method on isolated single piles and pile groups embedded in non-homogeneous three dimensional solid whose modulus of elasticity increases linearly with depth.
Grover K.S et al. (2015)	Analyzed the effect of stiffening on a single granular pile for both types of piles viz. floating and end bearing using elastic continuum approach.
Gupta P and Sharma J K (2017)	This analysis carried out study of non-homogeneous granular pile in homogeneous soil based on the continuum approach in terms of settlement influence factor, normalized axial load and mobilized stress distributions with depth and the percentage of applied load transferred to the base.
Madhav M R, et al. (2006)	Numerical solutions for the top displacement, normalised shear stresses, load distribution and percentage of load transferred to base are obtained for non-homogeneous floating and end-bearing granular piles based on elastic continuum approach.
Murali Krishna A. et al. (2006)	Discussed the ground treatment by rammed granular piles. It is observed that densification effect due to installation of granular pile is maximum near the periphery of pile and decreases with the distance from the pile.
Radoslaw L. Michalowski et al. (1993)	The bearing capacity of non homogeneous clay layers under embankments is given. The slip line method is used to calculate limit loads on layers of weak soil.
K. Rajyalakshmi et al. (2011)	The paper presents a method developed to estimate the bearing capacity of a strip footing on the surface of a reinforced granular bed over a finite layer of clay whose undrained strength increases linearly with depth, incorporating the contribution of granular fill, that of soft ground based on the Davis and Booker's theory and the axial tension in reinforcement.
Randolph, M. F. and Wroth, C. P. (1978)	The study given on analysis of deformation of vertically loaded piles in linear elastic soil. The application of method to pile design is discussed and design curves being sketched for different geometries in two typical soft clay deposits.
Vidyaranya B. et al. (2010)	Presents a method for estimating the ultimate pull out capacity of granular pile anchor in non-homogeneous soft ground.

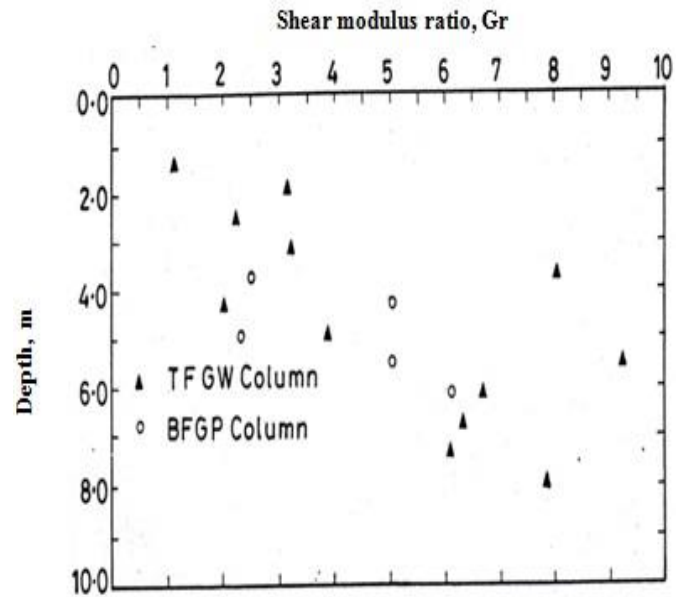


Figure 2 Variation of Shear modulus ratios with depth, Baez et al. (1995)

Single granular pile for both types of piles viz. floating and end bearing. The manifestation of the non-homogeneity of the granular pile could be an increase in its unit weight, reduction in the void ratio and increase in the modulus of deformation. A plate load test on the densified in situ soil (Figure 1 (a)), indicated settlement of about 6 mm under a load intensity of 400 kPa confirming the densification effect. Figure 1 (b) is an example of the densification effect in case of reclamation fills with 50 % of silt and clay fractions. The SPT (N) values increased by more than 100 % as a result of stone column installation. From the field study of Baez et al. (1995) on vibro-stone columns based on shear wave velocity test, the shear modulus of stone column material has been found to vary non-linearly with depth in the 'King Harbour test' as shown in Figure 2. The results indicate that shear modulus ratio, Gr (the ratio of shear modulus of stone column to that of improved soil) varies with depth between 1 and 8 for well graded stone columns, whereas varies between 1 and 6 for poorly graded stone columns. These results are in conformity of non-homogeneity of granular pile and soil. Thus, it is justified to take the variation of elastic modulus of granular pile and soil from linear to non-linear, which represents in-situ behavior closer and realistic. Figure 1 (c) depicts the observed shape of rammed stone columns which is given by De Cock et al. (1994). The diameter is in conformity with the CPT values of the in situ soil before treatment. Stone columns tend to have large diameter in softer strata rather than get densified. Since the in situ soil conditions in soft soils are non-homogeneous (both their undrained strength and the stiffness usually increase with depth) the granular piles installed in them become inherently non-homogeneous. Usage of non-homogeneous material at different stages of construction causes non-homogeneity of granular pile. From the numerous examples Nakayama et al. (1973); Loh (1982); Solymar et al. (1986) and Shamoto et al. (1997) of standard penetration tests, dynamic cone penetration tests and other in-situ tests carried out before and after treatment with granular piles, the densification effect can be noted to enhance the SPT values (N), unit weight, undrained strength and stiffness of the in situ soil (Figure 1 (b) and Figure 3 (a) & (b)). In Figure 3 (a) SPT value (N) of original ground increases from 5 at a depth of 1 meter to 10 to at a depth of 4 meters. After ground treatment the corresponding values are 12 and 22. In Figure 3 (b) similar trend in improvement of N-values of original ground is observed with sand compaction piles.

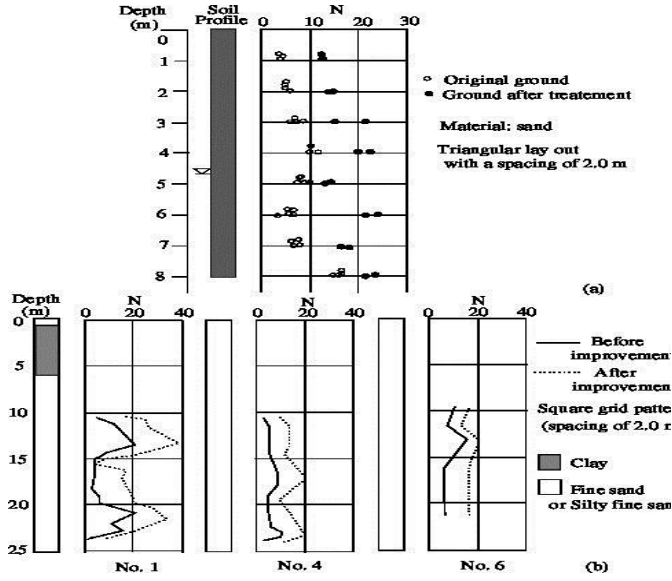


Figure 3 (a) Soil profile and comparison of N-values before and after treatment, Nakayama et al. (1973), (b) Typical Boring Logs of Chiba Prefecture Site, Shamoto et al. (1997)

Variations in these parameters may lead to non-homogeneity of granular pile and soil in terms of deformation modulus of both. In a study made by Madhav et al. (2006) the numerical solution for the top displacement, normalized shear stresses and percentage of load transferred to the base are obtained considering non-homogeneity of end-bearing granular pile in terms of its deformation modulus with the linear variation, using elastic continuum approach.

2. PROBLEM DEFINITION

Granular pile is discretised into 'n' cylindrical elements acted upon by shear stresses, τ , and with the base having a uniform pressure, p_b . The granular pile base is assumed to be smooth, across which the load is uniformly distributed. For approximating the non-homogeneity of granular pile is to consider its deformation modulus to increase non-linearly with depth from ground surface to its tip as shown in Figure 4. The deformation modulus $E_{gp}(z)$ at any depth z , from the top of the granular pile is

$$E_{gp}(z) = E_{gp0} \left\{ 1 + \alpha \frac{z}{L} + \delta \left(\frac{z}{L} \right)^2 \right\} \quad (1)$$

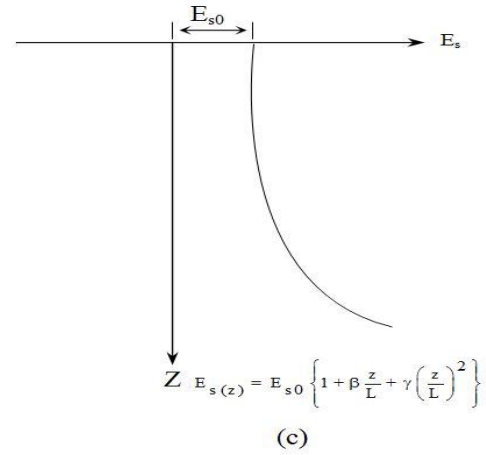
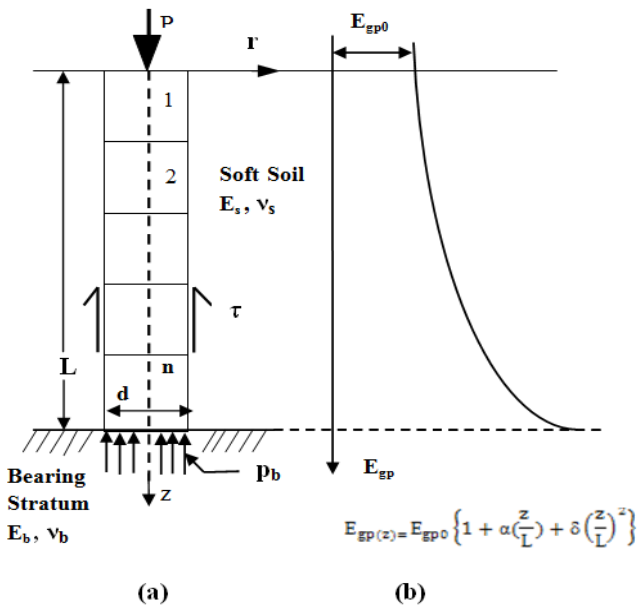


Figure 4 Definition Sketch (a) Non-homogeneous end bearing granular pile, (b) Variation of Modulus of deformation with depth for pile, (c) Variation of Modulus of deformation with depth for soil.

Where E_{gp0} is the deformation modulus at ground surface, α and δ are non-homogeneity parameters of pile can be expressed as:

$$\alpha = \frac{\left\{ \frac{E_{gp}(z)}{E_{gp0}} - 1 - \delta \left(\frac{z}{L} \right)^2 \right\}}{\left(\frac{z}{L} \right)} \quad (2)$$

$$\delta = \frac{\left\{ \frac{E_{gp}(z)}{E_{gp0}} - 1 - \alpha \left(\frac{z}{L} \right) \right\}}{\left(\frac{z}{L} \right)^2} \quad (3)$$

3. METHOD OF ANALYSIS

The soil displacements of the nodes on granular pile periphery and the centre of each element are evaluated based on the influence of the elemental shear stresses. Analysis is based on the continuum approach the basic assumptions in the analysis are: (1) The base of stone column/granular pile is assumed to be smooth and rigid across which the load is uniformly distributed as assumed by Madhav et al. (2006). (2) The disturbance effects in the in-situ soil due to the installation of granular piles are considered as this leads to non-homogeneity of soil in terms of its non-linear behavior which is due to installation and densification effect. (3) The settlement of the granular pile depends on its deformation modulus and geometry besides the magnitude of the load. Non-homogeneity of granular pile is considered in terms of its deformation modulus with the non-linear variation.

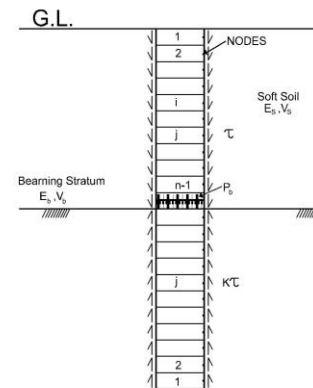


Figure 5 Mirror image techniques for Granular pile resting on bearing stratum

3.1 Soil Displacements

The method of analysis for consideration of non-homogeneity of soil is similar to that given by Poulos et al. (1979). He recommended the use of average of the soil modulus at influenced and influencing elements in the analysis, based on good agreement in the solutions with those from finite element analysis. The equivalent value of deformation modulus at node, i , for the influencing stress element, j , is

$$(E_{si})_{eq} = 0.5 [E_{si} + E_{sj}] \quad (4)$$

Where E_{si} and E_{sj} are the soil deformation modulus at nodes ' i ' and ' j ' respectively and with

$$E_{si} = E_{s0} \left(1 + \beta \frac{z_i}{L} + \gamma \left(\frac{z_i}{L} \right)^2 \right)$$

$$E_{sj} = E_{s0} \left(1 + \beta \frac{z_j}{L} + \gamma \left(\frac{z_j}{L} \right)^2 \right) \quad (5)$$

Where z_i and z_j are the depths of element ' i ' and ' j ' respectively. The equivalent deformation modulus of element ' i ' from Equations (4) & (5), is

$$(E_{si})_{eq} = E_{s0} \left[1 + 0.5\beta \frac{(z_i + z_j)}{L} + 0.5\gamma \left(\frac{z_i + z_j}{L} \right)^2 \right] \quad (6)$$

Thus the soil displacement equation for granular pile resting on a stiff bearing stratum is

$$\left\{ \rho^s \right\} = \left\{ \frac{S^s}{d} \right\} = \left[\left[I^{sp} \right] - \kappa \left[I^{spim} \right] \right] \left\{ \frac{\tau}{E_s} \right\} \quad (7)$$

Where $\{S^s\}$ and $\{\rho^s\}$ are soil displacement and normalised soil displacement vectors respectively. $\{\rho^s\}$ is of size ' n ' for end bearing GPs $\{\tau/E_s\}$ is column vector of size ' n ' of shaft stresses only, excluding the base pressure. To account for the influence of the bearing stratum, the mirror image approximation as explained by Mattes and Poulos (1969) is used. The influence of the mirror image elements is taken as, κ , times the influence of shear stresses on the real elements in the negative direction, where κ is a non-dimensional parameter that accounts for the compressibility of the base and lies between 0 and 1 for floating granular pile and granular pile resting on a rigid stratum respectively (Figure 5). $[I^{sp}]$ is a square matrix of soil displacement influence coefficients of size ' n ' for end bearing granular pile. $[I^{spim}]$ is a square matrix of soil displacement influence coefficients due to image elements of size ' n '.

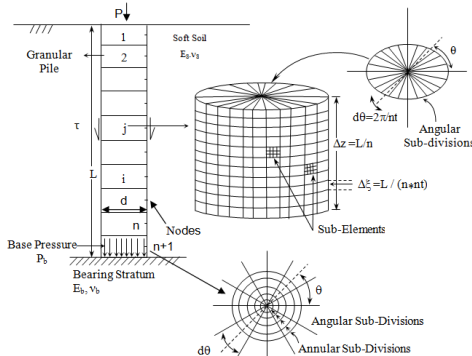


Figure 6 Pile discretisation scheme

3.2 Pile Displacements

Settlement of the base of a GP resting on a bearing stratum of finite compressibility is approximated by the equation for the displacement of a rigid circular disc on a semi-infinite mass as

$$\rho_b^p = \frac{S_b^p}{d} = \frac{P_b(1 - \nu_b^2)\pi/4}{E_b} \quad (8)$$

From the equilibrium equation, the base pressure is expressed in terms of shear stresses as

$$P_b = \frac{P}{\pi d^2/4} - \frac{4(L/d)}{n} \sum_{j=1}^n \tau_j \quad (9)$$

Thus the settlement of the base can be expressed in terms of the applied load and mobilized shear stresses (using Eq.s (8) & (9)) as

$$\rho_b^p = \left[\frac{P}{E_s \pi d^2/4} - \frac{4(L/d)}{n} \sum_{j=1}^n \frac{\tau_j}{E_s} \right] \times \frac{\pi(1 - \nu_b^2)}{4(E_b/E_s)} \quad (10)$$

Settlement of n^{th} element is estimated as the settlement of the base plus the settlement of the element due to the axial stress acting on it as

$$\rho_n^p = \rho_b^p + \frac{\sigma_n(\Delta z/2d)}{E_{gp}} \quad (11)$$

Where σ_n/E_{gp} is axial strain of the n^{th} element and Δz is element length. Settlement of any element ' i ' is estimated as the settlement of the ' $i+1$ ' plus the settlement of the element due to axial stress acting on it. Thus the settlement of any element i of granular pile is

$$\rho_i^p = \rho_b^p + \sum_{j=n}^{i+1} \frac{\sigma_j}{E_{gp0} \left[1 + \alpha \left(\frac{z_i}{1} \right) + \delta \left(\frac{z_i}{1} \right)^2 \right]} (\Delta z/d) + \frac{\sigma_i}{E_{gp0} \left[1 + \alpha \left(\frac{z_i}{1} \right) + \delta \left(\frac{z_i}{1} \right)^2 \right]} (\Delta z/2d) \quad (12)$$

The above set of displacement equations are expressed in matrix form as

$$\left\{ \rho^p \right\} = \rho_{b\{1\}} + [\Delta_1] \left\{ \frac{\sigma}{E_s} \right\} \quad (13)$$

Where $[\Delta_1]$ is upper triangular matrix as per Eq. (12) incorporating the non-homogeneity of the granular pile. Further using Eq. (10) for replacing the base displacement, Eq. (13) can be written as

$$\left\{ \rho^p \right\} = \frac{P(1 - \nu_b^2)}{(E_b/E_s)d^2E_s} \{1\} - \frac{\pi(L/d)(1 - \nu_b^2)}{n(E_b/E_s)} [1] \left\{ \frac{\tau}{E_s} \right\} + [\Delta_1] \left\{ \frac{\sigma}{E_s} \right\} \quad (14)$$

Where $\{1\}$ and $[1]$ are respectively column vector and square matrix of size ' n ' in which each term is unity. The shaft shear stresses and axial stresses of elements are related (based on equilibrium relationship) as

$$\sigma_i = \frac{P}{(\pi d^2 / 4)} - \sum_{j=1}^{i-1} \frac{4\tau_j L}{nd} - \frac{2\tau_i L}{nd} \quad (15)$$

The above equation may be written in matrix form for elements $i = 1$ to n as

$$\left\{ \frac{\sigma}{E_s} \right\} = \frac{P}{(\pi d^2 / 4) E_s} - \frac{4(L/d)}{n} [\Delta_2] \left\{ \frac{\tau}{E_s} \right\} \quad (16)$$

Where $[\Delta_2]$ is lower triangular matrix of size 'n' in which the diagonal and off diagonal terms are 0.5 and 1.0 respectively. Using the relationship between axial stresses and shaft shear stresses (Eq.16) the final form of displacement equations for elements $i = 1$ to n in terms of shaft shear stresses (Eq. 14) are

$$\rho^P = \{Y\} + [\Delta] \left\{ \frac{\tau}{E_s} \right\} \quad (17)$$

Where

$$\begin{aligned} \{Y\} &= \frac{P(1-\nu_b^2)}{(E_b/E_s)d^2 E_s} \{1\} + \frac{P}{(\pi d^2 / 4) E_s} [\Delta_1] \{1\} \\ [\Delta] &= -\frac{4(L/d)}{n} [\Delta_1] [\Delta_2] - \frac{\pi(L/d)(1-\nu_b^2)}{n(E_b/E_s)} [1] \end{aligned} \quad (18)$$

3.3 Compatibility of Displacements

Satisfying the compatibility of vertical displacements of the granular pile resting on stiff bearing stratum and the soil, solutions are obtained in terms of interface shear stresses and base pressure.

$$\{p^s\} = \{p^P\} \quad (19)$$

For granular pile resting on stiff bearing stratum (Eq.s (7) and (17)) the interface shear stresses are

$$\left\{ \frac{\tau}{E_s} \right\} = \left[\left[I^{sp} \right] - \kappa \left[I^{spim} \right] - [\Delta] \right]^{-1} \{Y\} \quad (20)$$

For estimation of κ , an iterative technique suggested by Poulos and Mattes (1969) is used. With an initial chosen value of κ , Eq.s (19) and (20) are solved to estimate the 'n' unknown shear stresses, τ , and base pressure, p_b . Having obtained the solution for chosen value of κ , a closer estimate of the correct value of κ is obtained by considering the compatibility between displacements of soil and the bearing stratum at the pile tip. The soil displacement at the pile tip is

$$\rho_b^s = \frac{S_b^s}{d} = \left\{ I_j^{sb} - \kappa I_j^{sbim} \right\} \left\{ \frac{\tau}{E_s} \right\} = \frac{\sum_{j=1}^n (I_j^{sb} - \kappa I_j^{sbim}) \tau_j}{E_s} \quad (21)$$

I_j^{sb} and I_j^{sbim} are the displacement influence coefficients for the tip due to shear stresses on real and imaginary elements j

respectively. However due to symmetry $I_j^{sb} = I_j^{sbim}$. Equating the soil displacement at the pile tip to the displacement of the base due to base stress, p_b (Eq. 8) the new value of the parameter, κ , is obtained as

$$\kappa = 1 - \frac{\pi(1-\nu_b^2)p_b}{4(E_b/E_s) \sum_{j=1}^n \tau_j I_j^{sb}} \quad (22)$$

Eq. (20) is solved iteratively using the new value of κ , and the process repeated until the required convergence is obtained for the value of κ .

The normalized top settlement of a single non-homogeneous End-bearing granular pile is obtained as

$$\rho_{top} = \frac{S_{top}}{d} = \frac{P}{\frac{\pi}{4} E_s d^2} I_{sp} \quad (23)$$

The top settlement of a single non-homogeneous End-bearing granular pile is obtained as

$$S_{top} = \frac{P}{\frac{\pi}{4} E_s d} I_{sp} \quad (24)$$

Where I_{sp} is settlement influence factor which depends on various parameters related to granular pile and soil. The overall response of the non-homogeneous granular pile is evaluated in terms of settlement influence factor, normalised shear stress and axial load distributions along granular pile - soil interface and percentage of load transferred to the base. Parameters affecting the overall response are (i) length to diameter ratio of the GP, (L/d) , (ii) the relative stiffness parameter, $K_{gp0} = (E_{gp0}/E_s)$, (iii) the relative stiffness of the bearing stratum E_b/E_{s0} (iv) the degree of non-homogeneity of granular pile, α and δ (v) Poisson's ratios of the soft soil, ν_s and of the base, ν_b , (vi) the degree of non-homogeneity of, β and γ .

In order to have the comparison in considering the average modulus of granular pile and soil with exact analysis the average value of deformation modulus of granular pile, E_{gp} is considered in pile displacement matrix and average deformation modulus of soil, E_s is considered in soil displacement matrix. The average value of deformation moduli of granular pile and soil are evaluated as

$$\begin{aligned} E_{gp}(z) &= E_{gp0} \left\{ 1 + \alpha \left(\frac{z}{L} \right) + \delta \left(\frac{z}{L} \right)^2 \right\} \\ E_{sz} &= E_{s0} \left\{ 1 + \beta \left(\frac{z}{L} \right) + \gamma \left(\frac{z}{L} \right)^2 \right\} \end{aligned} \quad (25)$$

For granular pile-

$$(E_{gp})_{av} = \frac{\int_0^L E_{gp0} \left\{ 1 + \alpha \left(\frac{z}{L} \right) + \delta \left(\frac{z}{L} \right)^2 \right\} dz}{L} \quad (26)$$

On integrating we get

$$(E_{gp})_{av} = E_{gp0} (1 + (\alpha/2) + (\delta/3)) \quad (27)$$

For soil-

$$(E_s)_{av} = \frac{\int_0^L E_{s0} \left\{ 1 + \beta \left(\frac{z}{L} \right) + \gamma \left(\frac{z}{L} \right)^2 \right\} dz}{L} \quad (28)$$

On integrating we get

$$(E_s)_{av} = E_{s0} (1 + (\beta/2) + (\gamma/3)) \quad (29)$$

4. RESULTS AND DISCUSSION

The results from the above analysis are validated with those of Poulos and Mattes (1969) and Madhav et al. (2006) for a single compressible non-homogeneous end bearing pile. The relative GP stiffness at base is defined as the ratio of the modulus of pile to that of the soil at base, i.e., $K_b = E_p/E_{sL}$. For granular pile relative stiffness K_{gp0} lies between 10 to 100 but for better understanding of results the analysis is carried out up to $K_{gp0}=10$ to 1000. The agreement between the results from the present analysis with those from Poulos and Mattes (1969) and Madhav et al. (2006) has been very close.

Table 1 Validation of Results with Madhav et al. (2006) and Poulos and Mattes (1969)

Present Analysis	Validation	References
End bearing pile (Non-homogeneous granular pile with linear variation in homogeneous soil conditions) $L/d = 10, v_s = 0.5, E_b/E_s = 100, \eta$ or $\alpha = 2, \delta = 0, \beta = 0, \gamma = 0, K_{gp0} = 50$ Settlement Influence Factor $(I_{sp}) = 0.088$	For same parameters Settlement Influence Factor $(I_{sp}) = 0.0883$	Madhav et al. (2006)
(b) End bearing pile $L/d = 50, v_s = 0.3, E_b/E_{sL} = 100, \alpha$ $= 0, \delta = 0, \gamma = 0, K_{gp0} = 200, \eta = 0.5$ or $\beta = 1$ Settlement Influence Factor $(I_{sp}) = 0.098$	For same parameters Settlement Influence Factor $(I_{sp}) = 0.0987$	Poulos and Mattes (1969)

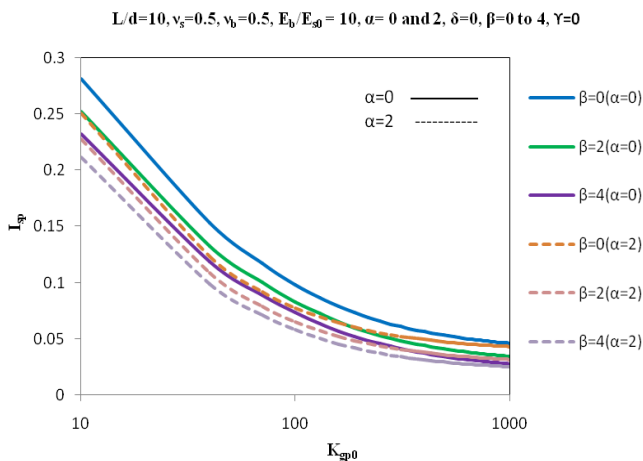


Figure 7 Variation of settlement influence factor 'I_{sp}' with relative stiffness parameter, K_{gp0} ($L/d=10$).

The variation of settlement influence factor 'I_{sp}' with relative stiffness parameter, K_{gp0} , is presented in Figure 7 for relative length,

$L/d = 10$ and relative stiffness of bearing stratum, $E_b/E_{s0} = 10$ with the influences of degrees of non-homogeneity of soil, β . With the increase of non-homogeneity of soil, the settlement influence factors reduces significantly in the stiffness range of $K_{gp0} = 10$ to 100. 'I_{sp}' values for non-homogeneous granular pile and $\beta = 0, 2$ and 4 are about 0.098, 0.083 and 0.073 for $K_{gp0} = 100$ ($\alpha=0$) and for $K_{gp0} = 100$ ($\alpha=2$) the settlement influence factors are 0.078, 0.065 and 0.058. With the increase of K_{gp0} settlement influence factors reduces.

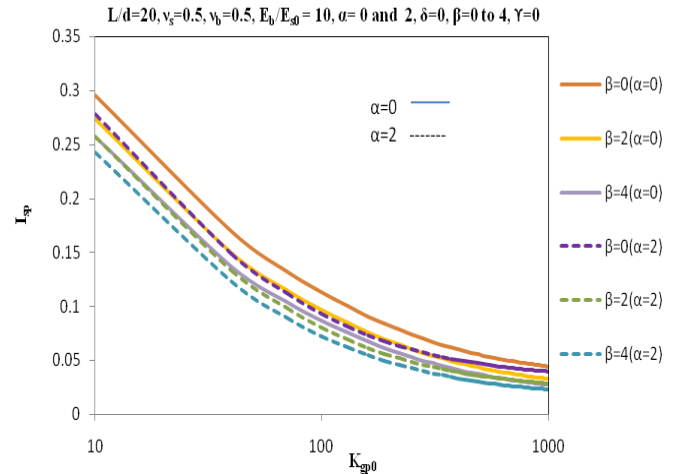


Figure 8 Variation of settlement influence factor 'I_{sp}' with relative stiffness parameter, K_{gp0} ($L/d=20$).

The effects of non-homogeneities of soil, β , and of granular pile, α , on the settlement influence factor is presented in Figure 8 for relatively longer end bearing granular pile resting on a stiff bearing stratum, $E_b/E_{s0} = 10$. The trends of curves are very similar to those of Figure 7. The 'I_{sp}' values for $K_{gp0} = 100$ ($\alpha=0$) and for $\beta = 0, 2$ and 4 are about 0.113, 0.096, and .086 and at $K_{gp0} = 100$ ($\alpha=2$) for $\beta = 0, 2$ and 4 are 0.093, 0.080 and 0.072 respectively. The reduction of settlement influence factors is observed to be more for non-homogeneity parameter, $\alpha=2$. With the increase of degree of non-homogeneity of granular pile, α from 0 to 2 the settlement influence factor decreases significantly for the shorter length GP ($L/d \leq 20$) because of higher values of deformation modulus of granular pile at all depths as compared to modulus of longer ones.

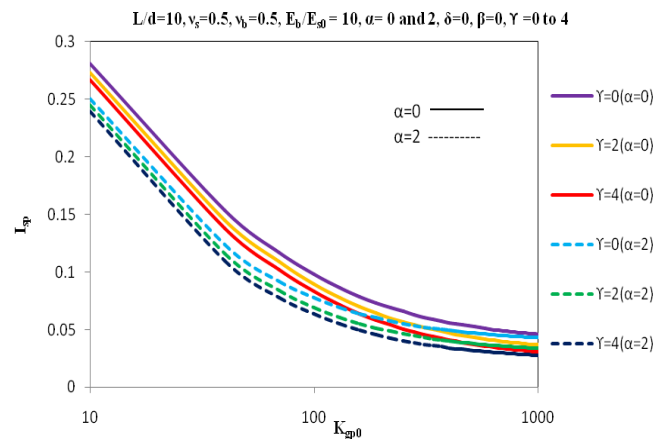


Figure 9 Variation of settlement influence factor 'I_{sp}' with relative stiffness parameter, K_{gp0} – effect of γ

The variation of settlement influence factor 'I_{sp}' with relative stiffness parameter, K_{gp0} , is presented in Figure 9 for relative length, $L/d = 10$ and relative stiffness of bearing stratum, $E_b/E_{s0} = 10$ with the influences of degree of non-homogeneity of soil, γ . 'I_{sp}' values

for non-homogeneous granular pile with non-homogeneity parameter, $\gamma = 0, 2$ and 4 are about $0.098, 0.089$ and 0.083 for $K_{gp0} = 100$ ($\alpha=0$). For $K_{gp0} = 100$ ($\alpha=2$) and non-homogeneity parameter, $\gamma = 0, 2$ and 4 , the settlement influence factor ' I_{sp} ' values are $0.077, 0.069$ and 0.063 . The rate of decrease of settlement with effect of degree of non-homogeneity, γ is less in comparison to the effect of non-homogeneity parameter, β .

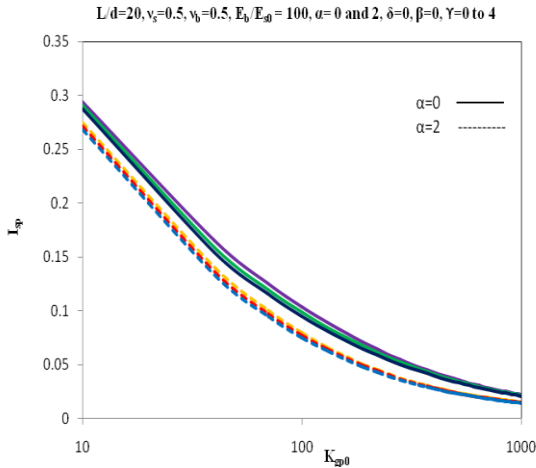


Figure 10 Variation of settlement influence factor ' I_{sp} ' with relative stiffness parameter, K_{gp0} ($L/d=20$) effect of non-linear non-homogeneity of soil, γ

Figure 10 shows the variation of settlement influence factor ' I_{sp} ' with relative stiffness parameter, K_{gp0} , for relative length, $L/d = 20$ and relative stiffness of bearing stratum, $E_b/E_{s0} = 10$ with the influences of degrees of non-linear non-homogeneity of soil, γ . For a given degree of linear non homogeneity parameter, α , a longer granular pile would have a relatively smaller moduli at all depths compared to a shorter one. A consequence of the above fact is that the effect of degree of non homogeneity on settlement influence factor I_{sp} decreases with increasing values of relative length L/d .

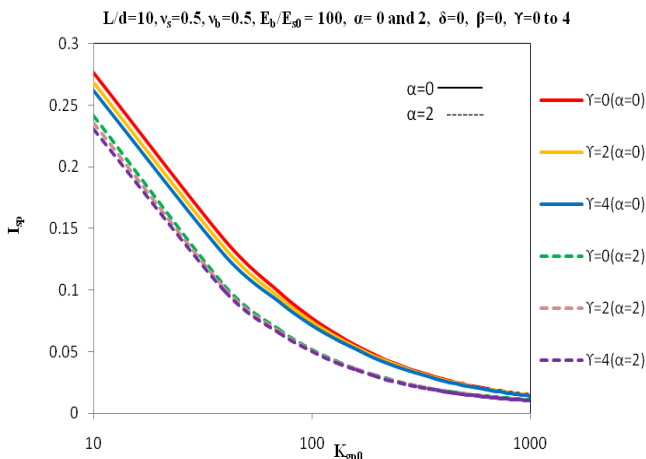


Figure 11 Variation of settlement influence factor ' I_{sp} ' with relative stiffness parameter, K_{gp0} ($E_b/E_{s0} = 100$)

The variation of settlement influence factor ' I_{sp} ' with relative stiffness parameter, K_{gp0} , is presented in Figure 11 for relative length, $L/d = 10$ and relative stiffness of bearing stratum, $E_b/E_{s0} = 100$ with the influences of degree of non linear non-homogeneity of soil, γ . With the increase of non-homogeneity of soil, the settlement influence factors reduces significantly in the stiffness range of $K_{gp0} = 10$ to 100 . ' I_{sp} ' values for non-homogeneous granular pile and $\gamma = 0, 2$, and 4 are about $0.077, 0.074$ and 0.072 for $K_{gp0} = 100$ ($\alpha=0$) and for $K_{gp0} = 100$ ($\alpha=2$) the settlement influence factors are $0.052,$

0.0512 and 0.0503 . With the increase of K_{gp0} settlement influence factors reduces. The rate of decrease of settlement increases with increase in relative stiffness of bearing stratum from $E_b/E_{s0} = 10$ to 100 .

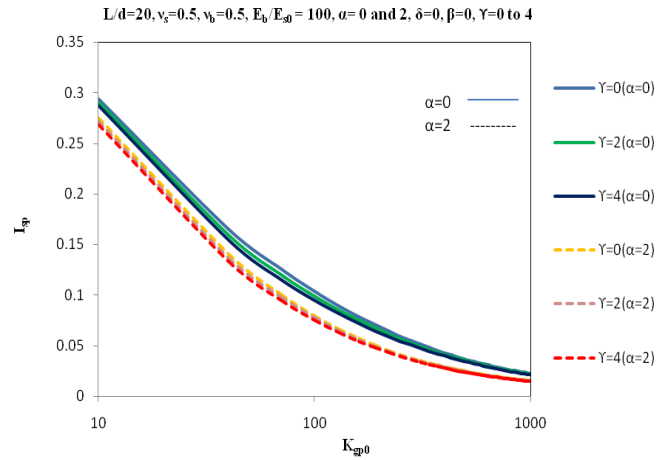


Figure 12 Variation settlement influence factor ' I_{sp} ' with relative Stiffness parameter, K_{gp0} , for relative length, ($L/d = 20$), ($E_b/E_{s0}=100$)

Figure 12 shows the variation of settlement influence factor ' I_{sp} ' with relative stiffness parameter, K_{gp0} , for relative length, $L/d = 20$ and relative stiffness of bearing stratum, $E_b/E_{s0} = 100$ with the influences of degree of non linear non-homogeneity of soil, γ . ' I_{sp} ' values for non-homogeneous granular pile and $\gamma = 0, 2$ and 4 are about $0.103, 0.098$ and 0.094 for $K_{gp0} = 100$ ($\alpha=0$) and for $K_{gp0} = 100$ ($\alpha=2$) the settlement influence factors are $0.079, 0.077$ and 0.075 . With the increase of K_{gp0} settlement influence factors reduces. The rate of decrease of settlement increases with increase in relative stiffness of bearing stratum from $E_b/E_{s0} = 10$ to 100 for the same relative length $L/d = 20$. Simultaneously it is also observed that the effect of degree of non homogeneity on settlement influence factor I_{sp} decreases with increasing values of relative length $L/d = 10$ to 20 .

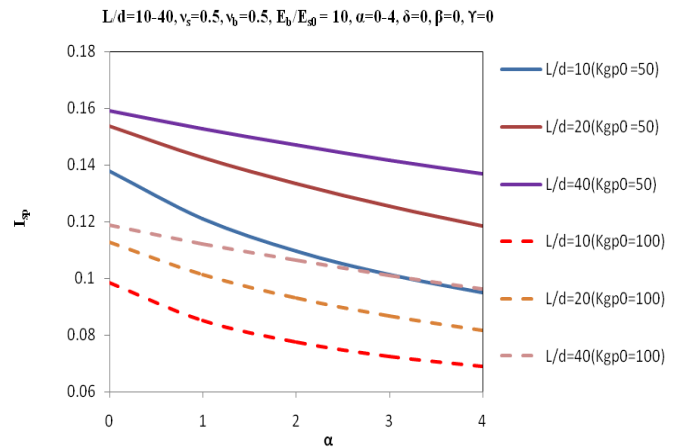


Figure 13 Variation of settlement influence factor ' I_{sp} ' with non-homogeneity parameter, α , ($E_b/E_{s0}=10$)

Variation of Settlement influence factor, I_{sp} with the degrees of linear non-homogeneity, α of granular pile is presented in Figure 13 for $E_b/E_{s0} = 10$ and for different relative length, L/d . The settlements of longer granular piles are more as compared to those for shorter ones due to the presence of bearing stratum at great depth for longer GPs. The increase of degree of non-homogeneity of granular pile decreases the settlement influence factors significantly for the shorter length of granular pile ($L/d \leq 20$) because of higher values of

deformation moduli of GP and soil for shorter GPs at all depths as compared to moduli longer ones. This reduction in settlement influence factor with non-homogeneity parameter, α decrease with the increase of relative stiffness parameter, K_{gp0} of granular pile.

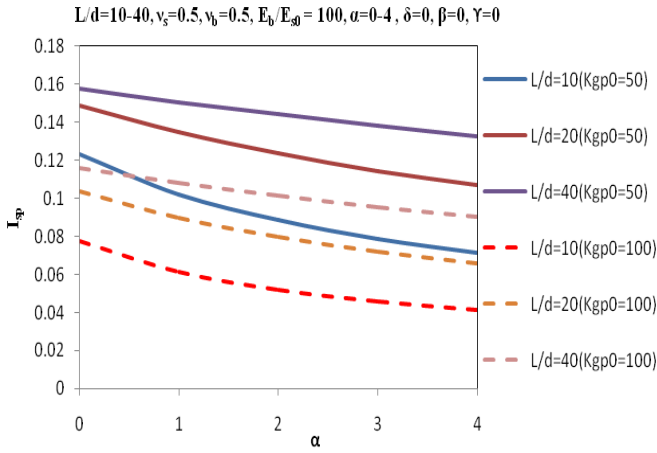


Figure 14 Variation of settlement influence factor ' I_{sp} ' with non-homogeneity parameter, α - effect of L/d ($E_b/E_{s0}=100$)

Figure 14 Shows Settlement influence factor with the degree of linear non-homogeneity of granular pile, α for $E_b/E_{s0} = 100$ and for different relative length, L/d . The increase of degree of non-homogeneity of granular pile decreases the settlement influence factors significantly. With increase in relative stiffness of bearing stratum, $E_b/E_{s0} = 10$ to 100 the rate of decrease of settlement influence factor increases.

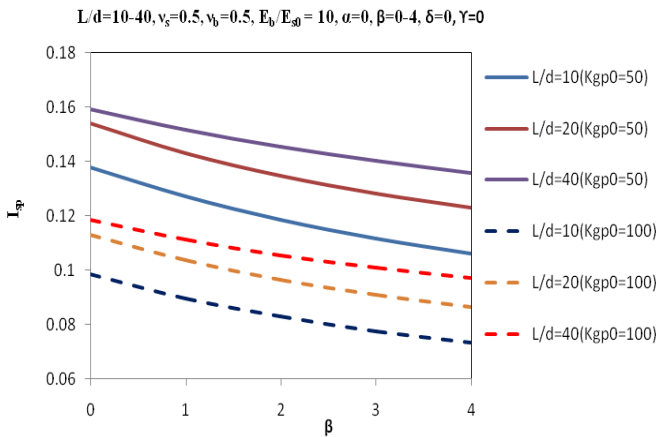


Figure 15 Variation of settlement influence factor ' I_{sp} ' with non-linear non-homogeneity parameter, β , ($E_b/E_{s0}=10$)

Figure 15 depicts the variation of settlement influence factor, I_{sp} with the degree of non-linear non-homogeneity of granular pile, β for $E_b/E_{s0} = 10$ and for different relative length, L/d . The settlements of longer granular piles are more as compared to those for shorter ones due to the presence of bearing stratum at great depth for longer granular piles. It can be seen that for higher values of relative length ($L/d > 20$) the rate of decrease of settlement influence factor with non-homogeneity parameters, α and β is almost same.

Variation of settlement influence factor with the degree of non-linear non-homogeneity of granular pile, β is shown in Figure 16 for $E_b/E_{s0} = 100$ and for different relative length, L/d . The trend is similar as shown in Figure 15. The settlements of longer granular piles are more as compared to those for shorter ones due to the presence of bearing stratum at great depth for longer granular piles. The rate of decrease of settlement influence factor with non-homogeneity parameters, β increases with increase in stiffness of bearing stratum $E_b/E_{s0} = 10$ to 100.

Variation of settlement influence factor with the degree of non-linear non-homogeneity of granular pile, δ is shown in Figure 17 for $E_b/E_{s0} = 10$ and for different relative length, L/d . The rate of decrease of settlement influence factor with non-homogeneity parameter, δ of granular pile increases. The settlements of longer granular piles are more as compared to those for shorter ones due to the presence of bearing stratum at great depth for longer granular piles.

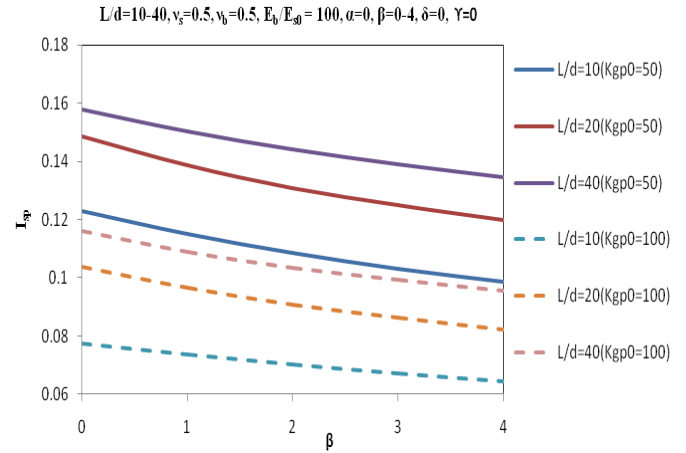


Figure 16 Variation of settlement influence factor ' I_{sp} ' with degree of non-homogeneity parameter of soil, β ($E_b/E_{s0}=100$)

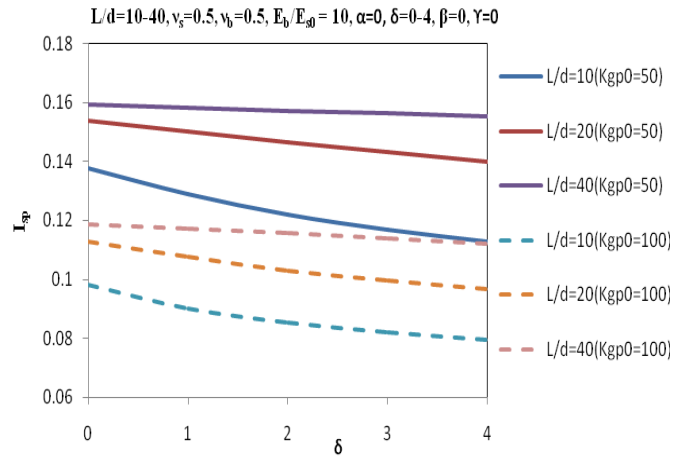


Figure 17 Variation of settlement influence factor ' I_{sp} ' with degree of non-linear non-homogeneity parameter of granular pile, δ

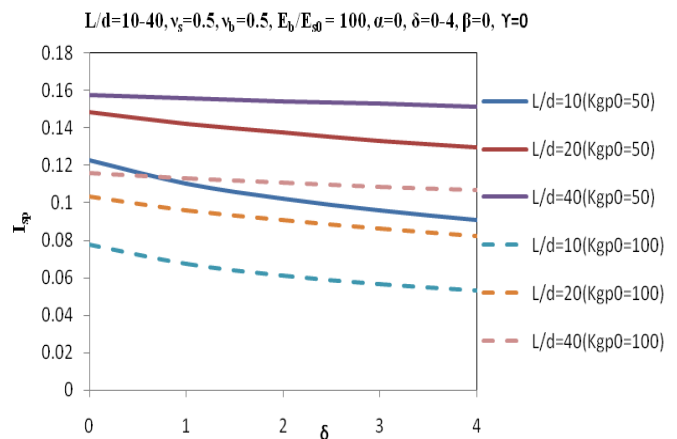


Figure 18 Variation of settlement influence factor ' I_{sp} ' with degree of non-linear non-homogeneity parameter of granular pile, δ , ($E_b/E_{s0}=100$)

Figure 18 shows the variation of settlement influence factor, I_{sp} with the degree of non-linear non-homogeneity of granular pile, δ for $E_b/E_{s0} = 100$ and for different relative length, L/d . The rate of decrease of settlement influence factor with non-homogeneity parameter, δ of granular pile increases. With increase in relative stiffness of bearing stratum $E_b/E_{s0} = 10$ to 100 the rate of decrease of settlement influence factor increases.

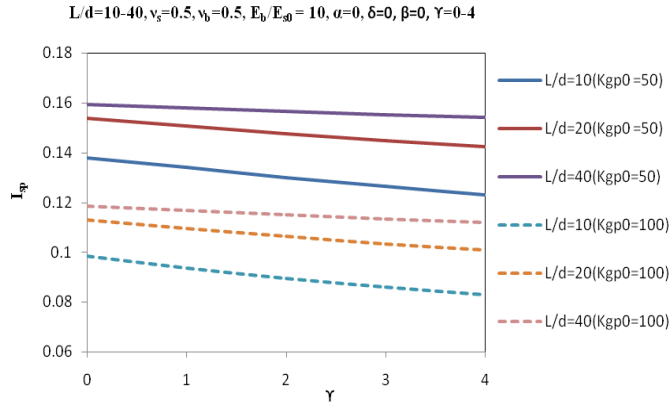


Figure 19 Variation of settlement influence factor 'I_{sp}' with degree of non-linear non-homogeneity parameter of soil, Y ($E_b/E_{s0} = 10$)

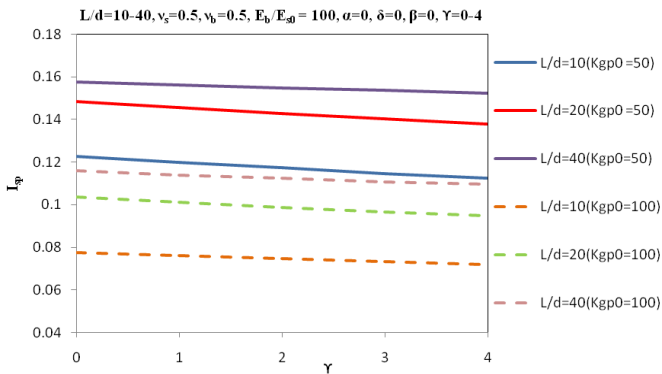


Figure 20 Variation of settlement influence factor 'I_{sp}' with degree of non-linear non-homogeneity parameter of soil, Y ($E_b/E_{s0} = 100$)

Variation of settlement influence factor with the degree of non-linear non-homogeneity of soil, Y is shown in Figure 19 and Figure 20 for $E_b/E_{s0} = 10$ and 100, for different relative length, L/d . The rate of decrease of settlement influence factor with non-linear non-homogeneity parameter, Y of soil increases. It is also observed that with increase in relative stiffness of bearing stratum $E_b/E_{s0} = 10$ to 100 the settlement influence factor decreases more.

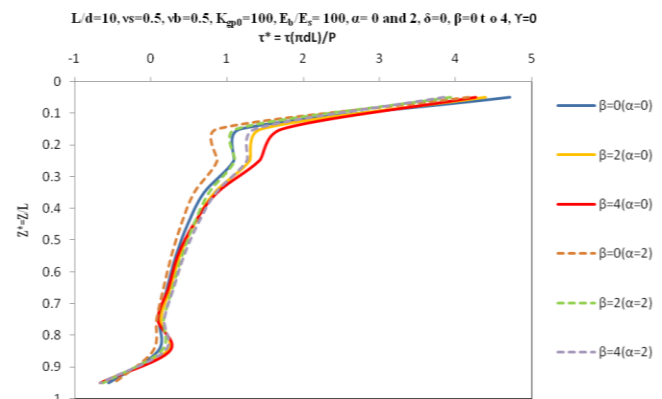


Figure 21 Variation of normalised shear stress, τ^* with normalised depth, Z^* with effect of non-homogeneity parameter for soil, β , ($\alpha=0$ and 2).

The variations of normalised shear stress with normalised depth is shown in Figure 21 for $L/d = 10$, $E_b/E_{s0} = 100$ and $K_{gp0} = 100$ along with the effect of non-homogeneity parameters of soil, β and of granular pile, α . With the increase of soil non-homogeneity, the shear stresses decrease in the 15 % top and 10 % bottom portions of granular pile while they increase in the rest (middle part) of its length. The increment in deformation modulus of soil with non-homogeneity parameter of soil, β in the middle part increases the shear stresses in that part to share more loads. Similar effects but relatively less in magnitudes for β ($\alpha=2$) are observed in the case of non-homogeneous granular pile. It can be said that due to non-homogeneity of granular pile, larger loads are transferred to the base resulting in a reduction of interfacial shear stresses over a remarkable length of granular pile. Shear stresses near to the base of granular pile are negative, due to soil surrounding the granular pile settles relatively more than the deformation of granular pile, i.e., an effect similar to a down drag due to presence of bearing stratum in order to satisfy the compatibility of displacements of granular pile and soil.

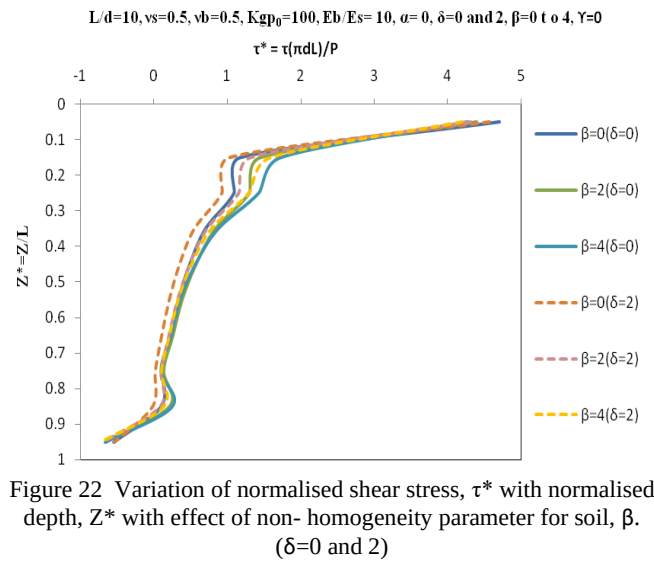


Figure 22 Variation of normalised shear stress, τ^* with normalised depth, Z^* with effect of non-homogeneity parameter for soil, β , ($\delta=0$ and 2)

The variations of normalised shear stress with normalised depth is shown in Figure 22 for $L/d = 10$, $E_b/E_{s0} = 10$ and $K_{gp0} = 100$ along with the effect of non-homogeneity parameters of soil, β and of granular pile, δ . With the increase of soil non-homogeneity, the shear stresses decrease in the top 10 % and 10 % bottom portions of granular pile while they increase in the rest (middle part) of its length.

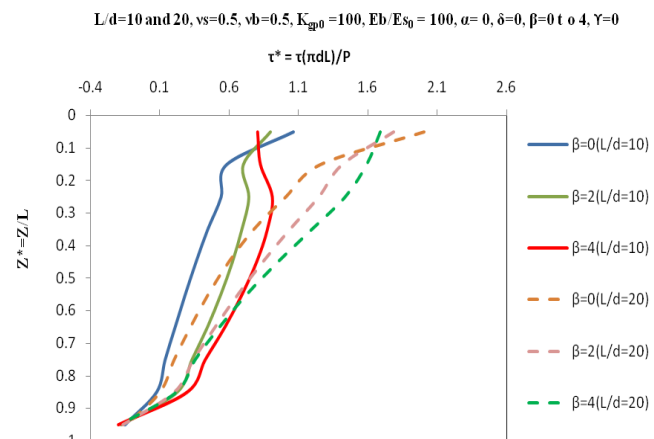


Figure 23 Variation of normalised shear stress, τ^* with normalised depth, Z^* with effect of non-homogeneity parameter for soil, β , ($L/d=10$ and 20)

The trends of these curves are similar for shorter ($L/d=10$) and longer ($L/d=20$) granular pile in Figure 23. The non-homogeneity of soil reduces the shear stresses in the upper region of soft soil along granular pile-soil interface and transfers them to the lower stiffer region of soil. For non-homogeneous granular pile, the non-homogeneity of soil, β in the range of $\beta = 2$ to 4, decreases the shear stresses along homogeneous GP in about 15% of its upper part and then suddenly increases at 25% depth and again decreases while in case of longer granular pile shear stresses decreases without any sudden increase at any depth of granular pile. Shear stresses decreases more for shorter pile in comparison to longer pile up to 65% depth of granular pile and then for more than 65% depth the decrease in shear stresses is almost same for shorter ($L/d=10$) and longer ($L/d=20$) granular pile.

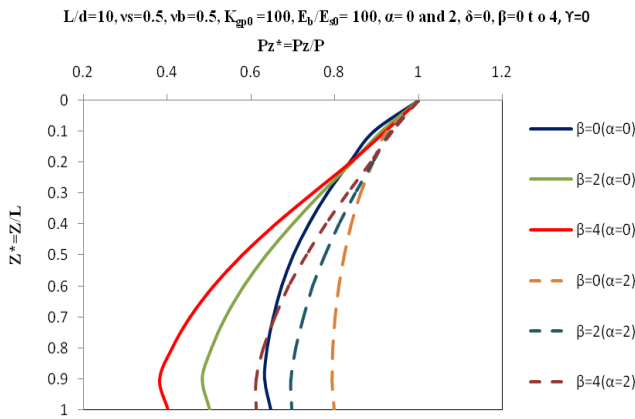


Figure 24 Normalised axial load, P_z^* of end bearing granular pile with normalised depth Z^*

Variation of normalised axial load, P_z^* ($= P_z/P$) of end bearing GP with normalised depth is presented in Figure 24. The axial load of a GP decreases with the increase of β at all depths in comparison to variations for homogeneous soil. The slight increase of normalised axial load in the upper part of granular pile for $\beta=0$ to 4 and $\alpha=0$, is due to the reduction of shear stresses in that part. The decrease in the axial load of granular pile with non-homogeneity parameter, β is due to increase in the shear stresses in the middle part of granular pile. Similar results are obtained for non-homogeneous granular pile but with less effect of β ($\alpha=2$). The reverse trend of increase of axial load with depth for $z^* > 0.90 L$ is because of the negative shear stresses near the stiff bearing stratum as seen in Figure 21 (pseudo down-drag effect). The values of P_z^* at $z^* = 0.6$ for $\beta = 0, 2$ and 4 are about 0.673, 0.582 and 0.510 for homogeneous ($\alpha = 0$) and 0.810, 0.748 and 0.690 for non-homogeneous ($\alpha = 2$) granular piles, respectively.

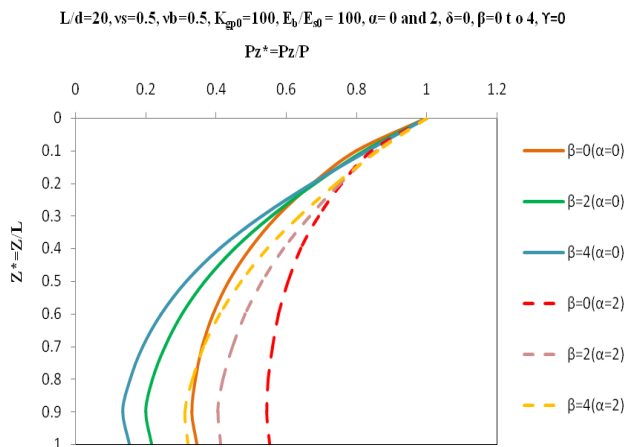


Figure 25 Normalised axial load, P_z^* of End bearing granular pile with normalised depth Z^* ($L/d=20$, $\alpha = 0$ and 2)

Variation of normalised axial load, P_z^* ($= P_z/P$) of end bearing GP with normalised depth is presented in Figure 25. The axial load of a granular pile decreases with the increase of β at all depths in comparison to variations for homogeneous soil. With increase in relative length $L/d=20$, the rate of decrease of normalised axial load increases in comparison to shorter pile ($L/d=10$). As in case of longer granular pile, a large part of the load is transferred through interface shear stresses due to great depth of the bearing stratum. Hence degree of non-homogeneity of soil has a marked influence on transfer of soil stresses to lower part of longer granular pile.

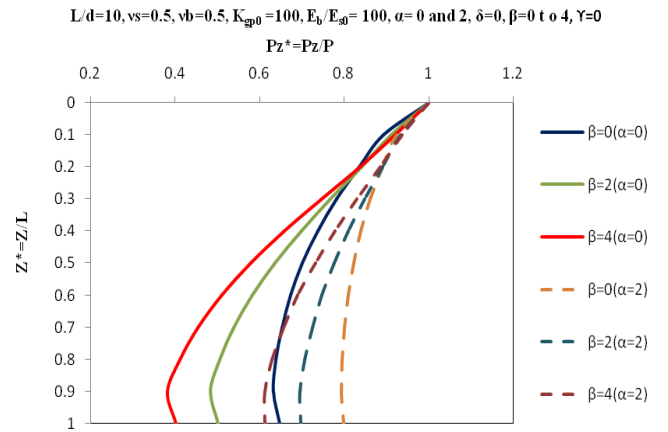


Figure 26 Normalised axial load, P_z^* of end bearing granular pile with normalised depth Z^* ($L/d=10$)

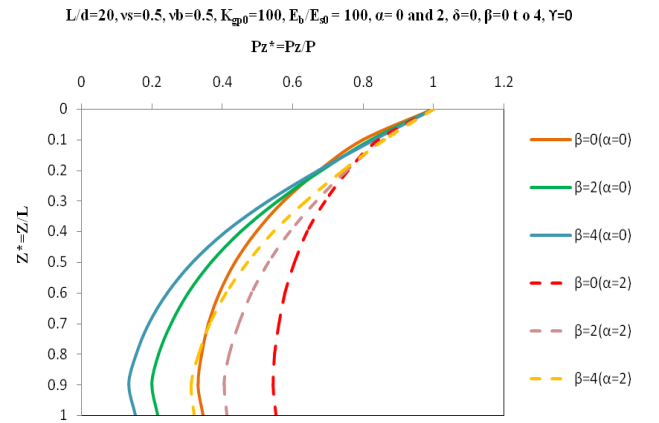


Figure 27 Normalised axial load, P_z^* of end bearing granular pile with normalised depth Z^* ($L/d=20$, $\delta = 0$ and 2)

Variation of normalised axial load, P_z^* ($= P_z/P$) of end bearing granular pile with normalised depth is shown in Figures 26 and 27. The trend is similar as shown in Figures 24 and 25. With the increase of β at all depths the axial load of a granular pile decreases in comparison to variations for homogeneous soil. The effect of non-linear non-homogeneity parameter, $\delta=2$ is more in comparison to linear non-homogeneity parameter, $\alpha=2$ as the rate of decrease of normalised axial load is more for $\delta=2$. With increase in relative length $L/d=20$, the rate of decrease of normalised axial load increases in comparison to shorter pile ($L/d=10$).

Figure 28 shows the variation of the percentage load transferred to granular pile base with stiffness parameter, K_{gp0} for $\alpha = 0$ and 2, $L/d = 10$ along with the effect of non-linear non-homogeneity parameter, γ . The load transferred to the base of granular pile, increases with the increase of stiffness of granular pile. The % load transferred to base decreases with the increase of γ , due to increment in shear stresses transferred to the middle stiff soil around granular pile. This reduction in percentage base load with non-linear non-homogeneity parameter, γ increases with the increase of stiffness parameter.

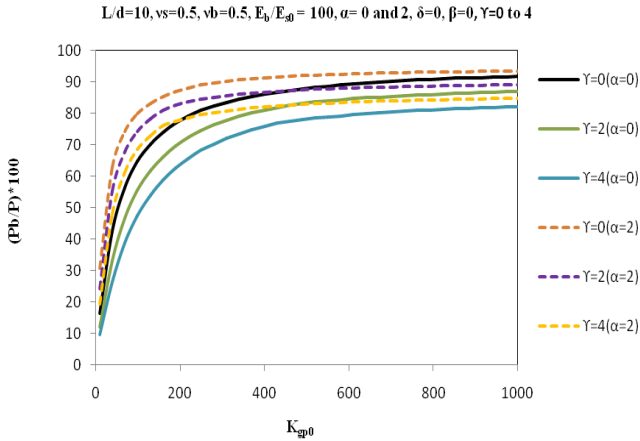


Figure 28 Variation of the percentage base load with relative stiffness parameter, K_{gp0} ($L/d=10$)

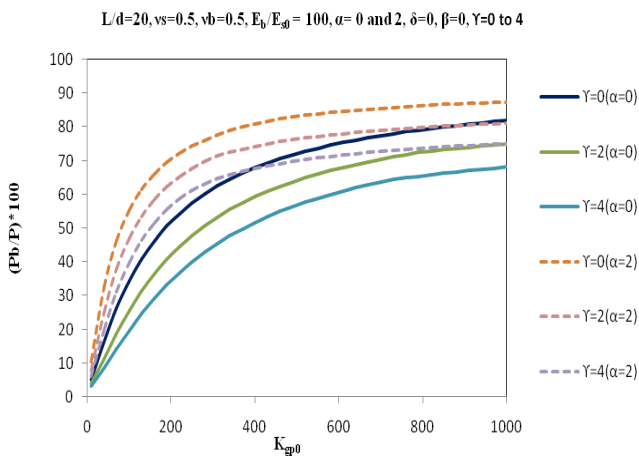


Figure 29 Variation of the percentage base load with relative stiffness parameter, K_{gp0} ($L/d=20$)

The variation of the percentage load transferred to granular pile base with stiffness parameter, K_{gp0} is presented in Figure 29 for $\alpha = 0$ and 2, $L/d = 20$ along with the effect of non-linear non-homogeneity parameter, Y . For the longer granular pile, the base load is very small in the lower range of the stiffness parameter, K_{gp0} , due to great depth of the bearing stratum resulting. Thus the effect of non-linear non-homogeneity parameter, Y is much less in transferring the load to the base. As the stiffness increases, the decrement in base load for longer GP ($L/d = 20$) with Y is more as compared to the decrement for shorter pile ($L/d = 10$).

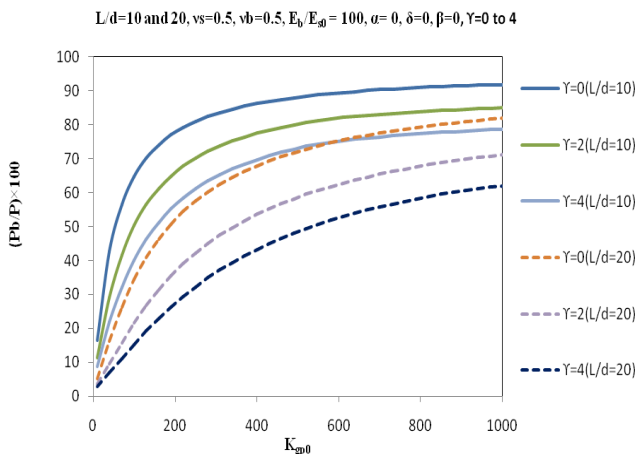


Figure 30 Variation of the percentage base load with relative stiffness parameter, K_{gp0} ($L/d=10$ and 20)

The variation of the percentage load transferred to granular pile base with stiffness parameter, K_{gp0} is presented in Figure 30 for $L/d = 10$ and 20 along with the effect of non-linear non-homogeneity parameter. For the longer granular pile, the base load is very small in the lower range of the stiffness parameter, K_{gp0} , due to great depth of the bearing stratum resulting. Thus the effect of Y is much less in transferring the load to the base.

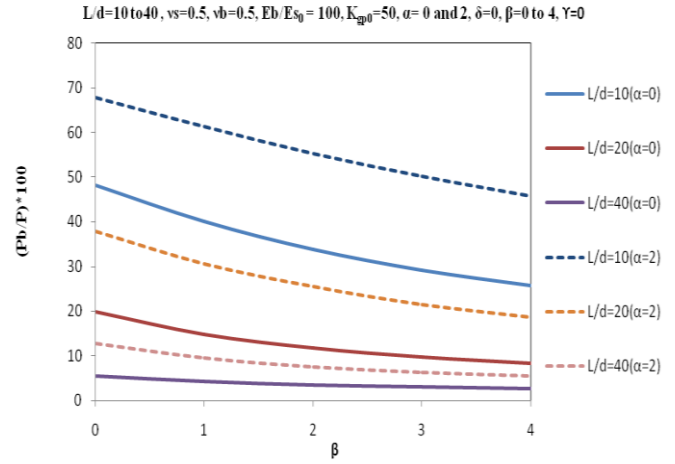


Figure 31 Variation of the percentage base load with non-homogeneity parameter, β ($K_{gp0}=50$)

In Figure 31 the base load increases with the increase of linear non-homogeneity parameter of granular pile, $\alpha = 0$ to 2, due to transfer of load from upper compressible part to stiffer lower portion of granular pile. The % base load for relative length $L/d=10$, $\alpha = 0$ and 2, are approximately 50.23 and 69.81 respectively for $\beta=2$, and for relative length $L/d=20$, $\alpha = 0$ and 2, are approximately 21.54 and 41.22 respectively for $\beta=2$. Similarly, with increase in non-homogeneity parameter, β of soil % base load decreases for all lengths of granular pile. It is also observed that with increase in relative length L/d 10 to 40 and $\beta = 0$ to 4 ($\alpha=0$ and 2) the base load decreases.

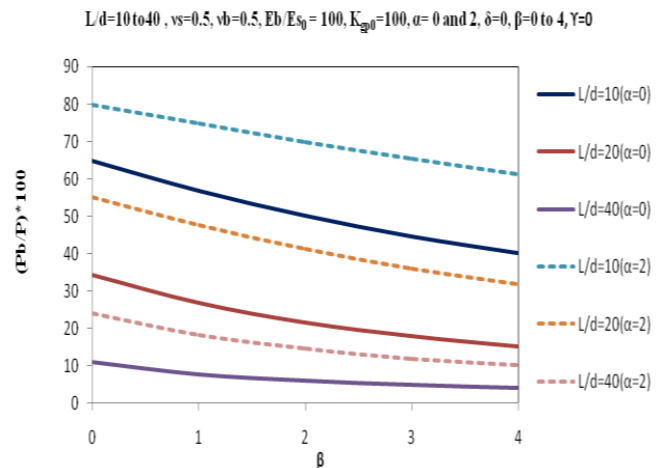


Figure 32 Variation of the percentage base load with non-homogeneity parameter, β with effect of relative length, L/d ($K_{gp0}=100$)

The effect of relative length of granular pile on the variation of percentage base load is depicted in Figure 32 with $\beta = 0$ to 4 for $K_{gp0}=100$ and $\alpha = 0$ & 2. The rate of decrease of % load transferred to the base with β is slightly more in the case of relative lengths, $L/d = 10$ to 40 and linear non homogeneity parameter, $\alpha=2$ of granular pile. For $K_{gp0} = 50$. The rate of decrease of percentage base load increases with the decrease of relative stiffness of granular pile.

Figure 33 and Figure 34 shows the variation of percentage base load, $(P_b/P) \times 100$, with relative stiffness of bearing stratum, E_b/E_{s0} , for different non-homogeneity parameter (β) and non linear non-homogeneity parameter, γ with effect of relative length of granular pile (L/d) for $K_{gp0} = 100$. The base load increases both with relative stiffness of the bearing stratum (E_b/E_{s0}) and the non-homogeneity parameter, β . The percentage decrement in base load for non-homogeneity parameter, β increasing from 0 to 2 is more in comparison to non-homogeneity parameter, γ increasing 0 to 2 for any relative length of the pile. In case of long granular pile, the percentage base load is less in comparison to the base load for a short granular pile. The effect of non-homogeneity of granular pile on base load increases with the increase in the relative stiffness of the bearing stratum. For $E_b/E_{s0} = 1000$ the bearing stratum is almost rigid and the percentage base load becomes nearly constant with further increase in E_b/E_{s0} .

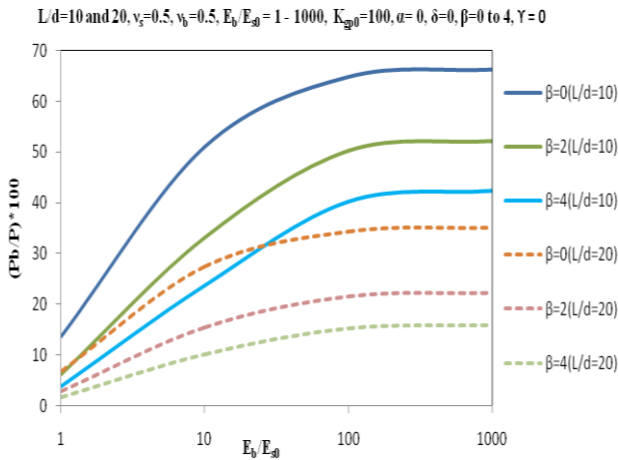


Figure 33 Variation of percentage base load, $(P_b/P) \times 100$, with relative stiffness of bearing stratum, E_b/E_{s0} , for different non-homogeneity parameter, β

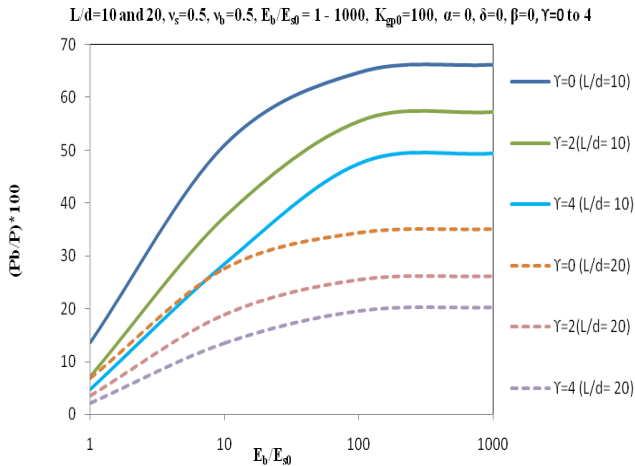


Figure 34 Variation of percentage base load, $(P_b/P) \times 100$, with relative stiffness of bearing stratum, E_b/E_{s0} , for different non-homogeneity parameter, γ

Figure 35 to Figure 39 shows the relative comparison for settlement influence factors for average and exact analysis for same set of parameters. The percentage difference in variation of settlement influence factors are given in Table 2. It can be seen that by average analysis the values of settlements influence factors are underestimated in comparison to exact analysis which is not suitable. From Figure 35 to Figure 39 and table it is concluded that by average moduli of granular pile and soil, the results are underestimated in comparison to the exact method. Hence the

method of averaging moduli is not suitable for estimating the non-linear behavior of soil and granular pile in terms of deformation modulus.

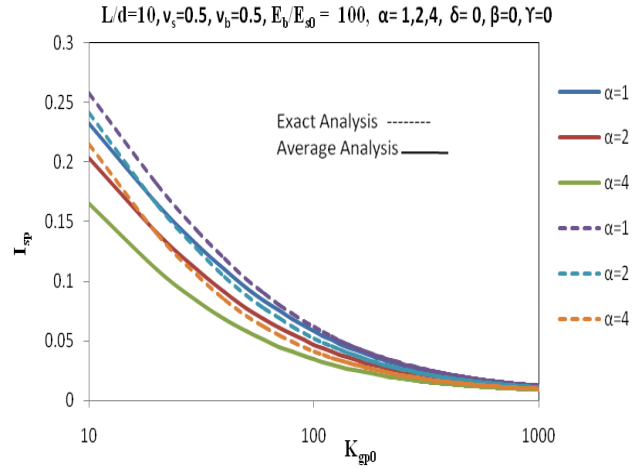


Figure 35 Comparative analyses for variation of settlement influence factor ' I_{sp} ' with relative stiffness parameter, K_{gp0} for GP with effect of linear non-homogeneity parameter, α

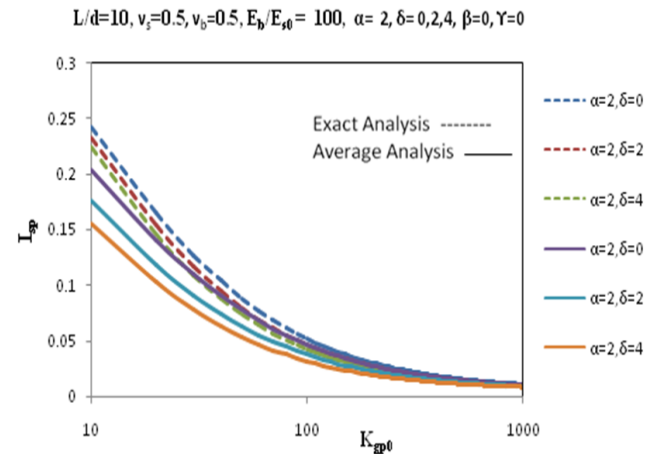


Figure 36 Comparative analyses for variation of settlement influence factor ' I_{sp} ' with relative stiffness parameter, K_{gp0} for granular pile with effect of non-homogeneity parameter, δ

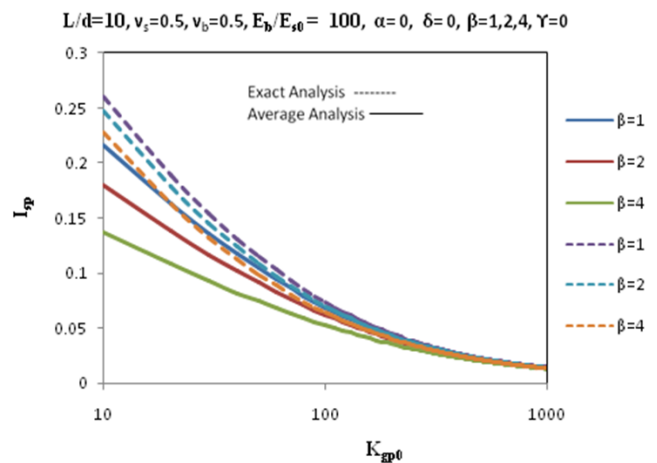


Figure 37 Comparative analysis for variation of settlement influence factor ' I_{sp} ' with relative stiffness parameter, K_{gp0} for soil with effect of linear non-homogeneity parameter, β ($\alpha=0$, $\delta=0$ and $\gamma=0$)

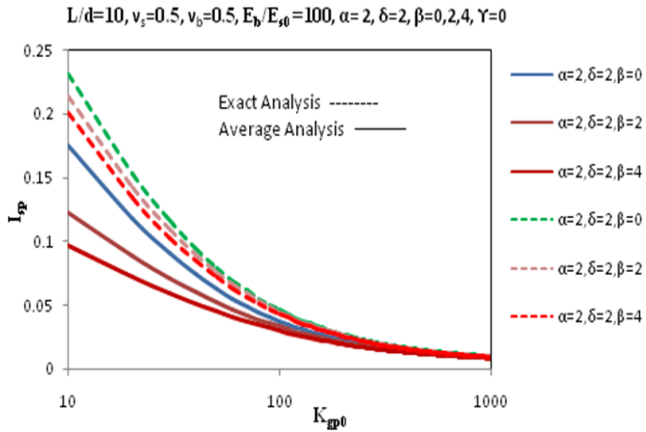


Figure 38 Comparative analysis for variation of settlement influence factor 'I_{sp}' with relative stiffness parameter, K_{gp0} for soil with effect of linear non-homogeneity parameter, β ($\alpha=2$, $\delta=2$ and $\gamma=0$)

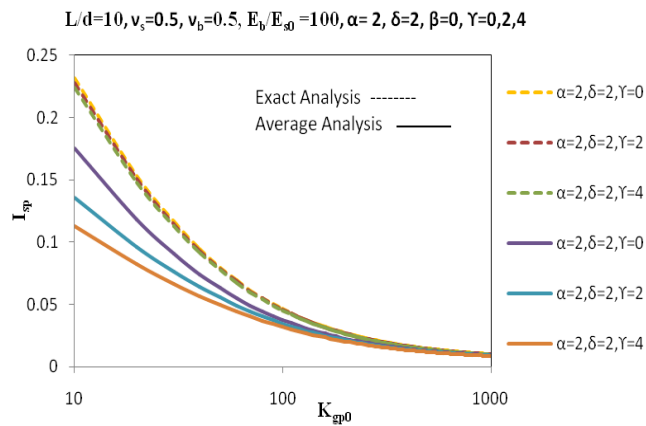


Figure 39 Comparative analysis for variation of settlement influence factor 'I_{sp}' with relative stiffness parameter, K_{gp0} ($E_b/E_{s0} = 100$) for soil with effect of non-linear non-homogeneity parameter, γ

Table 2 Comparative analysis for variation of settlement influence factor 'I_{sp}' with relative stiffness parameter, K_{gp0} for granular pile and soil

Figure 35 Non-homogeneity parameter of granular pile			
Relative stiffness, K _{gp0} =100	$\alpha=1$, $\delta=0$	$\alpha=2$, $\delta=0$	$\alpha=4$, $\delta=0$
Average Analysis	0.0578	0.0466	0.0343
Exact Analysis	0.0613	0.05203	0.0412
% Difference	6.14	11.7	20.16

Figure 36 Non-homogeneity parameters of granular pile			
Relative stiffness, K _{gp0} =100	$\alpha=2$, $\delta=0$	$\alpha=2$, $\delta=2$	$\alpha=2$, $\delta=4$
Average Analysis	0.0466	0.0375	0.0317
Exact Analysis	0.0520	0.04623	0.03936
% Difference	11.7	23.28	23.974

Figure 37 Non-homogeneity parameter of Soil			
Relative stiffness, K _{gp0} =100	$\beta=1$	$\beta=2$	$\beta=4$
Average Analysis	0.069	0.0624	0.0529
Exact Analysis	0.0736	0.0702	0.0645
% Difference	6.75	12.44	21.93

Figure 38 Linear and Non-linear Non-homogeneity Parameters of GP and soil			
Relative Stiffness, K _{gp0} =100	$\alpha=2$, $\delta=2$, $\beta=0$	$\alpha=2$, $\delta=2$, $\beta=2$	$\alpha=2$, $\delta=2$, $\beta=4$
Average Analysis	0.0375	0.0332	0.0299
Exact Analysis	0.04623	0.04465	0.04308
% Difference	23.28	34.5	44.08

Figure 39 Linear and Non-linear Non-homogeneity Parameters of GP and soil			
Relative Stiffness, K _{gp0} =100	$\alpha=2$, $\delta=2$, $\gamma=0$	$\alpha=2$, $\delta=2$, $\gamma=2$	$\alpha=2$, $\delta=2$, $\gamma=4$
Average Analysis	0.0375	0.0345	0.032
Exact Analysis	0.04623	0.0458	0.04524
% Difference	23.28	32.75	41.37

5. CONCLUSIONS

The present analysis of the behavior of non-homogeneous granular pile in non-homogeneous soil is carried out using elastic continuum approach and is based on finding out the stress system $\{\tau\}$, along the soil-granular pile interface and the base stress $\{p_b\}$, which satisfy the compatibility of displacements along the interface for no slip or yield condition as discussed by Mattes et al. (1969).

Formulation for pile elemental displacement equations incorporating the non-homogeneity parameter, α and δ for end bearing granular pile, in non-homogeneous soil having parameters β and γ is presented. Consideration of non-homogeneity of granular pile in the analysis reflects its true behaviour and accounts for the changes in the state of the end bearing granular pile and the in situ non-homogeneous soil due to installation, stiffening and improvement effects. The reductions in settlement for end-bearing non-homogeneous granular pile are in the range of 20 to 40 % with respect to the settlement of a homogeneous granular pile depending on the relative stiffness of the bearing stratum and the degrees of non-homogeneities, α and δ . Comparative study also made for average and exact analysis and it is found that by average moduli of granular pile and soil, the results are underestimated in comparison to the exact method. Hence it can be concluded that the method of averaging moduli is not suitable for estimating the non-linear behavior of soil and granular pile in terms of deformation modulus.

The effect of non-homogeneity parameters on settlement is shown in Table 3 and Table 4. It can be concluded that effect of linear non-homogeneity parameter of granular pile and soil is more pronounce in comparison to non-linear non-homogeneity parameter, in the decrement of settlement. It is also observed that with increase in non-homogeneity parameters of granular pile from 0 to 4, the decrease of settlement is more for linear non-homogeneity parameter relative to non-linear non-homogeneity parameter. Same trend is observed for non-homogeneity parameters of soil as shown in Table 3. Similarly, for soil with increase in non-homogeneity parameter from 0 to 4 the rate of decrease of settlement is more as shown in Table 4.

Table 3 Comparative effects of non-homogeneity parameters of granular pile on settlement influence factor in homogeneous soil			
L/d=10 (K _{gp0} =50)	% decrease in Settlement	L/d=10 (K _{gp0} =50)	% decrease in Settlement
$\delta=0$, $\beta=0$, $\gamma=0$		$\alpha=0$, $\delta=0$, $\gamma=0$	
α I _{sp}		δ I _{sp}	
0 0.1378	30.98%	0 0.13776	17.93%
4 0.0951		4 0.11283	

Table 4 Comparative effects of non-homogeneity parameters of soil on settlement influence factor in homogeneous granular pile			
L/d=10 (K _{gp0} =50)	% decrease in Settlement	L/d=10 (K _{gp0} =50)	% decrease in Settlement
$\alpha=0$, $\delta=0$, $\gamma=0$		$\alpha=0$, $\delta=0$, $\beta=0$	
β I _{sp}		γ I _{sp}	
0 0.13776	23.02%	0 0.13776	10.67%
4 0.10606		4 0.12298	

Non-homogeneity of end bearing granular pile has a marked influence on the variation of mobilized shear stresses along the granular pile-soil interface with depth. Shear stresses decreases more for a shorter pile in comparison to longer pile up to 65% depth of granular pile. In the region near the bearing stratum of end

bearing GP, the shear stresses are obtained negative, i.e., an effect similar to down drag. With the increase in relative length $L/d=20$, the rate of decrease of normalized axial load increases in comparison to a shorter pile ($L/d=10$). As in case of a longer granular pile, a large part of the load is transferred through interface shear stresses due to a great depth of the bearing stratum. The load transferred to the base of granular pile, increases with the increase of stiffness of granular pile.

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7. LIST OF SYMBOLS

GP	Granular Pile
L	Length of granular pile
D	Diameter of GP = (2a)
S	Spacing of GPs
P	Load on GP
E_{gp}	Deformation modulus of granular pile material
E_s, ν_s	Deformation modulus and Poisson's ratio of soil
E_{SL}	Deformation modulus of soil at the base
E_{s0}	Deformation modulus of soil at the surface
K_b	Granular pile stiffness at base = (E_{gp}/E_{SL})
K_{gp0}	Relative stiffness of granular pile = (E_{gp0}/E_s)
τ	Shear stresses at GP-soil interface
P_b	Pile base pressure
'n'	Total number of elements of GP
I_{sp}	Soil displacements influence factor
E_{gp0}	Stress-independent deformation modulus or deformation modulus at the top of granular pile
τ^*	Normalized shear stresses of GP = ($\tau / (P/\pi dL)$)
$z^* (= z/L)$	Normalized depth of GP
α and δ	Degrees of non-homogeneity of granular pile
β and γ	Degrees of non-homogeneity of soil

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