

Bearing Capacity of Shallow Foundations in Cohesive Soils- A State of the Art Literature Review

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ABSTRACT: The stability of shallow foundations primarily depends on the bearing capacity of cohesive soils. The precise estimation of bearing capacity ensures the resilience of infrastructure. In this regard, numerous studies have been conducted to assess the bearing capacity of such soils utilizing various parameters (e.g., c , ϕ , ψ). Terzaghi's bearing capacity equation served as a prelude to every static analysis technique. Some factors in modern forms of Terzaghi's equation, known as footing roughness and shape of the footing, substantially affect the ultimate bearing capacity of cohesive soils. As a few of the bearing capacity factors were formulated empirically, niches of soil and footing parameters exist where the bearing capacity predictions are unreliable. Some bearing capacity factors are sensitive to the scale of the footing. These need to be multiplied with appropriate scale functions to get accurate results in model tests. In footing with seismic loads, static analysis is not rigorous enough to determine seismic bearing capacity. This study was conducted to summarize the available methods of obtaining soil-bearing capacity. Equations for these studies are presented for comparison purposes. A direction for further investigation is paved in the corresponding sections. A Python module encapsulating code for non-finite Element Methods is developed and published in our Github (instructions for access at the end).

KEYWORDS: Bearing Capacity, Shallow Foundation, Resilience Infrastructures, Mohr-Coulomb Failure, and Economic Productivity

1. INTRODUCTION

The precise estimation of bearing capacity is essential in designing shallow footings (Pantelidis, 2024). It ensures the stability of buildings and brings economic productivity to the construction industry (Al-Janabi et al., 2024; Liang et al., 2021). Several bearing capacity equations proposed by various authors are available for calculating the ultimate bearing capacity of soil at the foundation level. These equations yield different bearing capacity values due to changes in fundamental assumptions. The changes in ultimate bearing capacity are more pronounced for soils with friction angles greater than 20° (Iqbal et al., 2023; Sim et al., 2024). Terzaghi presented the first bearing capacity theory for shallow foundations in the mid-19th century. There is still a need for refinements in particular niches of its applicability (Soubra, 1999). Current equations underestimate bearing capacity by as much as 322%, particularly in non-rectangular footings with extreme cases of bearing capacity parameters (Zhu, et al., 2011). These discrepancies owe their existence to several assumptions in the underlying theory. For instance, the failure criterion assumed by Terzaghi's and Meyerhof's bearing capacity equations ensures that they cannot be used for footings on sloping ground, as Zone II and Zone III failures are underestimated (Van Baars, 2014). Brinch-Hansen (1970) computed additional bearing capacity factors for ground and base inclination. As in previous research, these equations were generalized, resulting in regions in

their solution space where the solution deviates significantly from numerical studies (Van Baars, 2014). Saran-Agarwal (Saran & Agarwal, 1991) derived an expression for bearing capacity factors considering eccentric inclined loads. This significantly improved bearing capacity factors, as the ones provided by Terzaghi were lower-bound solutions.

Earlier studies proposed a general solution for bearing capacity. Such solutions significantly underestimate the bearing capacity in edge cases. Nowadays, research focuses on providing empirical bearing capacity factors for the Meyerhof or Brinch-Hansen equations that produce results consistent with numerical solutions, as seen in (Benmebarek et al., 2012; Georgiadis, 2010; Yang et al., 2021). Van Baars (2014) extended the shape factors to refine the failure spiral in the Brinch-Hansen bearing capacity equation. Leschinsky's bearing capacity correction factor is an alternative example of the phenomenon. A pseudo-static analysis is based on the aforementioned principle, in which Soubra, Jahanandish, and Sanghchuan-Ben presented seismic bearing capacity factors. Although a successful scheme for formulating a static bearing capacity model, recent studies have shown that introducing bearing capacity factors for dynamic forces is not practical (Yang et al., 2021).

2. THEORETICAL FRAMEWORK AND BACKGROUND

Bearing capacity theories are essential in geotechnical engineering for predicting the maximum load that soil can support without failure. Several theories have been developed over the years to estimate bearing capacity based on soil properties, foundation geometry, and loading conditions. Here are some prominent bearing capacity theories commonly used in practice:

2.1 Terzaghi

Terzaghi's theory is widely used and forms the basis of many geotechnical designs; it has certain limitations. For example, it assumes the soil is homogeneous and isotropic and may not be directly applicable to all soil types and conditions. Terzaghi (1943) developed a bearing capacity equation for a continuous shallow footing. Shallow footing was defined for the first time as a footing with a width greater than or equal to the depth of the foundation (Terzaghi, 1943). The area under the foundation was divided into three zones. In Zone I, the principal stress was vertical, whereas in Zone II, radial shear was the dominant component of the failure mechanism. The shape of the failure surface is given by (5), a logarithmic spiral described by Prandtl (Terzaghi, 1943). The contribution of Zone III to bearing capacity was due to Rankine's passive earth pressure, as seen in (4) (Terzaghi, 1943). The failure angle is a function of ϕ and a dimensionless constant λ . For frictional cohesive soil, the thickness of the overburdened soil layer was shallower compared to cohesionless soil (Yang et al., 2021). Note that Eq (2), (3), and (4) are deliberately in this generalized form. $\phi = 0^\circ$ in the case of this study.

$$q = c N_c + \gamma D N_q + \frac{\gamma B}{2} N_\gamma \quad (1)$$

$$N_c = \tan(\psi) + \frac{\cos(\psi - \phi)}{\sin(\phi) \cos(\psi)} \times (a_0^2(1 + \sin(\phi)) - 1) \quad (2)$$

$$N_q = \frac{\cos(\psi - \phi)}{\cos(\psi)} a_0^2 \tan\left(45^\circ + \frac{\phi}{2}\right) \quad (3)$$

$$N_\gamma = \frac{1}{2} \tan(\phi) \left(\frac{K_{p\gamma}}{\cos^2(\phi)} - 1 \right) \quad (4)$$

$$a_0 = e^{\left(\frac{3}{2}\pi + \frac{\phi}{2} - \psi\right) \tan(\phi)} \quad (5)$$

$$\psi = 45^\circ + \frac{\phi}{2} \quad (6)$$

2.2 Skempton

Skempton's bearing capacity is a significant advancement in geotechnical engineering for estimating the bearing capacity of shallow foundations on cohesive soils. Skempton's theory builds upon the earlier work of Karl Terzaghi and extends the understanding of soil behavior under loaded conditions. Skempton's theory is based on the concept of effective stress and the influence of pore water pressure on soil strength. It recognizes that the shear strength of cohesive soils depends on the total stress acting on the soil and the pore water pressure within the soil mass. The bearing capacity obtained from (7) was a relatively accurate estimation for a footing in homogeneous clay at an arbitrary depth. Generally, if settlement criteria do not govern, the safety factor was assumed to be 3 (Skempton, 1951). This equation was applicable to undrained cohesive soils. The shape factor (s_c) overestimates the bearing capacity of Tresca soil by 5%-12% depending on surface roughness (Gourvenec et al., 2006). A correction factor s_c (given in (10)) was introduced to account for this

deviation rough footing. Table 1 shows different variations of shape factors.

$$q = \frac{1}{F} \left(c N_c + p_0(N_q - 1) + \frac{\gamma B N_\gamma}{2} \right) + p \quad (7)$$

$$\phi = 0^\circ$$

$$N_{c(\text{rect})} = \left(0.84 + \frac{0.16B}{L} \right) N_{c \text{ square}} \quad (8)$$

$$N_c = \frac{4}{3} \ln \left(\frac{E}{c} + 1 \right) + 1 \quad (9)$$

Skempton adjusts (8), (9) linearly with $\frac{D}{B}$ (aspect ratio) (Gourvenec et al., 2006). For rough rectangular footings, the shape factor can be accurately predicted by

$$s_c = 1 + \frac{0.214B}{L} - \frac{0.067B^2}{L^2} \quad (10)$$

Multiplying (10) by the cohesive term in (7) gives an accurate q value for rough footing.

Table 1 Variation of shape factor

Depth	N_c	Footing Type
0	$N_{c \text{ strip}} = N_{CO} = 5$	Strip
	$N_{c \text{ strip}} = N_{CO} = 6$	Square/Circular
$\frac{D}{B} < 2.5$	$\left(1 + \frac{0.2D}{B} \right) N_{CO}$	All above
$\frac{D}{B} > 2.5$	$1.5 N_{CO}$	All above
$D = x$	$\left(1 + \frac{0.2B}{L} \right) N_{c \text{ strip}}$	Rectangular Footing

2.3 Meyerhof

Meyerhof's bearing capacity theory is widely used method in geotechnical engineering for estimating the ultimate bearing capacity of shallow foundations on cohesionless soils. Meyerhof's theory builds upon the earlier work of Karl Terzaghi and extends the understanding of bearing capacity calculations, particularly focusing on granular soils like sands and gravels. Meyerhof's theory is based on the concept of ultimate bearing capacity, which is the maximum load per unit area that a soil can withstand without failure. The theory considers both shear strength parameters (such as angle of internal friction, ϕ) and foundation geometry (such as width, B , and depth, D) in its calculations. Meyerhof introduced correction factors to refine the estimation of bearing capacity for different foundation shapes (such as square, rectangular, and circular) and depths. The ultimate bearing capacity (q_u) is calculated using parameters similar to those in Terzaghi's theory, but it includes additional factors to account for the shape and depth of the foundation. Although Meyerhof's theory is widely used and provides practical estimates of bearing capacity, it has certain limitations. For instance, the theory assumes that the failure surface beneath the foundation is a plane, which may not always be the case in practice. Additionally, the theory is more applicable to cohesionless soils and may require adjustments for cohesive soils.

Meyerhof (La Force Portante des Fondations sur Talus, 1957) also studied bearing capacity for a shallow and deep foundation where point loads act on the centre of homogenous soil. For shallow foundation, the soil was treated as a rigid body whereas, in

calculations for deep foundation, compressibility was considered. Mohr-Coulomb failure criteria were used to derive bearing capacity (Meyerhof, 2011). The developed theory was then validated against field and laboratory data. The resistance offered by the base of the shallow foundation is accurately predicted by the theory. In the case of deep foundation, base resistance is overestimated by (7) (La Force Portante des Fondations sur Talus, 1957). It applies to both shallow and deep foundation (Arya & Ameta, 2017).

$$q = \frac{1}{2} \gamma B N_{\gamma} s_{\gamma} d_{\gamma} i_{\gamma} + q_0 N_q s_q d_q i_q + c N_c s_c d_c i_c \quad (11)$$

Bearing Capacity Factors for Strip Footing

$$N_c = \cot(\phi) \left(\frac{(1 + \sin(\phi)) e^{2\theta \tan(\phi)}}{1 - \sin(\phi) \sin(2\eta + \phi)} - 1 \right) \quad (12)$$

$$N_q = \frac{(1 + \sin(\phi)) e^{2\theta \tan(\phi)}}{1 - \sin(\phi) \sin(2\eta + \phi)} \quad (13)$$

$$N_{\gamma} = \frac{4 P_p \sin(45^\circ + \frac{\phi}{2})}{\gamma B^2} - \frac{1}{2} \tan\left(45^\circ + \frac{\phi}{2}\right) \quad (11)$$

$$N_{\gamma} = (N_q - 1) \tan(1.4 \phi) \quad (12)$$

$$s_{\gamma} = s_q = 1 + 0.1 K_p \left(\frac{B}{L}\right) \sin(\phi) \quad \forall \phi > 0 \quad (13)$$

$$s_c = 1 + 0.2 K_p \left(\frac{B}{L}\right) \quad \forall B \leq L \quad (14)$$

$$i_q = i_c = \left(1 - \frac{\alpha^\circ}{90^\circ}\right)^2 \quad (15)$$

$$i_{\gamma} = \left(1 - \frac{\theta}{\phi}\right)^2 \quad \forall \phi > 0 \quad (18.1)$$

$$d_{\gamma} = d_q = 1 + 0.1 \sqrt{K_p \left(\frac{D}{B}\right)} \quad \forall \phi > 0^\circ \quad (16)$$

$$d_c = 1 + 0.2 \sqrt{K_p \left(\frac{d_f}{B}\right)} \quad \forall \phi > 0^\circ \quad (19.1)$$

Equations (14) and (15) describe the same entity. The latter is useful in manual calculations. In recent literature, a form of (15) exists where the multiplier of ϕ , that is 1.4 is taken as 1.

2.4 Vesic

Vesic's theory represents a significant advancement in bearing capacity analysis and builds upon the foundational work of Karl Terzaghi and Arthur B. Meyerhof. The theory also considers the influence of shear strength parameters, foundation geometry, and soil properties. Vesic (1973) revised the formula for bearing capacity factors, namely load inclination factors, ground inclination factors, and footing inclination factors. It was proven that shallow foundations are most susceptible when $H=0.5V$ (Taiebat & Carter, n.d.). Shape factors and depth factors are the similar to Eq 40-45 (Vesic, 1973). The calculation of bearing capacity is the same as (11). The formula for bearing capacity N_q, N_c factor is the same (12), (13) whereas N_{γ} is given as;

$$N_{\gamma} = 2(N_q + 1) \tan(\phi) \quad (17)$$

$$S'_{c(H)} = 0.2 \times \frac{B'}{L'} \quad (\phi = 0) \quad (18)$$

$$S_{c(H)} = 1 + \frac{N_q}{N_c} \times \frac{B'}{L'} \quad (19)$$

$$S_{c(V)} = 1 + \frac{N_q}{N_c} \times \frac{B}{L} \quad (20)$$

$$S_{q(H)} = 1 + \frac{B'}{L'} \times \sin(\phi) \quad (21)$$

$$S_{q(V)} = 1 + \frac{B}{L} \times \tan \phi \quad (22)$$

$$S_c = 1 \text{ (Strip Footing)} \quad (23)$$

$$s_{\gamma H} = 1 - 0.4 \frac{B'}{L'} \quad (24)$$

$$s_{\gamma V} = 1 - 0.4 \frac{B}{L} \geq 0.6 \quad (25)$$

$$k = \frac{D}{B} \quad \forall \frac{D}{B} \leq 1 \quad (26)$$

$$d'_c = 0.4 \times k \quad (\phi = 0^\circ) \quad (27)$$

$$k = \tan^{-1} \left(\frac{D}{B} \right) \quad \forall \frac{D}{B} > 1 \quad (28)$$

$$d_c = 1 + 0.4k \quad (29)$$

$$d_q = 1 + 2 \tan(\phi) (1 - \sin(\phi))^2 k \quad (30)$$

$$d_{\gamma} = 1 \quad \forall \phi \in \mathbb{R} \quad (31)$$

2.5 Brinch-Hansen

Brinch-Hansen's theory extends the classical Terzaghi and Meyerhof approaches by incorporating additional considerations related to soil strength and failure mechanisms. Brinch-Hansen's theory is based on fundamental principles of soil mechanics, with a particular focus on the strength characteristics of cohesive soils. The theory accounts for the effect of soil cohesion, angle of internal friction, and the distribution of stress beneath shallow foundations. Brinch-Hansen's theory recognizes multiple failure mechanisms in cohesive soils, including general and local shear failure. The theory aims to more accurately predict the shape and depth of the failure surface beneath the foundation. Earlier bearing capacity equations were developed for simple load cases. The footing load was assumed to be vertical and concentric, acting on a rectangular base. Brinch-Hansen (1970) introduced additional bearing capacity factors, namely the shape factor, depth factor, load inclination factor, base inclination factor, and ground inclination factor. Moreover, the corrections account for different foundation shapes by introducing the concept of an effective footing area as seen in (42), (43), (44) (Brinch Hansen, 1970).

$$q = \frac{1}{2} \gamma B N_{\gamma} s_{\gamma} d_{\gamma} i_{\gamma} b_{\gamma} g_{\gamma} + q_0 N_q s_q d_q i_q b_q g_q + c N_c s_c d_c i_c b_c g_c \quad (32)$$

$$N_q = e^{\pi \tan(\phi)} \tan^2 \left(45^\circ + \frac{\phi}{2} \right) \quad (33)$$

$$N_c = (N_q - 1) \cot(\phi) \quad (34)$$

$$N_\gamma = 1.5(N_q - 1) \tan(\phi) \quad (35)$$

$$s_q = 1 + \left(\frac{B}{L}\right) \sin(\phi) \quad (36)$$

$$s_c = 1 + \frac{N_q}{N_c} \times \frac{B'}{L'} \quad \forall B \leq L \quad (37)$$

$$s_\gamma = 1 - 0.4 \frac{B'}{L'} \quad (38)$$

$$i_c = i_q - \frac{1 - i_q}{N_q - 1} \quad (39)$$

$$i_q = \left(1 - \frac{\frac{1}{2}H}{V + A c \cot(\phi)}\right)^5 \quad (40)$$

$$i_\gamma = \left(1 - \frac{0.7 H}{V + A c \cot(\phi)}\right)^5 \quad (41)$$

$$d_q = 1 + 2 \tan(\phi) (1 - \sin(\phi)^2) \tan^{-1} \left(\frac{D}{B}\right) \quad \forall \frac{D}{B} \leq 1 \quad (42)$$

$$d_c = 0.4k \quad \phi = 0^\circ \quad (43)$$

$$d_\gamma = 1 \quad (44)$$

$$d_c = 1 + 0.4 \frac{D}{B} \quad \forall \phi > 0^\circ \quad (45)$$

$$N_{\gamma E} = \frac{2[P_{py} \cos(\alpha_1 - \phi) + P_{my} \cos(\alpha_2 - \phi_m)]}{\gamma B^2 \cos(i)} \quad (50)$$

$$N_{qE} = \frac{P_{pq} \cos(\alpha_1 - \phi) + P_{mq} \cos(\alpha_2 - \phi_m)}{\gamma D_f \cos(i)} \quad (51)$$

$$N_{cE} = \frac{P_{pc} \cos(\alpha_1 - \phi) + P_{mc} \cos(\alpha_2 - \phi_m)}{cB \cos(i)} + \frac{(1 + m)x_1 \sin \alpha_1 \sin \alpha_2}{\sin(\alpha_1 + \alpha_2) \cos(i)} \quad (52)$$

$$q = \frac{1}{2} \gamma B N_\gamma + \gamma D N_q + c N_c \quad (53)$$

2.7 Richards

Richards et al. (1993) explored the effects of seismic acceleration on bearing capacity. The foundation's failure during a seismic event was attributed to liquefaction. There have been instances in which field conditions indicated that liquefaction alone was an unlikely explanation. The only possible reason for the foundation settlement seems to be a decrease in bearing capacity due to increased pore water pressure. This theory gave conservative bearing capacity values (Richards et al., 1993).

$$N_{qE} = \frac{K_{PE}}{K_{AE}} \quad (54)$$

$$K_{AE} = \frac{\cos^2(\phi - \theta)}{\cos \theta \cos(\delta + \theta) \left(1 + \left(\sqrt{\frac{\sin(\phi + \delta) \sin(\phi - \theta)}{\cos(\delta + \theta)}}\right)^2\right)^2}$$

$$k = \tan^{-1} \left(\frac{D}{B}\right) \quad (46)$$

$$b_q = e^{-2\beta \tan(\phi)} \quad \forall \phi \neq 0 \quad (47)$$

$$b_\gamma = e^{-2.7\beta \tan(\phi)} \quad (48)$$

$$g_q = g_\gamma = (1 - 0.5 \tan(\beta))^5 \quad (49)$$

$$g_c = 1 - \frac{1 - i_q}{5.14 \tan(\phi)} \quad (52.1)$$

2.6 Saran-Agarwal

Saran-Agarwal (Saran & Agarwal, 1991) worked on bearing capacity of eccentrically and obliquely loaded footings. Shallow strip foundation with rough base was assumed for analysis where the weight of soil above the foundation was replaced by uniform surcharge load. Experimental work showed that the value of N_γ reported by Terzaghi's bearing capacity factor (N_γ) gave the lower bound solution for bearing capacity, hence underestimating the actual bearing capacity (Saran & Agarwal, 1991). Analysis of symmetrical and asymmetrical failure mechanisms was carried out on the basis of limit analysis theory. The failure mechanisms are considered for static and seismic conditions. The bearing capacity factors for both failure conditions are the same in case of static loading. In seismic case, the bearing capacity factors differ as much as 15% from classical values. The static bearing capacity factors in the equations were the same as Meyerhof. Seismic bearing capacity factors are expressed in terms of external work contributions which are rather lengthy and left here for brevity (Georgiadis, 2010).

$$N_{\gamma E} = \tan \rho_{AE} \left(\frac{K_{PE}}{K_{AE}} - 1\right) \quad (55)$$

$$N_{cE} = (N_q - 1) \cot(\phi) \quad (56)$$

Where K_{PE} and K_{AE} are dummy variables introduced by us for the sake of simplicity and are described in the next paragraph. It was assumed that $\delta = \frac{\phi}{2}$ gives the seismic bearing capacity factors N_{qE} and $N_{\gamma E}$ directly. While the value of N_{qE} was used to calculate N_{cE} (Richards et al., 1993). Here,

$$K_{PE} = \frac{\cos^2(\phi - \theta)}{\cos\theta \cos(\delta + \theta) \left(1 - \left(\sqrt{\frac{\sin(\phi + \delta) \sin(\phi - \theta)}{\cos(\delta + \theta)}} \right)^2 \right)}$$

$$\rho_{AE} = a + \tan^{-1} \left(\frac{\sqrt{(1 + \tan^2 a)[1 + \tan(\delta + \theta) \cot a]} - \tan a}{1 + \tan(\delta + \theta) (\tan a + \cot a)} \right)$$

$$\rho_{PE} = -a + \tan^{-1} \left(\frac{\sqrt{(1 + \tan^2 a)[1 + \tan(\delta - \theta) \cot a]} + \tan a}{1 + \tan(\delta + \theta) (\tan a + \cot a)} \right)$$

Finally, $P_{LE} = cN_{cE} + \gamma dN_{qE} + \frac{1}{2} \gamma B N_{\gamma E}$ (57)

An alternate equation for N_γ was presented that is easier to use. This equation gave neither unsafe N_γ values (as in (Terzaghi, 1943)) nor too conservative values (as in (Brinch Hansen, 1970; Meyerhof, 2011; Vesic, 1973)). The N_γ values obtained from $\psi = \phi$ are marginally larger than N_γ obtained from the Terzaghi bearing capacity equation (D. Y. Zhu, Lee, & Jiang, 2011).

$$N_\gamma = 2(N_q + 1) \tan(\phi)^{1.35} \quad (58)$$

$$N_\gamma = 2(N_q + 1) \tan(1.07\phi) \quad (59)$$

$$N_\gamma = 2(N_q + 1) \tan(\phi)^{1.45} \quad (60)$$

$$N_q = \tan^2 \left(\frac{\pi}{4} + \frac{\phi}{2} \right) \times e^{\pi \tan(\phi)} \quad (61)$$

The assumptions for (42), (43), (44) are applicable when $\psi = \phi$, $\psi = 45^\circ + \frac{\phi}{2}$ and $\{\psi | N_\gamma(\psi) = \min\}$, respectively. Method of characteristics (Sun et al., 2013) is a numerical approach at solving Terzaghi bearing capacity equation. In contrast with the method of superposition, the difference N_γ reported by this method varies from 45% to 83% in $\phi = 5^\circ$ to $\phi = 45^\circ$ range (Sun et al., 2013). The results of this study are not applicable to local shear failure.

2.8 Jahanandish, Georgiadis, Kumar

Jahanandish (Jahanandish & Keshavarz, 2005) took a classical approach at solving seismic bearing capacity factors on reinforced slope. Advantages of this analysis are that this model can define a heterogeneous seismic acceleration field around the footing. Granted that the authors have only provided design charts, it is a good approach nevertheless (Jahanandish & Keshavarz, 2005). An improvement to load inclination factors for footings on slopes in undrained conditions was made. Georgiadis (2010) developed an interaction curve between inclination and bearing capacity. The solution for inclination factors is presented in normalized form and thus cannot be used directly in the classic bearing capacity equation (Georgiadis, 2010). Kumar (Kumar & Khatri, 2011) extended the results on bearing capacity factors for circular footings to cohesive soils. The lower bound formulation assumes an axisymmetric soil element for Mohr-Coulomb failure criteria. The lower bound obtained was not rigorous enough as the equilibrium is satisfied only at centroids (Kumar & Khatri, 2011).

2.9 Van Baars

Brinch-Hansen and Meyerhof equations are well received theories for cohesive soil owing to the wide range of correction factors included in the equation. Van Baars (Van Baars, 2014) paper reported the effect evaluated of these bearing capacity factors to using numerical solutions. In particular, inclination and shape factors in the previously mentioned theories, are found to be under shooting. This was due to inappropriate approximation of logarithmic spiral in Zone-II of the failure wedge (Van Baars, 2014).

$$s_q = 1 + (1.5 \sin(\phi) + 3 \tan^3(\phi)) \times \frac{B}{L} \quad (62)$$

$$s_c \approx s_q = s_q + (0.2 - 0.1 \tan(\phi)) \times \frac{B}{L} \quad (63)$$

$$\phi = 0: i_q = \cos^2(\alpha) \times \frac{2 + \pi - 2\alpha}{2 + \pi} \quad (64)$$

$$\phi > 0: i_q = i_c \quad (65)$$

$$i_c = \cos^2(\alpha) \left(e^{-2\alpha \tan(\phi)} - \frac{2\alpha}{2 + \pi} e^{-\pi \tan(\phi)} \right) \quad (66)$$

It can be seen in Figure 1 and Figure 2, that the shape factor diverges almost linearly for Meyerhof and Hansen as compared to Van Baars where it diverges exponentially. For friction angles (ϕ) as low as 10° , Van Baars shape factors are 55% greater than the one given by (20), (21). For $\phi > 40^\circ$ this difference increased by 247%.

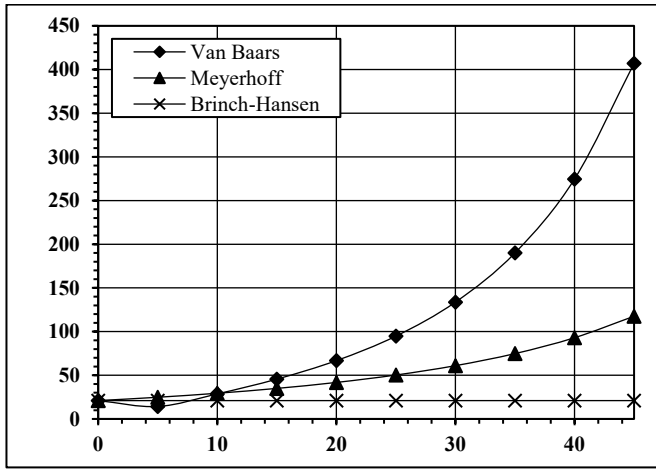


Figure 1 Comparison of shape factor s_c with (Brinch Hansen, 1970; G & Struct, n.d.).

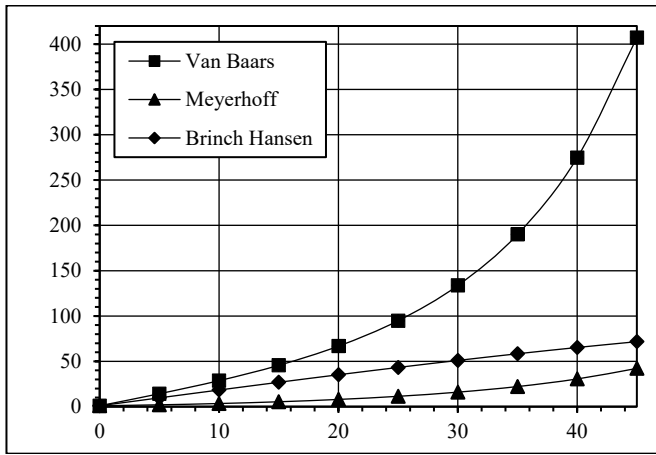


Figure 2 Comparison of shape factor s_q with (Brinch Hansen, 1970; G & Struct, n.d.).

Shallow footings on slopes are analyzed using the Meyerhof equation with modified bearing capacity factors to account for the slope. Earlier studies present specific bearing capacity factors that apply to either cohesive or non-cohesive soil. Discontinuity layout optimization was used to determine a reduction factor (RC) for the Meyerhof bearing capacity equation (Leshchinsky, 2015).

$$RC = \frac{q_{ult \text{ numerical}}}{q_{ult}} \quad (67)$$

2.10 Han-Xie

Han-Xie (Han et al., 2016) observed that the values N_c, N_γ, N_q obtained by the method of superposition are elevated when compared to numerical simulations. In cohesive soils, the equation for N_γ was lacking dominant soil parameters such as q, γ, c, B . In the case of no surcharge pressure, N_γ was consistent with (Martin & Martin, 2016) and (Smith, 2015). The relation between the bearing capacity factor of smooth and rough footing was stated as $N_{\gamma(\text{smooth})} = k N_{\gamma(\text{rough})}$ where k lies in the range $\left[\frac{1}{2}, 1\right]$. Surcharge ratio has a significant impact on the correctness of equations developed by Martin (2016) and Smith (2015). Smaller surcharge ratios result in prominent deviations from classical theories. The assumption that the base angle

of the active wedge under footing always equals to $45^\circ + \frac{\phi}{2}$ in (Zhu et al., 2011) leads to higher values of bearing capacity (Han et al., 2016).

$$N_\gamma = 2p_u - 2\lambda N_q \quad (68)$$

$$N_q = e^{\pi \tan(\phi)} \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \quad (69)$$

$$p_u = \frac{q_u + c \cot(\phi)}{\gamma B} \quad (70)$$

$$\lambda = \frac{q + c \cot(\phi)}{\gamma B} \quad (71)$$

As the surcharge ratio reaches $\lambda = 40$ in N_γ - λ space, the self-load factor becomes almost constant. There was no significant difference between previous work and the numerical simulations, as seen in Figure 3. At a smaller surcharge ratio, the difference between the proposed model and numerical simulation was as much as 322%. It is observed that the bearing capacity for circular footing is increased by at least 3%-10% (Gourvenec et al., 2006). Numerical simulation in Figure 3 refers to Han-Xie (Han et al., 2016).

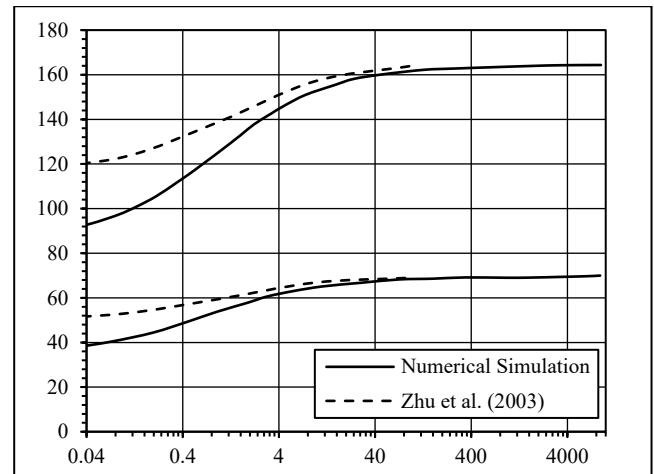


Figure 3 Comparison of Han-Xie (Han et al., 2016) numerical simulation with Zhu et al (Zhu et al., 2011).

2.11 Veiskarami, Eshkevari

Bearing capacity of footing on anisotropic soil was examined to produce design charts for anisotropic soil layer. Additional bearing capacity factors for anisotropy $\alpha_c, \alpha_q, \alpha_\gamma$ were introduced on the design charts. Other factors such as β_s and β_ϕ which represent the degree of anisotropy are calculated based on shear strength parameters (Veiskarami et al., 2017). The intention behind Eshkevari's investigation (Eshkevari et al., 2018) was to refine understanding of bearing capacity in layered soil profiles, particularly in sand over clay. The understanding of soil settlement was based on Terzaghi and Meyerhof models, which assumed the soil properties remained homogeneous and isotropic throughout the model (Shoaei et al., 2012). The developed plastic analysis was based on the aforementioned assumptions, leading designers to resort to numerical simulations or approximate bearing-capacity models. Eshkevari's model is an extended form of Terzaghi and Meyerhof's model. A new bearing capacity factor (K_{sr}) for shear failure was introduced. The

equation for K_{sr} were distinct for clay and sand layer (Taghvamanesh, 2021).

$$q_u B = \text{Min} \left[\gamma H^2 K_{sr} \tan(\phi') + N_c c_u (B + 2H \tan(\pm\theta)) + \gamma H^2 \tan(\pm\theta), q_t B \right] \quad (72)$$

For sand layer

$$\theta = A \ln \left(\frac{c_u}{\gamma H} \right) + B \quad (73)$$

$$A = 0.039 \ln(\tan(\phi')) - 0.164 \quad (74)$$

$$B = 0.597 \ln(\tan(\phi')) - 0.051 \quad (75)$$

$$K_{sr} = \frac{1}{\gamma H^2 \tan(\phi')} \left[\left(\frac{Q_U + Q_L}{2} \right) - N_c c_u (B + 2H \tan(\pm\theta)) - \gamma H^2 \tan(\theta) \right] \quad (76)$$

For clay layer,

$$K_{sr} = C \frac{c_u}{\gamma H} + 2 \quad (77)$$

$$C = -3.48 \tan(\phi') + 8.693 \quad (78)$$

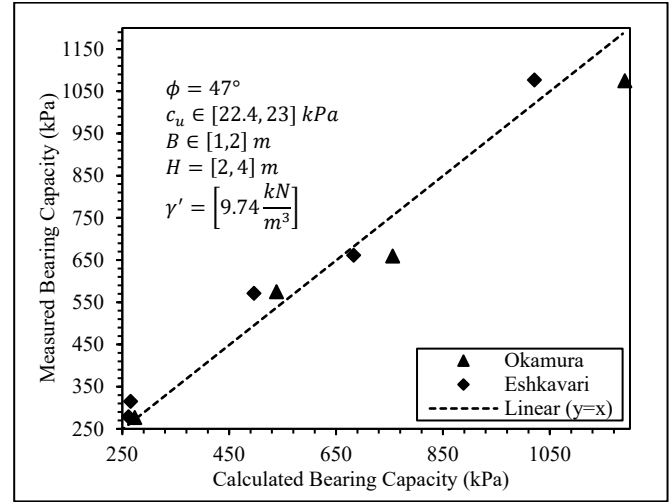


Figure 4 Comparison of Eshkevari model with Okamura (Eshkevari et al., 2018).

2.12 Shangchuan-Ben

Shangchuan-Ben (Yang et al., 2021) utilized upper bound limit analysis to arrive at seismic bearing capacity factors N_γ, N_c, N_q . This analysis considers pseudo-static loading on footing and soil. Earlier studies underestimated the ultimate bearing capacity of soil as they did not analyzed $k_h \in [0, 0.176]$ for general failure mechanism. Furthermore, at $k_h \geq 0.4$, gravity destabilizes the footing as N_γ approaches $\mathbb{R}^-$. The variation of bearing capacity factors with slope inclination is highly non-linear in larger values of k_h . Increase in slope angle causes seismic bearing capacity to decrease as compared to level ground. This is due the formation of smaller passive Zone under the footing. A limited amount of failure transitions are considered during the analysis (Local and General Failure) (Yang et al., 2021).

$$Q = c N_c + \frac{1}{2} \gamma B N_\gamma + \gamma D N_q \quad (79)$$

$$Q = Q_c + Q_\gamma + Q_q \quad (80)$$

$$N_c = \frac{\cos(\phi)}{((\sin(\alpha - \phi) + k_h \cos(\alpha - \phi)) v_{a,0})} \times \left(BC v_{a,0} + AC v_{1,a} + AD v_{c,n} + ED v_{c,0} + \frac{\delta\theta}{\cos(\phi)} \sum_{i=1}^n r_0 e^{\theta_i \tan(\phi)} v_{i,0} + \sum_{i=2}^n r_i v_{i,i-1} \right) \quad (81)$$

$$N_\gamma = - \frac{2}{(\sin(\alpha - \theta) + k_h \cos(\alpha - \phi)) v_{a,0} \gamma B} \times \left(W_A (\sin(\alpha - \phi) + k_h \cos(\alpha - \phi)) v_{a,0} + W_{c1} v_{c,0} (\sin(\alpha - \theta - \phi) + k_h \cos(\alpha - \theta - \phi)) + \sum_{i=1}^n \delta W_i v_{i,0} \right) \quad (82)$$

$$N_q = - \left[\frac{W_{c2} v_{c,0}}{((\sin(\alpha - \phi) + k_h \cos(\alpha - \phi)) v_{a,0} \gamma D)} \right] \times (\sin(\alpha - \theta - \phi) + k_h \cos(\alpha - \theta - \phi)) \quad (83)$$

Most tests were performed on small-scale models for cost efficiency. Literature showed that the bearing capacity values obtained from small scale-tests were significantly higher than field

values. A correction factor for this surge in bearing capacity was derived empirically from numerical studies (Zhu & Zhang, 2021).

$$\alpha = 0.96 - \frac{2.4B}{L_c} \quad (84)$$

Implementation for these models can be found at our repository Bearing Capacity Repository under the stage branch. The Python module mentioned is lacking unit tests and other important usability features (such as solvers for inverting equations) (Toudehdehghan, 2020).

3 RESULTS AND DISCUSSION

Terzaghi's bearing capacity equation is based on the shear strength of the underlying soil, which is divided into three zones. The factors used in the bearing capacity equation are functions of the friction angle of the soil. This method is the oldest and commonly used in the design of earth-supporting structures. In the case of the generalized form $\phi=0^\circ$, a simple equation is used to evaluate the bearing capacity. According to Meyerhof (La Force Portante des Fondations sur Talus, 1957), the bearing capacity of a shallow foundation is calculated using a point load in homogeneous soil, which is treated as a rigid body. The method uses Mohr-Coulomb failure criteria to derive bearing capacity (Meyerhof, 2011). The bearing capacity calculated by Terzaghi's Bearing Capacity will have universal values and lies between Meyerhof's bearing capacity. Higher values are given by Terzaghi's analysis. Furthermore, the main advantage is that Meyerhof's bearing capacity is for deep foundations as well as on slopes. This means that the bearing capacity for strip footings will be given by Terzaghi, and then modified for other foundation shapes.

A relatively accurate estimation of the bearing capacity of homogeneous clay at different depths may be obtained using Skempton (7). If settlement criteria do not govern, the factor of safety is assumed to be 3 (Skempton, 1951). This equation uses the undrained cohesive strength of clayey soil to evaluate the bearing capacity. The shape factor (s_c) overestimates the bearing capacity of Tresca soil by 5%-12% depending on surface roughness (G & Struct, n.d.). A correction factor s_c (given in (10)) was introduced to account for this deviation rough footing. The bearing capacity equation proposed by Hansen is an extension of Terzaghi's method, including foundations on slopes and other conditions not considered in Terzaghi's method. Brinch-Hansen (1970) introduced additional bearing capacity factors, namely the shape factor, depth factor, load inclination factor, base inclination factor, and ground inclination factor. Moreover, the corrections account for different foundation shapes by introducing the concept of an effective footing area, as seen in (42), (43), and (44) (Brinch Hansen, 1970). The Vesic method for evaluating bearing capacity is similar to the Hansen and Meyerhof methods. However, this method uses a slightly more accurate shear failure surface; it is an extension of the Terzaghi method to include slopes and inclined surfaces. This method is commonly used to evaluate bearing capacity in Marine and offshore structures. This method is more accurate than the Hansen method.

The bearing capacity of eccentrically and obliquely loaded footings was evaluated by Saran & Agarwal (1991) for shallow strip foundations with a rough base. The experimental work showed that the value of N_γ reported by Terzaghi's bearing capacity factor (N_γ) yielded the lower-bound solution for bearing capacity, hence underestimating the actual bearing capacity (Saran & Agarwal, 1991). The bearing capacity factors for seismic failure conditions are the same as those for static loading. In seismic cases, the bearing capacity factors differ as much as 15% from classical values. The static bearing capacity factors in the equations were the same as Meyerhof's. Richards et al. (1993) evaluated the effect of seismic loading on the bearing capacity of the foundation. The failure of the foundation during a seismic event was attributed to liquefaction. There have been

instances in which field conditions indicated that liquefaction alone was an unlikely explanation. The only possible reason for the settlement of the foundation appears to be a decrease in bearing capacity due to increased pore water pressure. This theory gave conservative bearing capacity values (Richards et al., 1993). The ultimate bearing capacity equation proposed by Richards et al. (1993) is based on plasticity theory. It is also easy to implement as the static and seismic bearing capacity equations are similar, except for the bearing capacity coefficients.

The Jahanandish (Jahanandish & Keshavarz, 2005) equation is a classical approach for determining seismic bearing capacity factors for reinforced slopes. The advantage of this analysis is that this model can define a heterogeneous seismic acceleration field around the footing. Although the authors have provided only design charts, the approach is nevertheless promising (Jahanandish & Keshavarz, 2005). The effect of inclined loading on the vertical and horizontal failure loads was studied by Georgiadis (2010) to calculate the bearing capacity of soil. A wide range of geometries and soil properties were considered using upper-bound plasticity and stress-field methods in finite element analysis, and the results were further validated against known solutions. The empirical equation for bearing capacity was proposed for the undrained inclined loading of a footing on near-slope ground, thereby improving the load inclination factors. Georgiadis (2010) developed an interaction curve between inclination and bearing capacity. The solution for inclination factors is presented in normalized form and thus cannot be used directly in the classic bearing capacity equation (Georgiadis, 2010).

Kumar (Kumar & Khatri, 2011) used lower-bound finite element analysis with linear programming to evaluate the bearing capacity factors of a circular footing due to cohesion, surcharge, and unit weight in $c-\phi$ soil, using different values of ϕ . Kumar (Kumar & Khatri, 2011) assumed an axisymmetric soil element in the lower-bound formulation and used the Mohr-Coulomb failure criterion. The lower bound obtained was not rigorous enough because the equilibrium is satisfied only at the centroids (Kumar & Khatri, 2011). In 1920, Prandtl evaluated the bearing capacity of a maximum strip load on a weightless infinite half-space using the sliding soil component into three zones: two triangular zones on the edges and a wedge-shaped zone in between the triangular zones that has a logarithmic spiral form. The solution was further extended by Reissner (1924) with a surrounding surcharge. The extended formulation has an additional bearing capacity coefficient for the soil weight and additional correction factors for inclined loads and non-infinite strip loads. Van Baars (2014) numerically solved for the stresses in the wedge zone and derived the corresponding bearing capacity coefficients, inclination, and shape factors. The inclination factors were also analytically solved. The author found that the previously mentioned theories underestimated the inclination and shape factors. This was due to the inappropriate approximation of the failure wedge's logarithmic spiral in Zone II (Van Baars, 2014).

The method proposed by Han-Xie et al. (2016), known as the method of characteristics (also called the slip-line method), is used to calculate the bearing capacity of strip footings in soil assumed to be rigid-plastic and employs the Mohr-Coulomb failure criterion. The authors proposed solution procedures by using a finite difference method suitable for both smooth and rough footings. The bearing capacity equation can strictly satisfy the required boundary conditions by accounting for the influence of the cohesion c , the friction angle ϕ , and the unit weight γ of the soil within a single failure mechanism. The bearing capacity equation for a strip footing on anisotropic soil was evaluated by Veiskarami et al. (2017) using a coupled analysis combining lower-bound and finite element linear programming techniques. Based on this study, design charts for the bearing capacity of anisotropic soil were developed. Additional bearing capacity factors for anisotropy α_c , α_q , and α_γ were introduced on the design

charts. Other factors, such as β_s and β_ϕ , representing the degree of anisotropy, are calculated based on shear strength parameters (Veiskarami et al., 2017).

The undrained bearing capacity equation proposed by Eshkevari (Eshkevari et al., 2018) was for strip footings on a sand clay layer. The rigorous upper and lower bound theorems of plasticity were used in order to evaluate the bearing capacity and identify the geometry of possible failure mechanisms. Then, the understanding of soil settlement was based on Terzaghi and Meyerhof's models, which assumed that soil properties remained homogeneous and isotropic throughout the model (Shoaei et al., 2012). The developed plastic analysis was based on the aforementioned assumptions, leading designers to resort to numerical simulations or approximate bearing-capacity models. Eshkevari's model is an extended form of Terzaghi and Meyerhof's model. A new bearing capacity factor (K_{sr}) for shear failure was introduced. The equation for K_{sr} was distinct for the clay and sand layers (Eshkevari et al., 2018). Shangchuan Yang et al. (Yang et al., 2021) evaluated the bearing capacity of shallow foundations near slopes under seismic loading conditions. The authors used analytical solutions based on upper-bound kinematic limit analysis to isolate both the mechanism of failure and the bearing capacity factors (N_c , N_γ , and N_q) for shallow foundations near slopes subject to seismic action without using the principles of superposition. Earlier studies underestimated the ultimate bearing capacity of soil as they did not analyze $k_h \in [0, 0.176]$ for the general failure mechanism. Furthermore, at $k_h \geq 0.4$, gravity destabilizes the footing as N_γ approaches R^- . The variation of bearing capacity factors with slope inclination is highly non-linear in larger values of k_h . An increase in slope angle reduces the seismic bearing capacity relative to level ground. This is due to the formation of a smaller passive Zone under the footing. A limited amount of failure transitions is considered during the analysis (Local and General Failure) (Yang et al., 2021). Most tests were performed on small-scale models to reduce costs. The literature showed that bearing capacity values obtained from small-scale tests were significantly higher than those from field tests. A correction factor for this surge in bearing capacity was derived empirically from numerical studies (Zhu & Zhang, 2021).

4 Comparative Analysis of the Reviewed Methods

The ultimate bearing capacity of a strip footing in cohesive soil was calculated using 12 different methods, each offering a unique perspective based on its assumptions, theoretical framework, and failure mechanisms. These methods range from classical approaches, such as Terzaghi's, to more advanced computational approaches, such as Eshkevari's. The variation in results highlights the impact of methodological differences on bearing capacity estimates, reflecting the suitability of different methods for different soil and footing conditions.

To illustrate these methods, a practical example as stated below was considered:

A strip footing for a residential building project is placed on cohesive soil (clay) with the following parameters: Soil Type: Cohesive soil with cohesion, $c = 25$ kPa, and an internal friction angle, $\phi = 15^\circ$. Footing Size: Strip foundation with a width $B = 2.5$ m and a depth $D = 1.2$. Loading Conditions: Applied vertical load, $P = 150$ kN/m, with no horizontal load or moment applied.

This example provides a clear and consistent basis for applying the 12 methods to compute the ultimate bearing capacity (q_{ult}), demonstrating how the results vary depending on the approach used. Table 2 summarizes the methods, key features, and their pros and cons.

Table 2 Summary of the methods, key features, and their pros and cons.

Methods	Pros	Cons
Terzaghi's Method	Simple, widely used, quick estimation.	Not applicable for sloped ground or complex load conditions.
Skempton's Method	Accurate for cohesive soils, particularly clays.	Assumes homogeneity, not suitable for complex soil profiles.
Meyerhof's Method	Versatile, suitable for inclined loads and shape effects.	Requires more data, more complex.
Vesic's Method	Higher accuracy in some cases, detailed analysis.	Computationally intensive, requires advanced knowledge.
Brinch-Hansen's Method	Accounts for shape and inclination, broad applicability.	More complex, requires detailed data.
Saran-Agarwal's Method	Flexible, better for non-homogeneous soils.	Less widely used, complex.
Richards' Method	Good for layered soils, detailed failure mechanisms.	Data-intensive, complex.
Jahanandish, Georgiadi, and Kumar's Methods	• Good for heterogeneous soils, includes depth and width factors. • More accurate for large foundations. • Good for cohesive soils with complex loading.	Computationally intensive. Not suitable for small foundations, complex. Requires detailed data, not universally applicable.
Van Baars	Suitable for various soil types, provides reliable results for complex cases.	Requires detailed soil input; less straightforward for simple applications.
Han-Xie's Method	Useful in seismic regions, dynamic load factors.	Only applicable to seismic regions, requires seismic data.
Veiskarami, Eshkevari's Method	Accounts for non-homogeneous and anisotropic soils. High accuracy with FEM analysis.	Complex, requires anisotropy data. Requires advanced computational tools and input data.
Shangchuan-Ben's Method	Useful for seismic regions, dynamic load consideration.	Requires seismic data, complex.

The variation in calculated ultimate bearing capacity across methods highlights the complexities of accurately assessing it. While methods like Terzaghi's and Meyerhof's offer a good starting point for simple designs, more advanced methods such as Vesic's, Richards', and Han-Xie's are better suited to complex conditions, including layered soils, deep foundations, and seismic loading.

The choice of method should depend on the site-specific conditions and the required level of accuracy. Methods like Skempton's and Brinch-Hansen's are best suited for cohesive soils

with lower variability, whereas the Richards' and Van Baars' methods offer enhanced reliability for stratified soils. Advanced methods like Eshkevari's and Shangchuan-Ben's are suitable for highly complex scenarios, but their computational intensity and data requirements can be a limiting factor for general use.

In practice, the use of simpler methods for preliminary designs followed by more complex methods for detailed analysis is often the most efficient approach.

5 CONCLUSIONS

A thorough review of multiple studies on the bearing capacities of cohesive soils was conducted. The key findings from this comprehensive analysis are summarized as follows:

- The critical finding of this analysis is that the calculated bearing capacity of soil is heavily influenced by the methodology employed, which in turn depends on the applicable design codes. Terzaghi's (1943) bearing capacity equation, while convenient for estimating ultimate bearing capacity (q_{ult}) where the depth-to-width ratio $D/B \leq 1$, is unsuitable for footings subjected to moments, horizontal loads, or placed on sloped ground. This limitation arises from the lack of load inclination factors in Terzaghi's original formulation, which does not account for the complex loading conditions in these cases. Despite its limitations, Terzaghi's equation is widely used due to its simplicity and ease of application.
- Similarly, the Meyerhof bearing capacity equation is not applicable to footings on sloped terrains, as the failure mechanism in such cases deviates from the classical three-zone failure concept. A more general approach, such as a Meyerhof-based formulation using a path integral method for failure surfaces, would be more appropriate for deriving a general bearing capacity equation that can account for the complexities associated with sloped footings.
- The method of superposition, which is typically applied in bearing capacity analyses, yields solutions based on the assumption of a specific value for the angle of internal friction (ϕ). In contrast, the method of characteristics achieves a more accurate result within the same range without assuming any fixed value for ϕ , thus providing a lower error bound.
- Brinch Hansen's bearing capacity equation, which incorporates shape and inclination factors from Van Baars and bearing capacity factors from Zhu and Han Xie, yields reliable results for both non-cohesive and cohesive soils, respectively. In cases of non-homogeneous cohesive soils, anisotropy factors derived from Veiskarami's study prove particularly suitable.
- While Eshkevari's equation provides a precise solution, its application is challenging due to the need for input from Finite Element Method (FEM) programs. This highlights the lack of variables associated with the free failure mechanism in Eshkevari's formulation, limiting its practical usability.
- In small-scale model footings, the observed bearing capacity is typically higher than that of the scaled model. To correct for errors in model tests, one can apply correction factors (87) by multiplying the corresponding bearing capacity factors by a specific value derived from the experimental setup.
- This study also concludes that bearing capacity factors derived from dynamic analysis offer greater reliability compared to those from pseudo-static studies. Most studies assume a direct relationship between vertical seismic

acceleration (K_v) and horizontal seismic acceleration (K_h), which restricts the applicability of the models to small intervals of K_v within the range of $K_v \in (1, 1.3)$.

Appendix A. (Nomenclature)

q	Bearing Capacity
N_γ	Self-load Factor
N_q	Surcharge Load Factor
N_c	Cohesion Factor
B	Width of Foundation
D	Depth of Foundation
L	Length of Foundation
L'	Effective Length of Footing
B'	Effective Width of Footing
A	Footing Area
a_0	Equation of log spiral
γ	Unit Weight of Soil
c_u	Undrained Shear Strength
s_n	Shape Factors ($n \in (c, \gamma, q)$)
i	Inclination Factors
d	Depth Factors
K_{py}	Passive Earth Coefficient
p, p_o	Overburden Pressure
F	Factor of Safety
ψ	Failure Wedge Angle
ϕ	Friction Angle
η	Angle of Plane Shear Zone
P_p	Equivalent Free Surface Stress
α	Inclination of Load
β	Inclination of Ground Level
H	Horizontal Load
V	Vertical Load
g	Ground Inclination Factor
$N_{\gamma E}$	Seismic Self-load Factor
N_{qE}	Seismic Surcharge Load Factor
N_{cE}	Seismic Cohesion Factor
α_1, α_2	Wedge Angles
ρ	Pressure Zone Angles
ϕ_m	Frictional Angle in Zone m
P_{m*}, P_{p*}	Bearing Capacity of Specific Zone
m, i	Zone Index
δ	Wall Friction
θ	Seismic Acceleration Ratio
ρ_n	Critical Rupture Angles
a	Acceleration
δ	Wall Friction
θ	Seismic Acceleration Ratio
P_{LE}	Ultimate Seismic Bearing Capacity
p_u	Normalized Bearing Capacity
q	Surcharge Load
λ	Surcharge Ratio
H	Sand Layer Thickness
K_{sr}	Shear Coefficient
W_i	Weight of Soil in Zone i
$v_{i,j}$	Velocity of i, j block
k_h	Horizontal Acceleration
L_c	Characteristic Length

Appendix B. Table 2 Summary of previous research papers.

S. No	Theory	Year	Soil	Analysis	Bearing Capacity Factors (N_c, N_q, N_γ) ¹	Soil Type	Inclination Factor (I_c, I_q, I_γ) ¹	Foundation Depth	Failure Criteria	Shape Factors (s_c, s_q, s_γ) ¹	Depth Factors (d_c, d_q, d_γ) ¹	Base Inclination Factor ¹	Ground Inclination Factor ¹	Programs Used
1	Terzaghi (Terzaghi, 1943)	1943	Cohesive & non-cohesive	Static	2,3,4	Isotropic	-	Shallow	Mohr-Coulomb	-	-	-	-	-
2	Meyerhof (Meyerhof, 2011)	1963	Cohesive & non-cohesive	Static	12 – 15	Isotropic	18	Shallow and Deep Foundation	Mohr-Coulomb	16, 17	19	-	-	-
3	Brinch Hansen (Brinch Hansen, 1970)	1970	Cohesive & non-cohesive	Static	36,37,38	Isotropic	42,43,44	Shallow and Deep Foundation	Mohr-Coulomb	39,40,41	45 – 49	50,51	52	-
4	Vesic (Vesic, 1973)	1973	Cohesive & non-cohesive	Static	17,18,2 –	Isotropic	42,43,44	Shallow Foundation	Mohr-Coulomb	21 – 28	30 – 34	50,51	52	-
5	Saran-Agarwal (Saran & Agarwal, 1991)	1991	Cohesive & non-cohesive	Static	53,54,55	Isotropic	-	Shallow	Mohr-Coulomb	-	-	-	-	-
6	Richards (Richards et al., 1993)	1993	Cohesive	Static	Seismic N_c, N_q, N_γ	Isotropic	-	Shallow	Mohr-Coulomb	-	-	-	-	Manual
7	Soubra (Soubra, 1999)	1999	Cohesive	Static & Pseudo-static	Seismic N_c, N_q, N_γ	Isotropic	-	Shallow	Mohr-Coulomb	-	-	-	-	Spreadsheet
8	Zhu (D. Y. Zhu, Lee, & Jiang, 2011)	2001	Cohesive	Static	61 – 64	Isotropic	-	Shallow	Mohr-Coulomb	-	-	-	-	-
9	M. Jahanandish (Jahanandish & Keshavarz, 2005)	2004	Cohesive & Non-cohesive	Pseudo-static	-	Isotropic with anisotropic continuum	-	Shallow Foundation	Michalowski Piecewise Mohr-Coulomb Criteria	-	-	-	-	-
10	Georgiadis (Georgiadis, 2010)	2009	Cohesive & Non-cohesive	Static	-	Isotropic	Green’s Formula	Shallow and Deep Foundation	Tresca	-	-	-	-	Plaxis 8.6
11	Van Baars (Van Baars, 2014)	2014	Cohesive	Static	12 – 15	Isotropic	69	Shallow Foundation	Mohr-Coulomb	65,66				Spreadsheet
12	Ben Leshchinsky (Leshchinsky, 2015)	2015	Cohesive & Non-cohesive	Static	12 – 15	Isotropic	-	Shallow Foundation	Mohr-Coulomb	-	-	-	-	Limit State GEO
13	Han-Xie (Han et al., 2016)	2016	Cohesive		67,68,10 71,72	Isotropic	-	Shallow and Deep Foundation	Mohr-Coulomb	-	-	-	-	Finite Difference
14	Veiskarami (Veiskarami et al., 2017)	2017	Cohesive & Non-cohesive	Static	-	Anisotropic	-	Shallow Foundation	Mohr-Coulomb	Design Chart	Design Chart	Design Chart	Design Chart	Self-Written Program
15	Eshkevari (Eshkevari et al., 2018)	2018	Cohesive & Non-cohesive	Static	75	Isotropic	-	Shallow and Deep Foundation	Mohr-Coulomb	-	-	-	-	GEO (LSG)
16	Sanghchuan-Ben [7]	2020	Cohesive	Pseudo-Static	84,85,86	Isotropic	-	Shallow	Mohr-Coulomb	-	-	-	-	-

¹ The numbers in this column refer to equation numbering.

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