Solution for Partially Penetrated Vertical Drain with Anchor Plate

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ABSTRACT: This paper presents a solution for partially penetrated prefabricated vertical drains (PVD) with less computational time and more accuracy than previous analytical solutions. A new impermeable boundary region at drain's end is introduced to simplify boundary conditions and reduce the number of variables. Moreover, Laplace transform technique is utilized in this solution which could help to estimate smooth excess pore pressures with depth even in the early stage of consolidation. Results of the new modified solution have been verified against results of the current literatures. A case study by vacuum consolidation technique is presented to verify the current solution. Excess pore water pressure in the soil and the drain and are well agreement with the proposed solution. Derived degree of consolidation from field settlements shows not much differences than current approach. Those verifications indicate that the proposed solution is an effective tool in solving the consolidation problems of partially penetrated vertical drain.

KEYWORDS: Partially penetrated prefabricated vertical drain, Anchor plate, Analytical solution, Laplace transform technique, Soft soil.

1. INTRODUCTION

Prefabricated vertical drains (PVDs) are widely used for soft soil improvement because it is a cost-effective and time-saving solution. There are different anchor's types depending on construction methods and installed machines (Bo et al., 2015). In specific, steel plate anchor is the most applicable type in the PVD's installation because it can be easily assembled and can protect soil intrusion into mandrel as shown in Figure 1. The anchor steel plate blocks drainage at the PVD's end, which creates an impermeable boundary condition. Nevertheless, extensive past studies mainly consider this region as a permeable or pervious drain's end (Tang and Onitsuka (1998 and 2001) , Indraratna et al..(2000, 2005 and 2008), Chai et al. (2001), Indraratna and Rujikiatkamjorn (2008), Ong et al. (2012), Zeng and Xie (2013) Voottipruex et al. (2014), Bergado et al. (2020) and Lam et al. (2015)). Various analyzing methods have been adopted such as analytical methods, semi-analytical methods, and finite element methods.

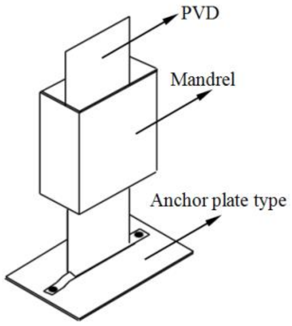


Figure 1 Mandrel and steel anchor plate.

Studies accounting for the impermeable boundary condition are relatively limited. For example, Nghia et al. (2018) considered the impermeable boundary condition into semi-analytical solution with Fourier series approximation technique and was verified by Finite element method (FEM). This research pointed out that the analytical solution by Fourier series approximation technique was inaccurate at the early stage of consolidation because of its fluctuations in estimating excess pore pressure. Hence, this paper presents a solution addressing the impermeable boundary condition at the PVD's end incorporated with Laplace transform technique for governing equations. Laplace transform technique can extend applications of analytical solutions to multi-ramp loading, multi-soil layers or even applied vacuum pressures (Geng et al. (2011, 2012) and Liu et al. (2014) Nghia-Nguyen et al. (2019)). Moreover, this technique could show less fluctuation in excess pore pressure estimation than the current analytical solution by Fourier series approximation technique (Geng et al. (2011)).

This paper presents a solution for penetrated vertical drain applying the Laplace transform technique in combination with the new impermeable drain's end boundary condition. Variation of the excess pore pressure during the consolidation process is further investigated. Past studies and a case history are employed to verify the application of the proposed solution. The general model for partially penetrated vertical drains is shown in Figure 2 (H = whole thickness of soil; h_1 = PVD length ; h_2 = length of soil without PVD; r_w = PVD radius; r_s = radius of smeared zone; r_e = equivalent radius of the influence zone; k_h = soil coefficient of horizontal permeability; k_{v1} = coefficient of vertical permeability of the soil with PVD; m_{v1} = coefficient of volume compressibility of the soil with PVD; k_s = coefficient of horizontal permeability of smeared zone; k_w = PVD coefficient of permeability; k_{v2} = coefficient of vertical permeability of soil without PVD; m_{v2} = coefficient of volume compressibility of the soil without PVD). There are two conditions of drainage boundaries including Pervious Top and Impervious Bottom (PTIB) and Pervious Top and Pervious Bottom (PTPB).

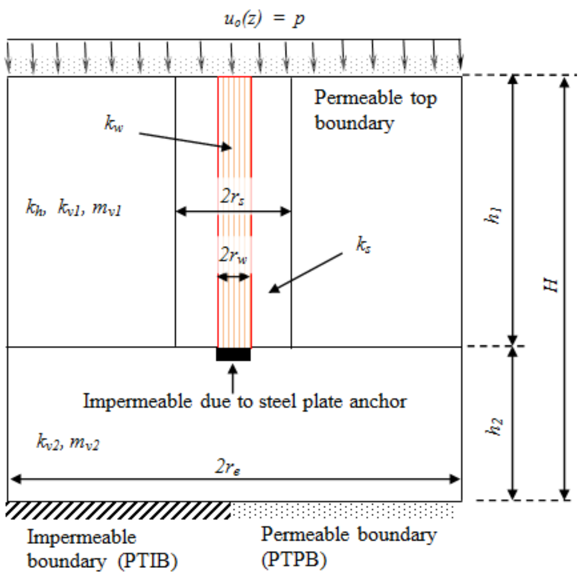


Figure 2 Analysis scheme of partially penetrated PVD

2. BASIC EQUATIONS

Govern equations used in this study are based on basic equations and assumptions by Tang and Onitsuka (1998 and 2001). For the section with vertical drains, $0 < z < h_1$, the governing partially differential equations for pore pressure dissipation are:

+ Smear zone, $r_w \le r \le r_s$

$$\frac{k_{s1}}{m_{v1}\gamma_w}\left(\frac{1}{r}\frac{\partial u_{s1}}{\partial r}+\frac{\partial^2 u_{s1}}{\partial r^2}\right)+\frac{k_{v1}}{m_{v1}\gamma_w}\frac{\partial \bar{u}_1}{\partial z^2}=\frac{\partial^2 \bar{u}_1}{\partial t} \quad (1)$$

+ Natural soil, $r_s \le r \le r_e$

$$\frac{k_{h1}}{m_{v1}\gamma_w}\left(\frac{1}{r}\frac{\partial u_{n1}}{\partial r}+\frac{\partial^2 u_{n1}}{\partial r^2}\right)+\frac{k_{v1}}{m_{v1}\gamma_w}\frac{\partial \bar{u}_1}{\partial z^2}=\frac{\partial^2 \bar{u}_1}{\partial t} \quad (2)$$

For the section without vertical drains, $h_1 \le z \le H$, the basic equation is:

$$\frac{k_{v2}}{m_{v2}\gamma_w}\frac{\partial^2 u_2}{\partial z^2}=\frac{\partial u_2}{\partial t} \quad (3)$$

where r = radial coordinate; z = vertical coordinate; t = time; γ_w = unit weight of water; $u_{s1}(r,z,t)$ excess pore water pressure in the smeared zone; $u_{n1}(r,z,t)$ = excess pore water pressure at any point in the natural soil zone of the section with vertical drains; $\bar{u}_1(z,t)$ = average excess pore water pressure of the section with vertical drains; and. $u_2(z,t)$ = excess pore water pressure of the section without vertical drains at any depth.

Average excess water pressure on horizontal plane is determined as:

$$\bar{u}_1=\frac{1}{\pi(r_e^2-r_w^2)}\left(\int_{r_w}^{r_s}2\pi r u_{s1}dr+\int_{rs}^{r_e}2\pi r u_{n1}dr\right) \quad (4)$$

The continuity conditions and boundary conditions are:

$$\frac{\partial^2 u_w}{\partial z^2}=-\frac{2}{r_w}\frac{k_{s1}}{k_w}\left(\frac{\partial u_{s1}}{\partial r}\right)\bigg|_{r=r_w} \quad (5)$$

$$\frac{\partial u_{n1}}{\partial r}\bigg|_{r=r_e}=0 \quad (6)$$

$$u_{s1}\big|_{r=r_w}=u_w\big|_{r=r_w} \quad (7)$$

$$u_{s1}\big|_{r=r_s}=u_{n1}\big|_{r=r_s} \quad (8)$$

$$k_{s1}\frac{\partial u_{s1}}{\partial r}\bigg|_{r=r_s}=k_{h1}\frac{\partial u_{n1}}{\partial r}\bigg|_{r=r_s} \quad (9)$$

$$u_w\big|_{z=0}=\begin{cases}0 & \text{Boundary case of pervious top}\\ P_{av}(t) & \text{Boundary case of vacuum pressure}\end{cases} \quad (10)$$

$$\bar{u}_1\big|_{z=0}=\begin{cases}0 & \text{Boundary case of pervious top}\\ P_{av}(t) & \text{Boundary case of vacuum pressure}\end{cases} \quad (11)$$

$$\frac{\partial u_2}{\partial z}\bigg|_{z=H}=0 \quad \text{(Impervious bottom)} \quad (12)$$

$$u_2\big|_{z=H}=0 \quad \text{(Pervious bottom)} \quad (13)$$

The new impermeable boundary condition at the PVD's end $z = h_1$ leads to the boundary and continuity conditions at this zone as:

$$\frac{\partial u_w}{\partial z}\bigg|_{z=h_1}=0 \quad \text{(Impermeable at the drain's end)} \quad (14)$$

$\bar{u}_1\big|_{z=h_1}=u_2\big|_{z=h_1}$ (15) (Equal pore pressure of sections with vertical drain and without vertical drain)

$k_{v1}\pi(r_e^2-r_w^2)\frac{\partial \bar{u}_1}{\partial z}\bigg|_{z=h_1}=k_{v2}\pi r_e^2\frac{\partial u_2}{\partial z}\bigg|_{z=h_1}$ (16) (Continuity condition)

3. ANALYTICAL SOLUTION

Substitute (5) into (1) to obtain the partially differential equation for only $u_w(z,t)$ (Tang and Onitsuka(2001))

$$c_{v1}\frac{\partial^4 u_w(z,t)}{\partial z^4}-\frac{\partial^3 u_w(z,t)}{\partial z^2\partial t}-c_{h1}\varphi_1\frac{\partial^2 u_w(z,t)}{\partial z^2}+\varphi_2\frac{\partial u_w(z,t)}{\partial t}=\varphi_2\frac{\partial \sigma(t)}{\partial t} \quad (17)$$

Excess pore pressure in the drain and average excess pore pressure in soil is defined as.

$$\frac{\partial^2 u_w(z,t)}{\partial z^2}=-\varphi_2(\bar{u}_1(z,t)-u_w(z,t))=0 \quad (18)$$

where

$$c_{v1}=\frac{k_{v1}}{m_{v1}\gamma_w}, c_{h1}=\frac{k_{h1}}{m_{v1}\gamma_w}, s=\frac{r_s}{r_w}, n=\frac{r_e}{r_w},$$

$$\varphi_1=\frac{2}{r_e^2 F}\left[1+(n^2-1)\frac{k_{v1}}{k_w}\right], \varphi_2=(n^2-1)\frac{2}{F}\frac{k_{h1}}{k_w}\left(\frac{1}{r_e}\right)^2,$$

$$F=\left(\ln\frac{n}{s}+\frac{k_{h1}}{k_{s1}}\ln s-\frac{3}{4}\right)\frac{n^2}{n^2-1}+\frac{s^2}{n^2-1}\left(1-\frac{s^2}{4n^2}\right)+\frac{k_{h1}}{k_{s1}}\frac{1}{n^2-1}\left(1-\frac{1}{4n^2}\right)$$

Further, Laplace transform technique is applied to equation (15) yield.

$$c_{v1}\frac{\partial^4 Lu_w(z,s)}{\partial z^4}-(c_{h1}\varphi_1+s)\frac{\partial^2 Lu_w(z,s)}{\partial z^2}+(\varphi_2 s)Lu_w(z,s)-\varphi_2 u_o=s\sigma(\text{s}) \quad (19)$$

where

$Lu_w(z,s)$ is the Laplace transform of $u_w(z,t)$

$u_w(z,0)=u_o$ is the initial pore pressure distribution in soil

$\sigma(s)$ is the Laplace transform of surcharge load $\sigma(t)$

Solution of the equation (19) is

$$Lu_w(z,s) = X_1e^{a_1z} + X_2e^{-a_1z} + X_3e^{a_2z} + X_4e^{-a_2z} + \frac{u_o}{s} - \sigma(s) \quad (20)$$

where

$$a_1 = \sqrt{\frac{c_{h1}\varphi_1 + s - \sqrt{(c_{h1}\varphi_1 + s)^2 - 4\varphi_2 c_{v1}s}}{2c_{v1}}}$$

$$a_2 = \sqrt{\frac{c_{h1}\varphi_1 + s + \sqrt{(c_{h1}\varphi_1 + s)^2 - 4\varphi_2 c_{v1}s}}{2c_{v1}}}$$

In addition, Laplace transform technique is also applied to equation (16) yield.

$$\frac{\partial^2 Lu_w(z,s)}{\partial z^2} + \varphi_2 L\bar{u}_1(z,s) - \varphi_2 Lu_w(z,s) = 0 \quad (21)$$

where $L\bar{u}_1(z,s)$ is Laplace transform of $\bar{u}_1(z,s)$

Substitute equation (20) into (21) to obtain the Laplace transform of average excess pore pressure distribution in the soil with PVD.

$$L\bar{u}_1(z,s) = \left(1-\frac{a_1^2}{\varphi_2}\right)X_1e^{a_1z} + \left(1-\frac{a_1^2}{\varphi_2}\right)X_2e^{-a_1z} + \left(1-\frac{a_2^2}{\varphi_2}\right)X_3e^{a_2z} + \left(1-\frac{a_2^2}{\varphi_2}\right)X_4e^{-a_2z} + \frac{u_o}{s} - \sigma(s) = 0 \quad (22)$$

Using Laplace transform technique for equation (4) to obtain to excess pore pressure in the soil without PVD

$$c_{v2}\frac{\partial^2 Lu_2(z,s)}{\partial z^2} = s \times Lu_2(z,s) - u_o + s\sigma(s) \quad (23)$$

where $Lu_2(z,s)$ is the Laplace transform of $u_2(z,t)$

Partially derivative equation (23) can be derived as:

$$Lu_2(z,s) = X_5e^{b_1z} + X_6e^{-b_1z} + \frac{u_o}{s} - \sigma(s) \quad (24)$$

where $b_1 = \sqrt{\frac{s}{c_{v2}}}$

Solutions for the average excess pore water pressure and the average degree of consolidation are presented in Appendix A. The average excess pore water pressures in the soil with PVD and without PVD can be determined in equations (A4) and (A5), respectively. Moreover, the average degree of consolidation can be obtained by equation (A6).

4. VALIDATION AND DISCUSSION

The proposed solution is validated for the excess pore pressure and degree of consolidation. Pore pressure dissipation estimated by the proposed method are compared with results obtained by two previous works of Nghia et al. (2018) and Tang and Onitsuka (1998). Figures 3(a) and 3(b) show the comparison of the excess pore water pressures at degree of consolidation of 50% (U=50%) obtained by the three solutions. Tang and Onitsuka's approach shows the excess pore pressure) reduces rapidly at the PVD's end (z = 0.8H) while it continuously increases at this boundary in other two solutions. The unrealistic situation may come from Tang and Onitsuka (1998)'s assumption that the pore pressure of the drain at the PVD's end is equal to the pore pressure of the soil without PVD ($z=h_1, u_2=u_w$). Meanwhile, other two solutions consider the new impermeable region boundary ($z=h_1, \frac{\partial u_w}{\partial z}=0$). In addition, Figure 3(a) and 3(b) also indicate the excess pore pressure by Nghia'solution (2018) is more fluctuated than that obtained by the proposed solution. In other words, the current approach can predict a smooth excess pore pressure distribution with depth.

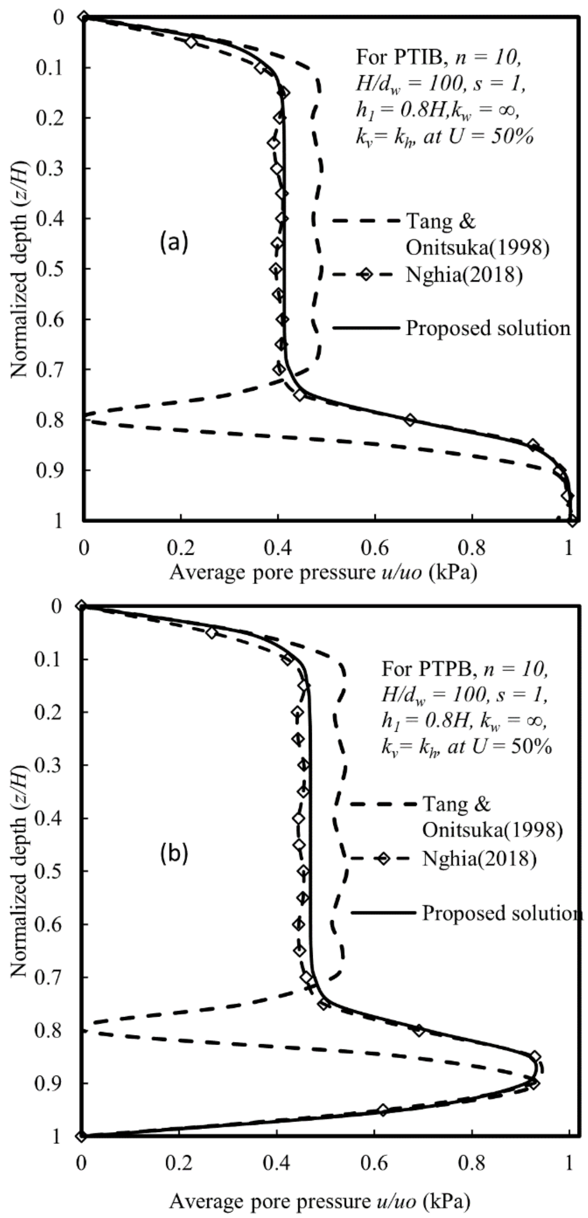


Figure 3 (a) Comparison pore pressure in case impervious bottom (PTIB), (b) Comparison pore pressure in case pervious bottom (PTPB)

Figure 4 shows comparisons in terms of three different consolidation degrees at U=30%, 50%, and 80% of the proposed solution and Nghia's solution (2018) for two boundary condition cases of PTIB and PTPB. Apparently, the pore pressures determined by the two techniques are in close agreement especially in large consolidation degree such as U=80%. While at the early stage of consolidation of U=30%, the pore pressure by Nghia's solution (2018) scatter significantly in comparison with pore pressure solved by the proposed solution.

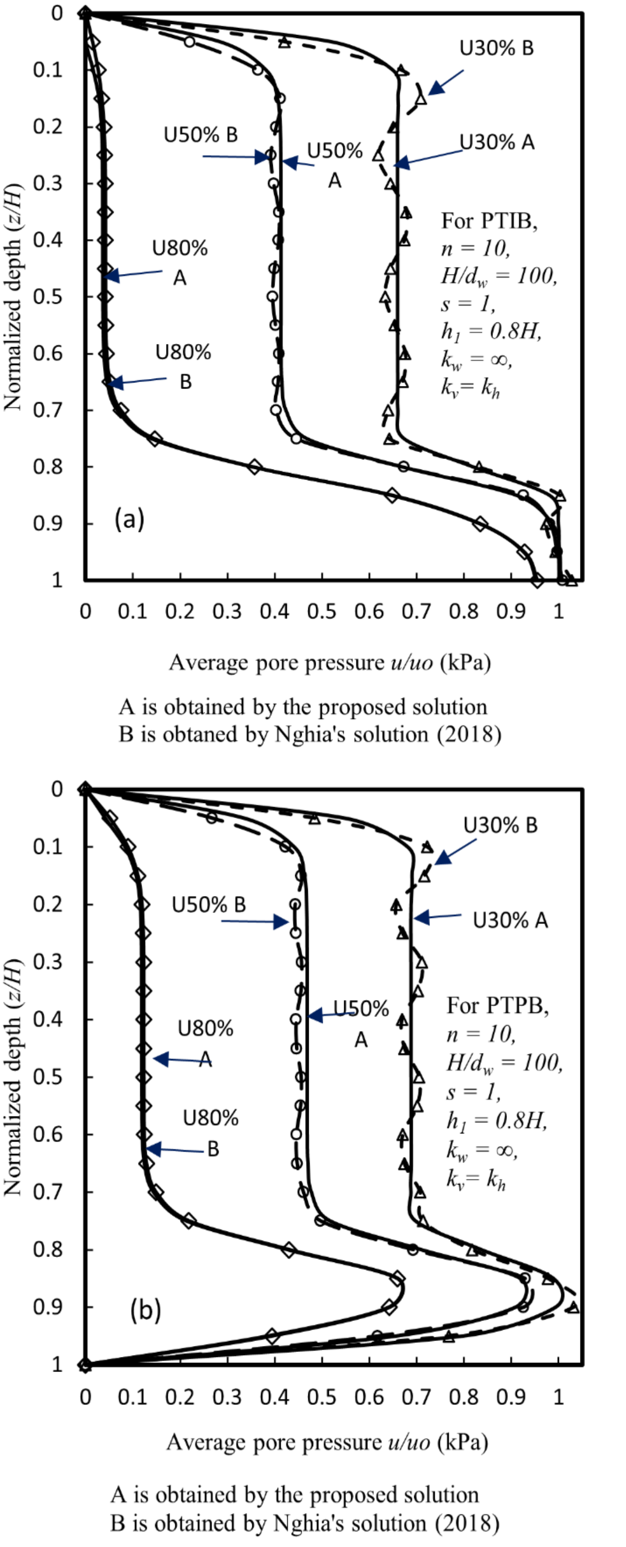


Figure 4 (a) Pore pressure with depth by present solution in case impervious bottom (PTIB), (b) Pore pressure with depth by present solution in case pervious bottom (PTPB)

Moreover, in order to investigate dissipation process of the excess pore pressure with depth and time, the pore pressure variation obtained by the proposed solution for two cases of PTIB and PTPB is mapped with the 10% interval of consolidation degree from 10% to 95% (Figure 5). It can be seen that the soil with PVD consolidates considerably faster than the soil without PVD. For example, under PTIB condition from early stage of consolidation to U=70% the excess pressure in PVD dissipates significantly meanwhile the water pressure without PVD does not experience any change. On the other hand, from U=70% to U=95% the soil without PVD starts dissipating at higher speed and at U=95% the remaining excess pore pressure is about 30% of the original value. As a result, one can visually understand the consolidation process along the depth in the soil with and without PVD and can determine the necessary depth of the PVD for a particular soil improvement project. It is noted that former analytical solutions such as Nghia's solution (2018) are hard to create the excess pore pressure contour because of their fluctuations.

In addition to excess pore pressure, an estimated solution proposed by Zeng and Xie (1989) for the overall average consolidation degree is employed to compare with the present solution and Tang and Onitsuka's solution (1998). The overall average degree of consolidation for the whole thickness of soil of Zeng and Xie (1989) is derived as:

$$\bar{U}_p = \rho\bar{U}_{rz} + (1-\rho)\bar{U}_z \tag{25}$$

where,

$\bar{U}_{rz}$ = average degree of consolidation for the section with vertical drains.

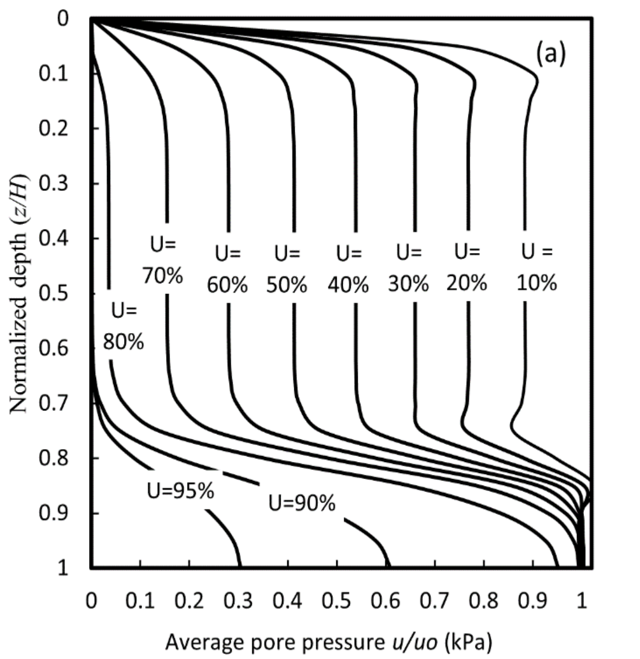
$\bar{U}_z$ = average degree of consolidation for the section without vertical drains.

$\rho = \dfrac{h_1}{H}$ the penetration fraction.

The average consolidation for the section with vertical drains

$$\bar{U}_{rz} = 1 - \frac{8}{\pi^2} e^{-\beta_{rz} t} \tag{26}$$

The average consolidation for the section without vertical drains



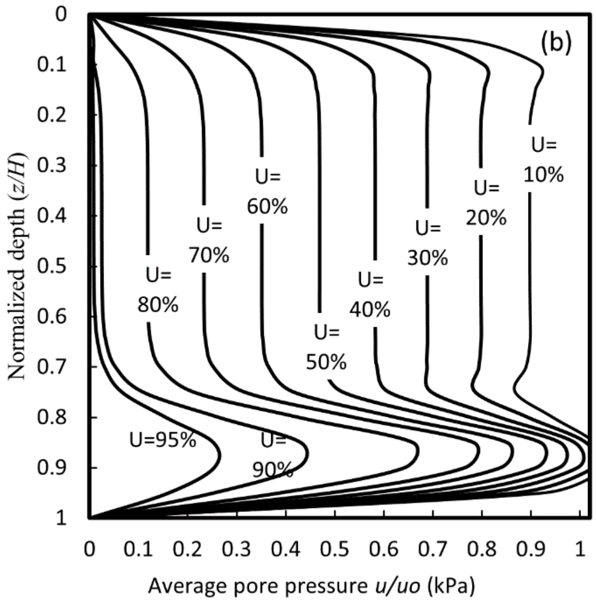


Figure 5 (a) Pore pressure with depth with various consolidation degree by the proposed solution for PTIB, (b) Pore pressure with depth with varied consolidation degree by the proposed solution for PTPB

$$\bar{U}_z = 1 - \frac{8}{\pi^2} e^{-\beta'_z t} \tag{27}$$

where: $\beta'_z = \dfrac{\beta_z}{(1-\alpha\rho)^2}, \beta_z = \dfrac{\pi^2}{4}\dfrac{1}{H^2}\dfrac{k_v}{m_v\gamma_w}, \alpha = 1 - \sqrt{\dfrac{\beta_z}{\beta_{rz}}},$

$$\beta_r = \frac{8}{(2r_e)^2(F+\pi G)}\frac{k_h}{m_v\gamma_w}, \beta_{rz} = \beta_r + \beta_z, G = \frac{k_h}{k_w}\left(\frac{H}{2r_w}\right)^2$$

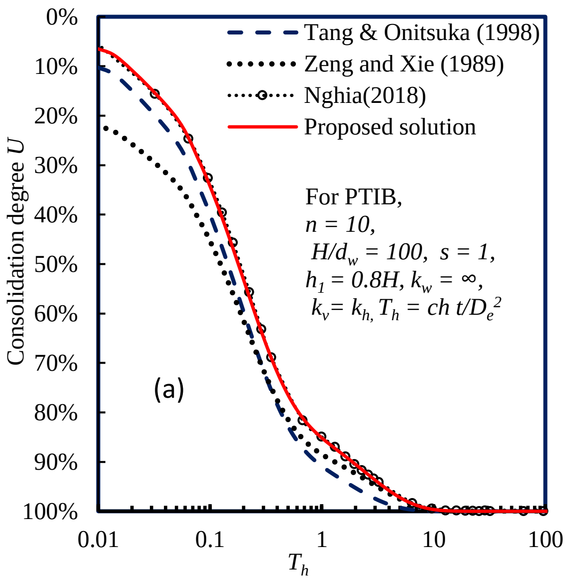
G is well resistance factor

The above expressions are used for the impervious bottom (PTIB) condition. For the pervious bottom (PTPB), the drainage distance, H, in the above expressions should be changed to $\dfrac{H}{2}$.

Zeng and Xia's solution (1989) is presented in two scenarios of without well resistance (Figure 6a) and with well resistance (Figure 6b). The input parameters are also included in these figures. In general, the consolidation degree of the current approach is slower than that from previous solutions of Tang and Onitsuka (1998) and Zeng and Xia (1989). This may come from the assumption of the impermeable boundary at the PVD's end. In contrast, discrepancy between the proposed method and Nghia et al. (2018) is negligible. It is because both methods are derived from the same partially differential equations, the same boundaries and initial conditions. The pore pressure dissipation from Nghia et al. (2018) by the Fourier series approximation technique could be fluctuated; nevertheless, the proposed method and Nghia et al. (2018) lead to the same the total excess pore pressure.

5. A CASE STUDY

A road project as shown in Figure 7 namely R3 Project in District 2, Ho Chi Minh City, Vietnam locating on the soft soil of Sai Gon-Dong Nai River Delta (SDRD) where have been extensively studied with PVD treatment method by Long et al. (2006, 2013 and 2016). R3 project utilized PVD under vacuum consolidation in combination with surcharge preloading to treat the soft soil layer. The PVD was installed with steel anchor plate. The general soil profile including a 22m-thick soft clay layer underlayed by the silty sand layer. At the



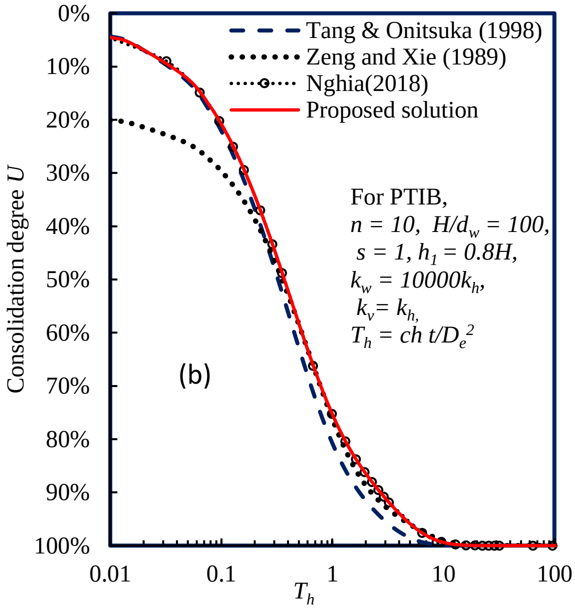


Figure 6 (a) Comparison of average degree of consolidation without well resistance, (b) Comparison of average degree of consolidation with well resistance

section km+2.470 (middle of the road R3 project), the thickness of soft clay layer is 20m; PVD length is only 18m to protect PVD from pressure leaking to silty sand layer as illustrated in Figure 8. PVD was installed in a triangle pattern with spacing of $S = 0.9$ (m); therefore, the influence diameter is $D_e = 1.05S = 0.945$ (m). Soil/drain's properties are summarized in Table 1. The coefficient of vertical consolidation of the soft clay layer $c_v = 1.68\ (\text{m}^2/\text{year})$ is taken from oedometer tests. The coefficient of horizontal consolidation is assumed as double as that value in the vertical consolidation $c_h = 2c_v = 3.36\ (\text{m}^2/\text{year})$. Ratio of permeabilities in the natural zone and the smear zone is assumed to be $k_h/k_s = 5$.

The current section is studied by the proposed solution and a study of Geng et al. (2011). It is because Geng et al. (2011) also applied Laplace transform technique for solution of partially PVD under vacuum consolidation. It is noted that Geng et al. (2011) considered the region around the drain's end as pervious while the proposed solution assumes it as impervious. Further, the non PVD zone was assumed by a virtual drain by Geng et al. (2011). Subsequently, solution from Geng et al. (2011) contains 8 unknown variables; while current solution contains only 6 unknown variables denoted as matrix

X in equations (A1) and (A2). Obviously, the more unknown variables are incorporated into the solution the more computation time is required. Therefore, the proposed solution may save more computation time than Geng et al. (2011).

Earth surcharge and vacuum pressure sequence is presented in Figure 9. Surcharge loading started with a small loading of about 9 kPa which is considered as the existing backfill layer for PVD platform. The next stage of the surcharge increased to 26 kPa at day 35 days after the vacuum pump operation which is the time for vacuum leaking check. Subsequently, the surcharge continuously increased to maximum loading of 81 kPa at 146 days. The maximum load was maintained until the final construction stage. The vacuum pressuremeter was recorded immediately after vacuum pumps were operated. The minimum vacuum pressure under membrane was quickly obtained with -75 (kPa) at 5 days. This minimum vacuum pressure was, then, maintained during the treatment process.

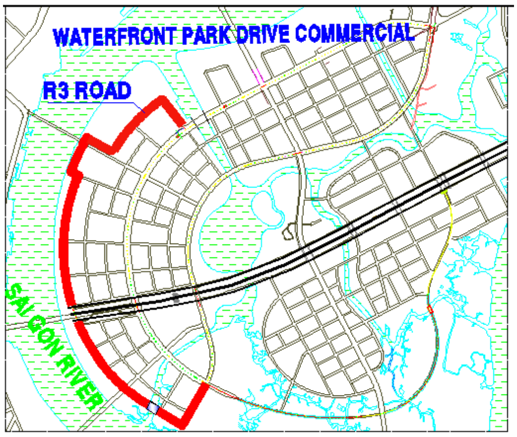


Figure 7 R3 project location

Computational time for determining excess pore pressure $\bar{u}_1(z,t)$ at one specific depth and time ($\bar{u}_1(1,1)$ for instance) by the proposed solution and Geng et al. (2011) is respectively 31.52 seconds and 2025.23 seconds with the same computer and the same Laplace inverse technique. Apparently, the new approach is definitely faster about 62 times than the solution of Geng et al. (2011). As a consequence, the new approach can be time-saving to obtain all output data.

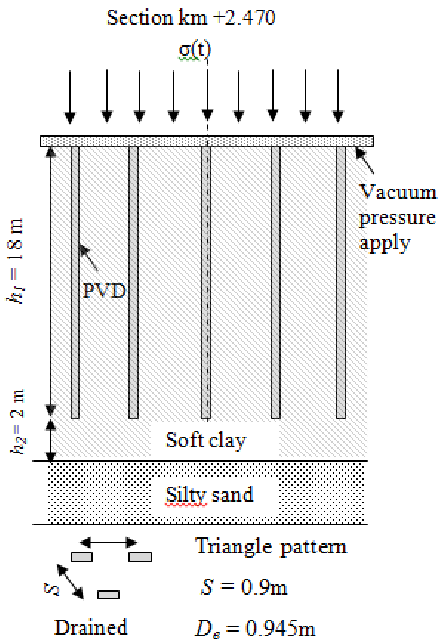


Figure 8 Section of km +2.470

Table 1 Soil/drain's properties

Parameters of soft clay and drain	**Values**
c_v (m^2/year)	1.68
c_h (m^2/year)	3.36
d_w (m)	0.033
D_e (m)	0.945
d_s (m)	0.24
k_h/k_s	5.0
k_w/k_h	100000
m_v (m^2/kN)	1/1200
γ_w (kN/m^3)	9.81

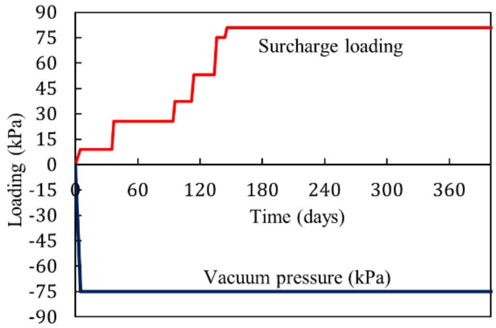


Figure 9 Loading sequence at km+2.470

The excess pore pressure monitored by piezometers at elevation -1m, -7m, and -13m (equivalent with the depth of 2m, 8m and 14m below the ground level, respectively) are compared with solutions obtained by the proposed approach and Geng et al. (2011) (Figure 10). In general, the predicted pore pressure by the proposed method can approximate well the in-situ measurements at various depths especially from early consolidation stage up to 180 days. Nonetheless, after 180 days to the end of the treatment process the discrepancy between predicted pore pressure and field data become more obvious. It may attribute to blocking sediment or equipment problems. Moreover, it is observed that the predicted excess pore pressure by the proposed method is closer to the field data than by Geng et al. (2011). Plus, Geng et al. (2011) shows faster dissipation than that from the proposed approach and faster than the in-situ measurement. It may be due to different assumption of drainage condition at the drain's end by two methods. Thus, the field data implies that considering the impervious boundary at the drain's end is more reasonable in the design than pervious boundary.

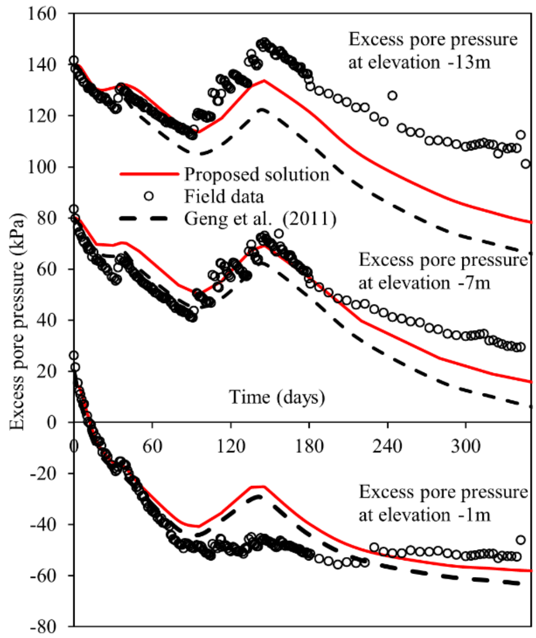


Figure 10 Comparison excess pore pressures

Complete excess pore pressure dissipation along the depth after the maximum surcharge loading at 146 days is presented in Figure 11. It can be seen that the pore pressure on the top boundary quickly dissipate and reach the vacuum pressure of -75 kPa at the top membrane. The pore pressure becomes constant after the depth of 2.0 m to the PVD's end. Moreover, the soil below the PVD's end indicates a slower water dissipation than the soil with PVD. At 450 days, almost the excess pore pressure in the zone with PVD reached to value of -75 kPa meanwhile the zone without PVD excess pore pressure shows a smooth transition from -75 kPa to 0 kPa. This curve is derived based on an approach of Chai et al. (2005).

In addition, degree of consolidation was determined by effective pressure which is defined as a subtraction of surcharge loading and excess pore pressure in soil.

$$\begin{cases} \sigma'(z,t) = \sigma(t) - \bar{u}_1(z,t), \; z \le h_1 \\ \sigma'(z,t) = \sigma(t) - \bar{u}_2(z,t), \; z > h_1 \end{cases} \tag{28}$$

The degree of consolidation is then computed as

$$U(t) = \frac{\int_{z=0}^{z=H} \sigma'(z,t)dz}{h_1(q_u + |P_{av}|) + h_2(q_u + |P_{av}|/2)} \tag{29}$$

where

q_u is the maximum surcharge loading

P_{av} is the minimum vacuum pressure

h_1 is the depth of zone with PVD

h_2 is the depth of zone without PVD

Figure 12 shows the comparison of degree of consolidation by the proposed solution, Geng et al. (2011), and the field data in which the degree of consolidation of field data is derived from settlement measurement. At early stage, the field data show slightly faster consolidation than the proposed solution. Nevertheless, after 120 days the predicted consolidation degree properly approximate the field monitoring. On the other hand, solution by Geng et al. (2011) matches the field measurement before 120 days and shows faster consolidation degree afterward. Nonetheless, the discrepancy between predicted solutions and in-situ measurements are relatively small. Thus, the proposed solution is applicable to be adopted in actual soil treatment projects with PVD and even the vacuum pressure.

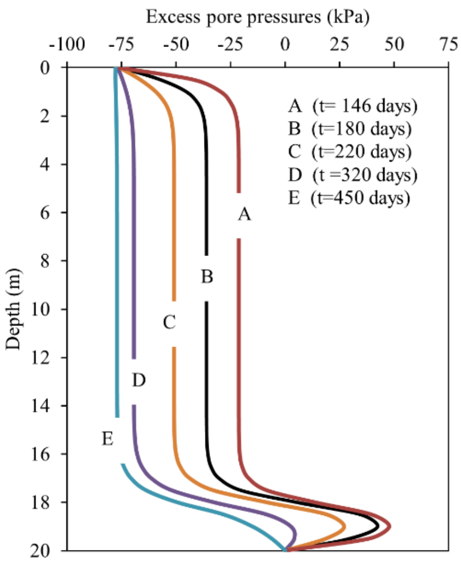


Figure 11 Dissipation of excess pore pressures

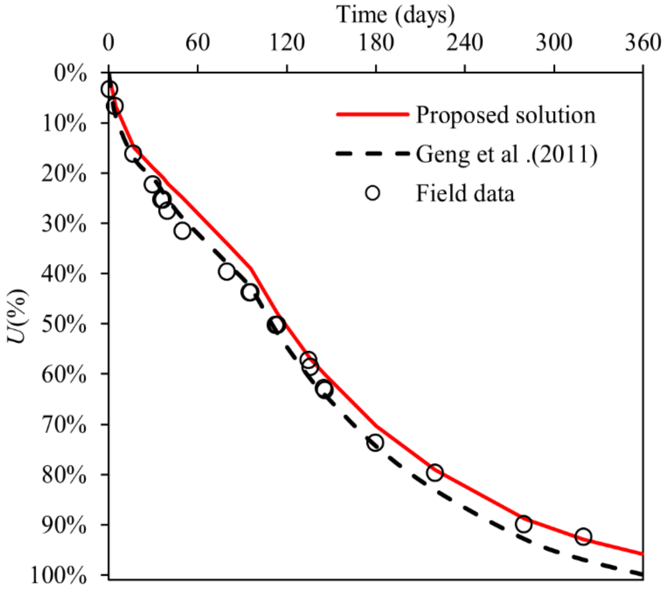


Figure 12 Comparison of degree of consolidation

6. SUMMARY AND CONCLUSION

The paper proposed a new solution adopting a new boundary approach and the Laplace transform technique to solve the consolidation problem of partially penetrated prefabricated vertical drains. The following conclusions can be drawn:

1. The new solution with Laplace transform technique including the new impermeable boundary region at the PVD's end due to the anchor plate appeared to be more accurate than the solution with Fourier series approximation technique in estimating the excess pore pressure with depth and time. The excess pore pressures by Fourier series approximation technique was fluctuated at the early stage of consolidation process while those obtained with Laplace transform technique is a smooth curve during the consolidation process.
2. The new solution lead to a lower degree of consolidation than previous solutions of Tang and Onitsuka (1998) and Zeng and Xia's solution (1989) due to the consideration of the new impermeable region at the PVD's end. Further, the average consolidation degrees obtained by Laplace technique (LTT) and Fourier series approximation technique (FT) converged at the same line because both solutions are derived from the same partially differential equations and the same boundary conditions.
3. In a case study of R3 road project in Ho Chi Minh City, Vietnam, the proposed solution is faster than the solution by Geng et al. (2011) about 62 times with the same computer configuration and the same Laplace inverse technique. It can save more calculation time for engineers. The in-situ excess pore pressure dissipated slower than predicted values. Moreover, the solution from Geng et al. (2011) showed faster dissipation than the proposed solution. It suggested that considering impervious boundary condition at the PVD's end is reasonable to have accurate prediction in the consolidation process. Plus, the current approach shows very good agreement with field data in term of degree of consolidation. Excess pore pressure by current solution can be explained both with time and depth in which one can easily observe the consolidation process that is also a strong point of the present solution.

7. ACKNOWLEDGEMENTS

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8. APPENDIX

The appendix is derived solutions to determine excess pore pressure and average degree of consolidation with different boundary conditions.

8.1 For Pervious Top and Impervious Bottom boundary conditions (PTIB)

The six boundary conditions (10), (11), (12), (14), and (15) and the three Laplace equations (20), (22), and (24) are combined to obtain the (6, 6) matrix equation.

$$S_{PTIB}.X^T = Q_{PTIB} \quad \text{(A1)}$$

where

$$S_{PTIB} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & S_{24} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & 0 & 0 \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix}$$

$$X = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \end{bmatrix}$$

$$Q_{PTIB} = \begin{bmatrix} -\frac{u_o}{s} + \sigma(s) & -\frac{u_o}{s} + \sigma(s) & 0 & 0 & 0 & 0 \end{bmatrix}$$

In case of vacuum pressure

$$Q_{PTIB} = \begin{bmatrix} -\frac{u_o}{s} + \sigma(s) & -\frac{u_o}{s} + \sigma(s) & P_{va}(s) & P_{va}(s) & 0 & 0 \end{bmatrix}$$

$P_{va}(s)$ = Laplace transform of $P_{va}(t)$

$S_{21} = \left(1 - \frac{a_1^2}{\varphi_2}\right)$, $S_{22} = \left(1 - \frac{a_1^2}{\varphi_2}\right)$, $S_{23} = \left(1 - \frac{a_2^2}{\varphi_2}\right)$, $S_{24} = \left(1 - \frac{a_2^2}{\varphi_2}\right)$,

$S_{35} = b_1 e^{b_1 H}$, $S_{36} = -b_1 e^{-b_1 H}$, $S_{41} = a_1 e^{a_1 h_1}$, $S_{42} = -a_1 e^{-a_1 h_1}$,

$S_{43} = a_2 e^{a_2 h_1}$, $S_{44} = -a_2 e^{-a_2 h_1}$, $S_{51} = \left(1 - \frac{a_1^2}{\varphi_2}\right) e^{a_1 h_1}$,

$S_{52} = \left(1 - \frac{a_1^2}{\varphi_2}\right) e^{-a_1 h_1}$, $S_{53} = \left(1 - \frac{a_2^2}{\varphi_2}\right) e^{a_2 h_1}$, $S_{54} = \left(1 - \frac{a_2^2}{\varphi_2}\right) e^{-a_2 h_1}$,

$S_{55} = -e^{b_1 h_1}$, $S_{56} = -e^{-b_1 h_1}$, $S_{61} = \left(1 - \frac{1}{n^2}\right)\left(1 - \frac{a_1^2}{\varphi_2}\right) a_1 e^{a_1 h_1}$,

$S_{62} = -\left(1 - \frac{1}{n^2}\right)\left(1 - \frac{a_1^2}{\varphi_2}\right) a_1 e^{-a_1 h_1}$, $S_{63} = \left(1 - \frac{1}{n^2}\right)\left(1 - \frac{a_2^2}{\varphi_2}\right) a_2 e^{a_2 h_1}$,

$S_{64} = -\left(1 - \frac{1}{n^2}\right)\left(1 - \frac{a_2^2}{\varphi_2}\right) a_2 e^{-a_2 h_1}$, $S_{65} = -b_1 e^{b_1 h_1}$, $S_{66} = b_1 e^{-b_1 h_1}$

8.2 For Pervious Top and Pervious Bottom boundary conditions (PTPB)

Apply six boundary conditions of equations (10), (11), (13), (14), and (15) to three Laplace equations (20), (22), and (24) to obtain the matrix (6, 6) system of equations

$$S_{PTPB}.X^T = Q_{PTPB} \quad \text{(A2)}$$

where

$$S_{PTPB} = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 \\ S_{21} & S_{22} & S_{23} & S_{24} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & 0 & 0 \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{bmatrix}$$

$$X = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 \end{bmatrix}$$

$$Q_{PTPB} = \begin{bmatrix} -\frac{u_o}{s} + \sigma(s) & -\frac{u_o}{s} + \sigma(s) & -\frac{u_o}{s} + \sigma(s) & 0 & 0 & 0 \end{bmatrix}$$

$S_{21} = \left(1 - \frac{a_1^2}{\varphi_2}\right)$, $S_{22} = \left(1 - \frac{a_1^2}{\varphi_2}\right)$, $S_{23} = \left(1 - \frac{a_2^2}{\varphi_2}\right)$,

$S_{24} = \left(1 - \frac{a_2^2}{\varphi_2}\right)$, $S_{35} = e^{b_1 H}$, $S_{36} = e^{-b_1 H}$, $S_{41} = a_1 e^{a_1 h_1}$,

$S_{42} = -a_1 e^{-a_1 h_1}$, $S_{43} = a_2 e^{a_2 h_1}$, $S_{44} = -a_2 e^{-a_2 h_1}$,

$S_{51} = \left(1 - \frac{a_1^2}{\varphi_2}\right) e^{a_1 h_1}$, $S_{52} = \left(1 - \frac{a_1^2}{\varphi_2}\right) e^{-a_1 h_1}$, $S_{53} = \left(1 - \frac{a_2^2}{\varphi_2}\right) e^{a_2 h_1}$,

$S_{54} = \left(1 - \frac{a_2^2}{\varphi_2}\right) e^{-a_2 h_1}$, $S_{55} = -e^{b_1 h_1}$, $S_{56} = -e^{-b_1 h_1}$,

$S_{61} = \left(1 - \frac{1}{n^2}\right)\left(1 - \frac{a_1^2}{\varphi_2}\right) a_1 e^{a_1 h_1}$, $S_{62} = -\left(1 - \frac{1}{n^2}\right)\left(1 - \frac{a_1^2}{\varphi_2}\right) a_1 e^{-a_1 h_1}$,

$S_{63} = \left(1 - \frac{1}{n^2}\right)\left(1 - \frac{a_2^2}{\varphi_2}\right) a_2 e^{a_2 h_1}$, $S_{64} = -\left(1 - \frac{1}{n^2}\right)\left(1 - \frac{a_2^2}{\varphi_2}\right) a_2 e^{-a_2 h_1}$,

$S_{65} = -b_1 e^{b_1 h_1}$, $S_{66} = b_1 e^{-b_1 h_1}$

The unknown X which is obtained by the above matrix equation is substituted to the Laplace equations (20), (22) and (24). An inverse Laplace transform technique using the approximate numerical method of Weeks (1966) has applied to get excess pore pressure.

$$Lu_w(z,s) \rightarrow \textit{Inverse Laplace by Weeks Method} \rightarrow u_w(z,t) \quad \text{(A3)}$$

$$L\bar{u}_1(z,s) \rightarrow \textit{Inverse Laplace by Weeks Method} \rightarrow \bar{u}_1(z,t) \quad \text{(A4)}$$

$$Lu_2(z,s) \rightarrow \textit{Inverse Laplace by Weeks Method} \rightarrow u_2(z,t) \quad \text{(A5)}$$

The average consolidation degree of whole soil is first integrated in Laplace form.

$$LU(s) = 1 - \frac{\int_0^{h_1} L\bar{u}_1(s,z)dz + \int_{h_1}^{H} Lu_2(s,z)dz}{Hu_o} = 1 - \frac{A_1X_1 + A_2X_2 + A_3X_3 + A_4X_4 + A_5X_5 + A_6X_6 + Hu_o}{Hu_o} \quad \text{(A6)}$$

where

$A_1 = \frac{1}{a_1}\left(e^{a_1 h_1} - 1\right)\left(1 - \frac{a_1^2}{\varphi_2}\right)$, $A_2 = -\frac{1}{a_1}\left(e^{-a_1 h_1} - 1\right)\left(1 - \frac{a_1^2}{\varphi_2}\right)$,

$$A_3 = \frac{1}{a_2}\left(e^{a_2 h_1} - 1\right)\left(1 - \frac{a_2^2}{\varphi_2}\right) \quad , \quad A_4 = -\frac{1}{a_2}\left(e^{-a_2 h_1} - 1\right)\left(1 - \frac{a_2^2}{\varphi_2}\right) \quad ,$$

$$A_5 = \frac{1}{b_1}\left(e^{b_1 H} - e^{b_1 h_1}\right), \qquad A_6 = \frac{1}{b_1}\left(e^{-b_1 h_1} - e^{-b_1 H}\right)$$

Then applying inverse Laplace technique by Weeks method (1966) for (30) to obtain the average consolidation degree $LU(s) \rightarrow Inverse\ Laplace\ by\ Weeks\ Method \rightarrow U(t)$

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