Geotechnical Engineering Journal of SEAGS & AGSSEA

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Study on the Mathematical Model of Gas Preferential Flow of MSW

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ABSTRACT: Municipal solid waste (MSW) has various components, different sizes, and strong heterogeneity. The significant preferential flow effect of MSW directly affects the migration path of landfill gas. In order to study the gas preferential flow characteristics of MSW, a mathematical model for describing gas preferential flow in the pore area and the fractured area was established. The whole process of gas passing through MSW sample was simulated by the established model. The simulation results showed that gas flow is not only related to the total permeability and total porosity of MSW but also needs to consider the changes in the permeability and porosity of the pore area and fracture area. The peak value of the gas breakthrough curve increases with the ratio of fracture to pore permeability, and the value of the gas flow rate at the outlet increased from 0.025 L/s to 1.22 L/s, respectively. Meanwhile, the time of gas passing through the MSW sample decreases with the fracture to pore permeability ratio. The peak value of the gas breakthrough curve decreases gradually with the decrease of the proportion of fracture area in pore space, and the value of the gas flow rate at the outlet increased from 0.025 L/s to 0.277 L/s, respectively.

KEYWORDS: Municipal Solid Waste, Landfill Gas, Preferential Flow, Mathematical Model, Numerical Simulation.

1. INTRODUCTION

The treatment methods of municipal solid waste (MSW) around the world include landfill, incineration, composting, etc. Landfill is the main way of MSW disposal in China (Ke et al., 2023, Shu et al., 2023). The main input substances of landfill are domestic waste, and the main output substances are landfill gas and leachate (Zeng and Ma., 2021). MSW is a typical solid, liquid, and gas three-phase porous medium, which has the characteristics of complex composition, macropores, and strong anisotropy (Woodman et al., 2013; Wu et al., 2018). Pores and fractures are the material basis and prerequisite for the formation of preferential flow, which in turn promotes the further development of pores and fractures. Preferential flow is a significant flow pattern in MSW, which is mainly due to the presence of matrix regions and macropores of MSW. Preferential flow has been identified as an important flow pattern in MSW (Feng et al., 2018; Zhang et al., 2019a). The preferential flow in MSW is the heterogeneous penetration with macropores as the preferred path. Therefore, it is of great theoretical significance and practical value to carry out theoretical research on pores, cracks, and preferential flow in MSW for pollution control and safe operation of landfills. A lot of scholars have carried out experimental studies on macropore and priority flow by means of staining tracer test, microscope section method, CT scan, and penetration test (Rosqvistand Destouni., 2000; Weiler & Naef., 2003; Rosqvist et al., 2005; Woodman et al., 2013; Zhang et al. 2019a; Zhang et al. 2019b; Liu Liu et al., 2016; Zeng & Hu., 2024). The study of a mathematical model of gas preferential flow is not enough. For this purpose, a pore-fracture dual permeability model was established based on dual media seepage theory to describe the phenomenon of gas preferential flow in MSW. The reliability of the mathematical model was verified by comparing the outflow rate of laboratory test data and simulation results.

2. MATHEMATICAL MODEL OF GAS PREFERENTIAL FLOW IN MSW

The theory of dual medium seepage in oil and gas reservoirs is also applicable to landfills. The dual medium is divided into a fracture system and a pore system. Landfill gas not only migrates and flows in fractured areas but also in porous areas. The main difference between dual medium and general porous medium is that there are two permeability and two porosities at any point (particle volume) in the dual medium. The fracture-pore dual media can be simplified abstractly into various geological models, such as the Warren-Root model, Kazemi model, De Swaan Model, and Factal Model. The Warren-Root model is a geological model that reduces the matrix to regular hexahedrons cut by orthogonal fractures in the same direction as the principal permeability and assumes that the width of the fractures is constant. The Kazemi model simplifies the matrix to be divided by a set of fractures in parallel bedding, and the model is composed of horizontal fractures and a horizontal matrix. The De Swaan Model assumes that the matrix is not a parallelepiped but a circular sphere. The Factal Model has some similarities between the whole and the part, and the fractal dimension is different from the heterogeneous characteristics of the matrix. The most representative of the four models is the Warren-Root model (Nie *et al*., 2012).

Gas flowed not only in fractured and porous areas, but also between the two areas. The dual permeability model is written in the form of double permeability as Equation 1 and Equation 2.

$$\frac{\partial}{\partial t}(\rho_f \varphi_f) = \nabla(\rho_f \frac{k_f}{\mu} \cdot \nabla(P_f)) + Q_{mf} \quad (1)$$

$$\frac{\partial}{\partial t}(\rho_m \varphi_m) = \nabla(\rho_m \frac{k_m}{\mu} \cdot \nabla(P_m)) - Q_{mf} \quad (2)$$

where, the subscript f represents the fracture area and the subscript m represents the pore area; ρ_f and ρ_m are the gas density of fracture and pore area respectively; φ_f and φ_m are the gas porosity of fracture and pore area respectively; k_f and k_m are the gas permeability of fracture and pore area respectively, m^2; μ was the viscosity coefficient of the gas, $\mu \cdot Pa/s$; P_f and P_m are the gas

https://doi.org/10.14456/seagj.2025.4 Published on 18 September 2025

pressure of fracture and pore area respectively, Pa; Q_{mf} is the gas exchange term between pore and fracture area.

There is a relationship between the total permeability and the permeability of the two areas by Equation 3.

$$k_t = w_f \cdot k_f + (1 - w_f) \cdot k_m \quad (3)$$

where, k_t was the total permeability, it can be obtained through laboratory test. w_f was the ratio of fracture flow space to total flow space, $0 < w_f < 1$.

There is a relationship between the total porosity and the porosity of the two areas as Equation 4.

$$\varphi_t = w_f \cdot \varphi_f + (1 - w_f) \cdot \varphi_m \quad (4)$$

where, φ_t is the total porosity, it can be obtained through laboratory tests.

The gas exchange term Q_{mf} can be expressed as Equation 5:

$$Q_{mf} = \delta \frac{\rho k_m}{\mu} (P_m - P_f) \quad (5)$$

where, δ is the shape factor, $\delta = \pi^2(\frac{1}{L_x^2} + \frac{1}{L_y^2} + \frac{1}{L_z^2})$; L_x,L_y,L_z is the length of the specimen in the direction of x, y, z.

3. NUMERICAL SIMULATION PROCESSOF GAS PREFERENTIAL FLOW

The two Darcy flow models were established by COMSOL Multiphysics software to represent the constitutive equations of fluid flow in the fracture area and pore area, respectively. The numerical simulation model was established, and the initial conditions and boundary conditions of the model were set. The numerical simulation model was calculated, and the results of the calculation were recorded. The parameters of the gas preferential flow model were optimized by comparing them with the results of the laboratory test. The test equipment, test method and test data were described in detail in the literature by Zeng & Hu. (2024).

3.1 Numerical simulation model

The numerical model of one-dimensional vertical gas transport through municipal solid waste samples was simulated by COMSOL Multiphysics. The fluid flow in the earth science module is selected, and two Darcy flow models are established to represent the constitutive equations of fluid flow in the fracture region and the pore region respectively. The height of the numerical simulation model was H, which was equal to the height of the MSW sample 300 mm. The diameter of the sample cylinder was 100 mm. The top ports (gas outflow) and bottom (gas inflow) ports of the sample cylinder were connected by a conduit with an inner diameter of 2 mm, so the pressure boundary width of the top ports and bottom ports was 2 mm. The simulation diagram is shown in Figure 1.

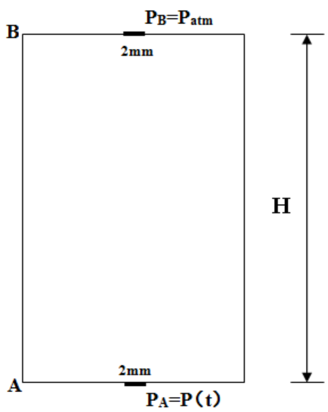

Figure 1 Schematic diagram of numerical simulation diagram

3.2 Parameter setting

The unsteady mathematical model of Darcy's law in COMSOL Multiphysics software is as Equation 6:

$$\delta_S S \partial p/\partial t + \nabla[-\delta_k (k_S/\eta)(\nabla p + \rho_f g \nabla D)] = \delta_Q Q_S \quad (6)$$

where, the values of the proportional coefficients δ_S、δ_k、δ_Q are 1.

The relationship between the parameter S and porosity was as Equation 7.

$$S = \frac{\varphi M}{RT} \quad (7)$$

The gas permeation medium used in the experiment is nitrogen, and the molar mass M of nitrogen was 16 g/mol. R is the gas constant, and the value is equal to 8.314 J/(mol • K). T is thermodynamic temperature, and the value is equal to 293 K.`

It should be noted that the format and dimension of the control equation in the software should be the same when setting the parameters, and the permeability in the control equation of fracture area and pore area should be set to the corresponding sum value respectively.

3.3 Boundary conditions

The first boundary condition type was used at the inlet and outlet of the sample cylinder in the numerical simulation. The inlet of the MSW sample cylinder was connected with the inlet of the gas standard pressure chamber through a valve. The gas flow rate at the inlet of the MSW sample cylinder was changed with time. The boundary condition at gas inflow was simplified as the pressure boundary, denoted as P_A, which was the pressure value monitored by the pressure sensor, and its function form was generally exponential attenuation P(t). The expression of P(t) can be input into COMSOL Multiphysics with Matlab language format by the expression fitted out through the laboratory test data. The pressure boundary at gas outflow of the MSW sample cylinder P_B was atmospheric pressure, and the initial outlet pressure of MSW sample cylinder was atmospheric.

3.4 Postprocessing

The profile parameters were selected during post-processing to determine the required coordinates and time of the breakthrough curve data, which is the gas vertical velocity in the vertical section of the model at all times. The parameters of the dual permeability model were determined by comparing the calculated breakthrough curve results with those obtained from the test data.

4 RESULTS AND ANALYSIS OF NUMERICAL SIMULATION

4.1 Change rules of gas preferential flow under different permeability ratios

Figure 2 showed that the gas breakthrough curve distribution under different permeability ratios when the volume ratio was 0.1. The gas flow rate at the outlet was simulated for permeability ratios of 5, 10, 20, 50, 100, 150 and 200. The total permeability of the MSW sample remained constant, and the influence of different permeability ratios of the fracture to the pore space on gas breakthrough curves was analyzed.

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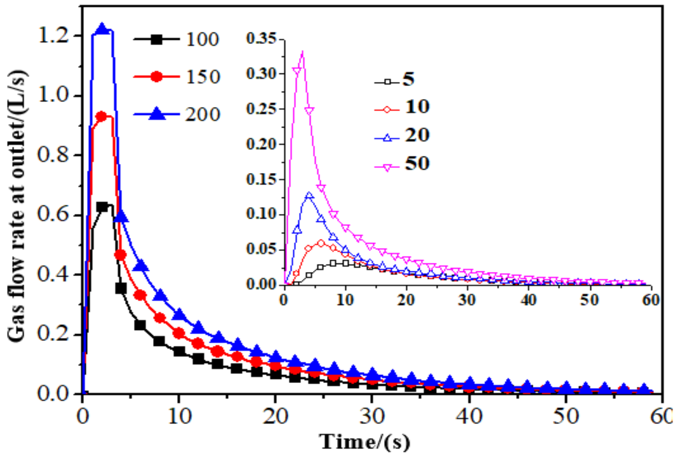

Figure 2 Simulated breakthrough curves under different k_f/k_m

It can be seen from Figure 2 that the larger the permeability ratio of fracture area to pore area, the larger the peak value of the gas breakthrough curve and the shorter the breakthrough time. This was mainly due to the increase of permeability in the fracture area with the increase of the ratio, which leads to the acceleration of gas flow rate from the pore area, thus speeding up the speed of gas passing through the MSW sample and shortening the breakthrough time.

4.2 Change rules of the breakthrough curve under different volume ratio

Figure 3 shows that the distribution of the gas breakthrough curve varied W_f values when the permeability ratio was 10.

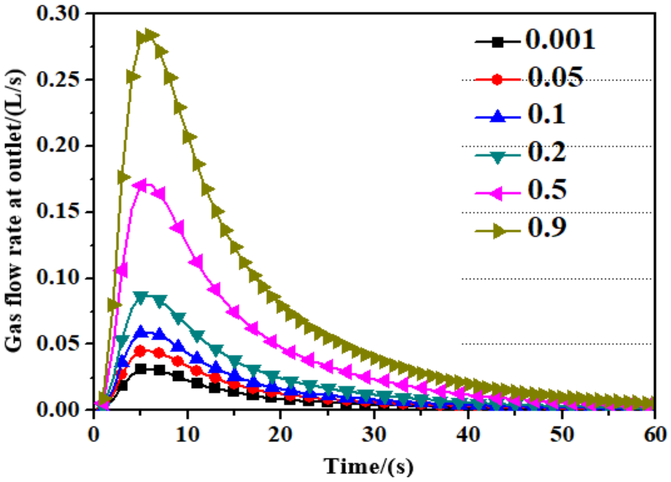

Figure 3 Breakthrough curves under different w_f

It can be seen from Figure 3 that with the decrease of w_f, the proportion of fracture area in pore space decreases, resulting in the decrease of permeability in fracture area and the decrease of gas outflow at the outlet. Therefore, the peak value of breakthrough curve decreased with the decrease of w_f.

4.3 Parameter determination of gas preferential flow under different moisture content

Figure 4 shows that the comparison between the simulated values and the laboratory test results of MSW sample breakthrough curve under different moisture content. Where, C_v was the calculated value of the model.

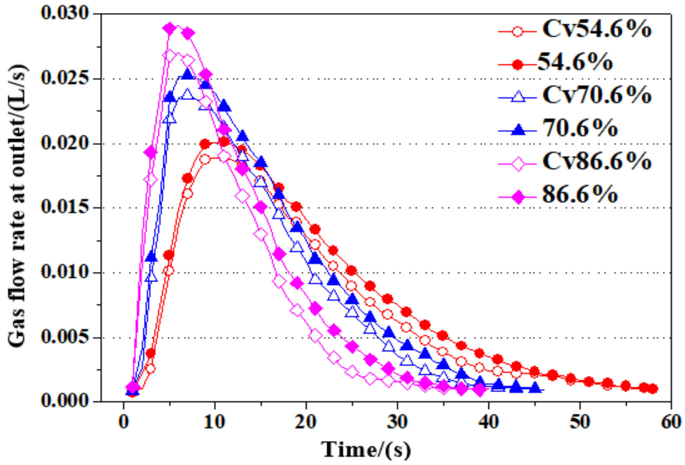

Figure 4 Simulated and experimental results of the breakthrough curves under different moisture content

It can be seen from Figure 4 that the optimal calculation value of the model was the same as the variation trend of the gas flow rate monitored by the laboratory test, but the value was slightly smaller than the gas flow rate measured by the laboratory test. It might be that the actual condition was simplified in the model calculation.

The parameters in the dual permeability model of the MSW sample with different moisture content were determined through the optimal simulation parameters of the breakthrough curve. The model values were shown in Table 1, and the unit of k_f and k_m were 10^{-12} m^2.

Table 1 Results of dual-permeability model parameters under different moisture content

Moisture Content	k_f	k_m	$\frac{k_f}{k_m}$	φ_f	φ_m	w_f
54.6%	3.319	0.849	3.91	0.094	0.626	0.1004
70.6%	3.167	0.691	4.58	0.073	0.659	0.0992
86.6%	2.902	0.555	5.23	0.035	0.393	0.0989

It can be seen from Table 1 that with the increase of moisture content, φ_f and k_m decreases continuously. The permeability ratio k_f/k_m increased with the increase in moisture content, while the porosity ratio φ_f/φ_m decreased with the increase of moisture content. This was mainly because the increase in moisture content made the non-flow space in the fracture area occupied by water, which led to the decrease of effective flow space for the dominant flow channel.

The method of inverting the permeability and porosity of the fracture area and pore area of the model was reliable through the dual permeability model constructed combined with the laboratory test data of MSW sample permeability, porosity and gas breakthrough curve.

5. CONCLUSIONS

The main conclusions of this manuscript were as follows:

(1) A dual-permeability model of gas preferential flow in MSW was established based on the pore-fracture seepagein porous media theory.

(2) The whole process of gas passing through the MSW sample was simulated by the constructed dual permeability model, and the influence of fracture ratio and so on in the double-porosity medium on the gas migration rule was analysed.

(3) The larger the permeability ratio between the ffracture area and the pore area, the larger the peak value of the gas breakthrough curve, and the shorter the time of the gas breakthrough MSW

sample. Gas flow in MSW samples has obvious heterogeneous characteristics. The pore structure has a direct influence on the gas flow state. The change in gas flow is not only related to the total permeability and total porosity of MSW but also needs to consider the changes in the permeability and porosity of the pore area and fracture area.

(4) The proportion of the fracture area in the pore space decreases with the decrease of the porosity ratio. The peak value of the gas breakthrough curve decreases gradually due to the decrease of the permeability characteristic of the fracture area.

The applicability and reliability of the dual permeability model were evaluated by comparing the outflow rate of the monitoring data and the simulation results.

6. ACKNOWLEDGMENTS

This research was financially supported by Xiangyang Innovation and Development Joint Fund project of Hubei Province Natural Science Foundation, grant number2024AFD036, and the Open Fund for Hubei Provincial Engineering Research Center of Slope Habitat Construction Technique Using Cement-based Materials, grant number 2022SNJ08.

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