

3-D Bearing Capacity Analysis of Square Foundation in Spatially Variable Cohesive-Frictional Soil

Tarek Kormi^{1,2}, Amal Hentati³, Mbarka Selmi³, and Julien Baroth⁴

¹*Applied Mechanics Laboratory (LASMAP), Ecole Polytechnique de Tunisie, University of Carthage, La Marsa, 2078, Tunisia*

²*National Engineering School of Gabès (ENIG), Rue Omar Ibn ElKhattab, 6029, University of Gabès, Tunisia*

³*Modélisation en Génie Civil et Environnement (MGCE), National Engineering School of Gabès (ENIG), Rue Omar Ibn ElKhattab, 6029, University of Gabès, Tunisia*

⁴*Univ. Grenoble Alpes, CNRS, Grenoble INP, Institute of Engineering, UMR 5521, 3SR, 38000, Grenoble, France
E-mail: tarekkormi@gmail.com*

ABSTRACT: Natural soil variability is a well-known issue in geotechnical design particularly in footing design. When subsoil must be characterized in terms of mechanical properties, random finite element method (RFEM) can be effectively adopted. This method had been widely considered in two-dimensional case (plane strain condition) focusing on soil strength variability in two dimensions and assuming no variability along the direction perpendicular to the considered plane by default. This limitation can significantly influence probabilistic bearing capacity evaluations and emphasizes the importance of three-dimensional (3-D) analysis in such stochastic characterization. The 3-D analysis in stochastic context has not fully explored as such analysis is time-consuming and different stochastic conditions can be considered. The purpose of this paper is to use RFEM for the bearing capacity assessment of a square shallow foundation to endorse our understanding of the influence of 3-D analysis on bearing capacity and on failure mechanisms. In this study, soil shear strength properties (i.e. soil cohesion and friction angle) are assumed to be independent and correlated random fields. Results show that the influence of correlation between shear strength properties can be significant and lead to more realistic estimates of bearing capacity. A probability distribution of bearing capacity was then estimated and used to calibrate safety factors for reliability purposes.

KEYWORDS: Random Finite Element Method, Random Fields, Spatial Variability, and Foundation Reliability.

1. INTRODUCTION

Predicting the bearing capacity of shallow foundations is one of the most serious problems in geotechnical engineering. Several approaches to this topic were developed in the literature. The conventional approach in structural analysis and design follows a deterministic framework, where characteristic values of soil properties are typically considered. Uncertainties in soil properties or loading conditions are accounted for indirectly through partial safety factors, calibrated using a semi-probabilistic method (Massih et al., 2008).

However, it is widely recognized in geotechnical engineering that soils are inherently heterogeneous materials, and their properties exhibit significant spatial variability throughout the soil mass. As a result, the determination of soil parameters is subject to uncertainty, since measured values cannot fully capture the natural fluctuations of soil properties (Cherubini et al., 1993; Phoon et al., 1999; Kawa & Pula, 2020; Pieczyńska-Kozłowska & Vessia, 2022). This spatial variability, characterized by a so-called scale of fluctuation, influences soil mechanical behaviour and the interaction between soils and structures in a range of geotechnical scenarios such as bearing capacity of strip footings and slope stability (Griffiths & Fenton, 2001).

As an alternative to deterministic approaches for structural analyses and designs, one can use probabilistic approaches that directly account for the inherent uncertainty of each variable in the problem under consideration. The probabilistic approaches had been studied extensively due to the increase of computational capacities, introducing advanced finite element analysis (FEA) software to solve numerical problems of the subsoil media (e.g., Fenton & Griffiths, 2003; Popescu et al., 2005; Cassidy et al., 2013; Jiang et al., 2014; Kawa & Pula, 2020; Li et al., 2021).

In geotechnical engineering, probabilistic analysis of spatial variability problems is often carried out using Monte Carlo simulation, employing a random field coupled with a finite element or finite difference method. The majority of the studies employed two-dimensional (2-D) random fields and plane-strain conditions despite soil spatial variability being naturally three-dimensional (3-D). Full 3-D probabilistic (random field) analyses of footings are surprisingly scarce as the random 3-D analyses of such nonlinear problems become time-consuming.

Alternative ways to find bearing capacity solutions of square footings resting on 3D spatially variable soils exist: one is to derive a relationship between the 2-D plane strain bearing capacity solution, which assumes an infinite scale of fluctuation in the out-of-plane direction, and the full 3-D solution [although this is a costly exercise, because no simple relationship exists (Li et al., 2021)].

Probabilistic approaches have shown the potential of these methods to rigorously account for uncertainties in soil properties, particularly in the bearing capacities of strip footings. It is found that soil spatial variability can dramatically reduce the foundation bearing capacity (e.g., Popescu et al., 2005; Li et al., 2015; Hentati et al., 2018; Selmi et al., 2019; Kormi et al., 2019).

Consequently, the resulting safety factor is not a consistent measure of risk evaluation since foundations with the same safety factor may exhibit different risk levels depending on the variability in soil properties (Li & Lumb, 1987; Hentati et al., 2018; Selmi et al., 2019; Kormi et al., 2019). Therefore, a statistical description of these later and a probability calculation constitute extremely precious tools to quantify soil uncertainties and to incorporate them into foundations designs and analyses (Cherubini, 1997). For that, a considerable attention is attached in the literature to the reliability theory application to foundations bearing capacity. Reliability analysis

involves estimating small failure probabilities or, conversely, extreme quantiles.

Ching (2009) has shown that a relationship between reliability and factor of safety can be determined via Monte Carlo simulation or, when small failure probability is of concern, via Subset Simulation. Once the relationship between reliability and factor of safety is determined, one can achieve a reliability-based design by using a safety-factor approach. Also, practical civil engineers who are not familiar with reliability concepts can easily achieve reliability-based design by using the equivalence.

In this paper, a random finite element method was used to assess the effect of soil variability on the performance of square shallow foundations. For this, 3-D stationary random fields were simulated to account for the inherent spatial variability of both soil cohesion and friction angle. Two sets of simulations are considered in this paper: in the first set of simulations, soil cohesion and friction angle have been assumed to be independent random fields and in the second set the above shear strength parameters have been considered to be correlated with a cross-correlation coefficient $\rho_{c,\phi} = -0.25$. It is worth mentioning that a negative correlation between cohesion and friction angle is mainly reported in the literature (Imanzadeh et al., 2020), with values ranging from -0.1 to -0.87 (Wu, 2013). A finite element method was then applied using the commercial software Abaqus to evaluate the foundation bearing capacity. Approximations of both foundation reliability index and probability of failure were finally determined using a Monte Carlo simulation method. The application of the methodology to a square shallow foundation subjected to vertically centered loading demonstrated the insufficiency of the conventional deterministic approach for predicting the foundation's bearing capacity and failure mechanism.

2. METHODOLOGY

2.1 Random Fields Simulation

Log-normally distributed random fields are selected to simulate the positive values of shear strength parameters including the soil cohesion (c) and friction angle (ϕ) with mean values μ_c and μ_ϕ , and coefficients of variation COV_c and COV_ϕ , respectively.

The spatial variability of a random parameter can be described statistically by its mean value, variance, and scale of fluctuation δ which defines the distances over which there is significant correlation between material property values (Vanmarcke, 1977).

This autocorrelation structure of a soil property is described by its autocorrelation function $\rho(\cdot)$. A set of autocorrelation functions of soil properties were presented by Popescu (1995), Vanmarcke (1983), Griffiths et al. (2004) and Rackwitz (2000), among others. In this study, a square exponential autocorrelation function (also called Markovian) is adopted. Its expression is given by:

$$\rho(\alpha_x, \alpha_y, \alpha_z) = e^{-2\sqrt{\left(\frac{\alpha_x}{\delta_x}\right)^2 + \left(\frac{\alpha_y}{\delta_y}\right)^2 + \left(\frac{\alpha_z}{\delta_z}\right)^2}} \quad (1)$$

where: α_x , α_y and α_z are the distances between two points in the x , y horizontal directions and the z vertical direction, respectively. δ_x , δ_y and δ_z are the scales of fluctuation in the x , y horizontal directions and z vertical direction, respectively. In the present study, the same scale of fluctuation is assumed in both horizontal directions.

2.2 Performance Function

A reliability-based analysis requires the definition of a performance function G . The performance function used in this study is defined with respect to the bearing failure of the soil. It is given as follows:

$$G = \frac{Q_l}{Q} - 1 = F_p - 1 \quad (2)$$

where Q_l represents the ultimate foundation load, Q the applied load and F_p is the safety factor in a deterministic context.

$G(X) = 0$ is the boundary between the safety and failure regions described mathematically by the events $\{G(X) > 0\}$ and $\{G(X) < 0\}$, respectively (Melchers, 1999).

To evaluate an estimate of the probability of failure P_f , the expected value and standard deviation of the performance function need to be determined.

2.3 Monte Carlo Simulation

Although computationally expensive, Monte Carlo Simulation (MCS) can capture the influence of inherent soil spatial variability on the bearing capacity of shallow foundations, where failure mechanisms can change from one realization of random soil properties to another (Popescu et al., 2005).

In fact, the Monte Carlo method simulates realizations of the random parameters. For each realization, the performance function is evaluated. The process is repeated, then, n times to obtain the expected value and standard deviation of the performance function as follows:

$$\mu_G = \frac{1}{n} \sum_{i=1}^n G_i \quad (3)$$

$$\sigma_G = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (G_i - \mu_G)^2} \quad (4)$$

2.4 Reliability Index and Failure Probability

Once the expected value and standard deviation of the performance function have been determined, the probability of failure P_f can then be estimated using the cumulative distribution function of the standard normal distribution Φ evaluated at $-\beta_c$:

$$P_f = (-\beta_c) \quad (5)$$

where β_c denotes the reliability index which was originally proposed by Cornell (1969) for load and resistance reliability analysis, and was generalized for geotechnical reliability-based design because of its analytical simplicity (Christian et al., 1994; Duncan, 2000). However, the Cornell index is subjected to several limitations and, in particular, is sensitive to different but equivalent formulations of the performance function (Phoon et al., 2022).

To overcome these limitations, Hasofer and Lind (1974) introduced the first-order reliability method (FORM) for normal variables. In the space of uncorrelated standard normal variables

$u = [u_1, u_2, \dots, u_n]^T$, the Hasofer-Lind reliability index β_{HL} , defined as the minimum distance from the origin to the failure surface in the space of reduced variables, is adopted in this paper.

3. NUMERICAL MODEL

The FE analysis model used in this paper is illustrated in Figure 1 where a 3-D soil model is assumed using the commercial Abaqus finite element package. A square shallow foundation with a width $B=1\text{m}$, subjected to a centered vertical loading is considered. To ensure no boundary effects, a soil width of $5B$ and a thickness of $3B$ were considered. The chosen soil domain (width = $5B$, depth = $3B$) was verified to eliminate boundary effects on bearing capacity under centered vertical loading, consistent with standard recommendations (e.g., Loukidis & Salgado, 2009). Zero-displacement boundary conditions prevented out-of-plane displacements of the lateral soil boundaries. The bottom boundary was fixed in all three directions while the top surface was assumed to be free. The footing was modelled as a rigid body with a reference point at its center. Therefore, a fully bonded foundation/soil interface was assumed to be appropriate to prevent slip and separation in the current study.

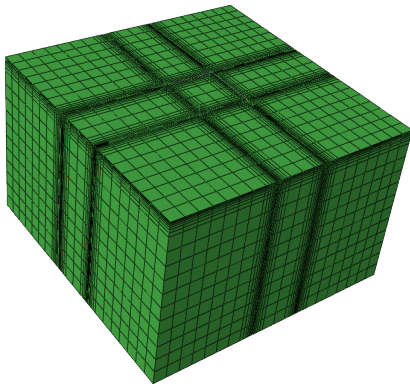


Figure 1 FE meshing used into Abaqus 6.13

The soil domain was divided into 43800 3-D meshes of C3D8I type in total. In each element, the soil properties were constant and defined by a cohesion c and a friction angle ϕ . However, these properties varied from element to element, reflecting the soil's spatial variability in both horizontal and vertical directions. In this analysis, a more refined mesh was used below and in the vicinity of the footing to improve the numerical accuracy of the solution. Although the importance of mesh refinement in determining the accuracy of the results is recognized, it is chosen in reliability analyses to preserve the distribution and the initial autocorrelation structure of the considered random fields. Corresponding to the analyses performed by Griffiths and Fenton (2001, 2004) and Ching and Phoon (2013), the ratio of the finite element size to the scale of fluctuation has to be sufficiently small to ensure that the variance function approaches 1. Therefore, the maximum width of one element is limited to the half of the scale of fluctuation (Der Kiureghian & Ke, 1988).

The soil was modeled with an elasto-perfectly plastic assumption of the Mohr-Coulomb constitutive law. The elastic behavior was defined by a Poisson's ratio $\nu = 0.2$ and a Young's modulus $E = 240$ MPa. The soil unit weight was assumed to be deterministic with a constant value $\gamma = 20$ kN/m³.

The shear strength parameters including the soil cohesion (c) and a friction angle (ϕ) were modeled as lognormally distributed random fields with means μ_c and μ_ϕ , and coefficients of variation COV_c and COV_ϕ , respectively.

In this study, the mean values μ_c and μ_ϕ were held constant at 30 kPa and 20°, respectively, and COV_c and COV_ϕ were assumed at 20% and 10%, respectively. Moreover, two configurations were considered: the first one assuming that c and ϕ are independent random fields and the second configuration presuming that c and ϕ are correlated with a cross-correlation coefficient $\rho_{c,\phi} = -0.25$. In the current study, an anisotropic behavior of the soil is assumed, characterized by a small vertical scale of fluctuation $\delta_v = 1$ m and a larger horizontal scale of fluctuation $\delta_h = 20$ m due to uniform deposition. These values are consistent with typical values reported by Phoon and Kulhawy (1999) and Bienen et al. (2012). It is worth highlighting that, in practice, the probabilistic distribution parameters (mean, COV, scale of fluctuation, and cross-correlation) should be derived from site-specific in-situ tests such as CPTu, piezocone, or other relevant geotechnical soundings, rather than relying solely on generic values from the literature.

Although the footing size (1 m × 1 m) appears small compared to the horizontal scale of fluctuation ($\delta_h = 20$ m), this ratio ($B/\delta_h = 0.05$) is representative of highly stratified deposits where soil properties are nearly constant over the footing width.

MATLAB code has been developed to generate random fields of shear strength. Importing these fields into Abaqus has been performed using a Python code based on the Local Average Subdivision algorithm (Fenton & Vanmarcke, 1990).

Generating Abaqus configurations using the conventional Monte Carlo method is very time-consuming. For this, a second MATLAB code has been developed to generate files with the .inp extension as input files for Abaqus.

Figure 2 and Figure 3 show, respectively, realizations of soil friction angle and cohesion for the first soil configuration (i.e. independent c and ϕ). It can be seen from these figures that the soil tends to be stratified, as the horizontal fluctuation scale is much larger than the vertical one for both cohesion and friction angle random fields.

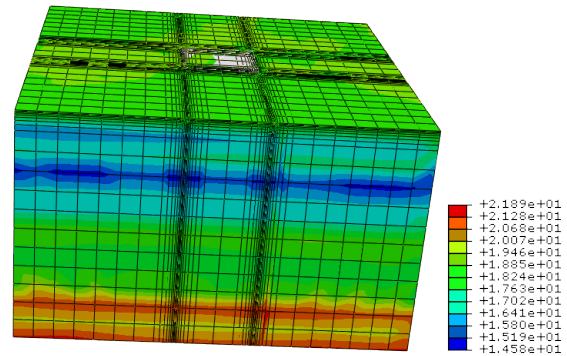


Figure 2 Realization of the random field modeling the friction angle (Configuration1)

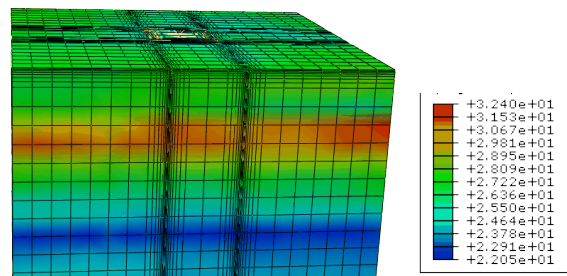


Figure 3 Realization of the random field modeling the cohesion (Configuration1)

A post-processing MATLAB code has also been developed to extract load-displacement data from Abaqus results files with the .odb extension, in order to perform statistical analysis.

4. RESULTS AND DISCUSSIONS

4.1 Model Validation

Even if the problem considered in this study is a static problem, Abaqus/explicit, i.e., integrating an explicit procedure, is used to reduce the computational cost. A first deterministic analysis had been carried out to assess the capacity of the model to estimate the ultimate vertical load of the square footing.

The load-displacement curve for the reference point of the footing is provided in Figure 4, showing that the ultimate vertical load for the deterministic configuration is about $Q_l = 645$ kN and associated with the failure mechanism represented in Figure 5. This ultimate load is compatible with the one obtained using the Eurocode standards. The plateau of the curve in Figure 4 clearly shows that the ultimate vertical load is reached after a foundation settlement of about 4.5 cm. The failure mechanism is marked by punching shear failure which is a typical mechanism for compressible soils.

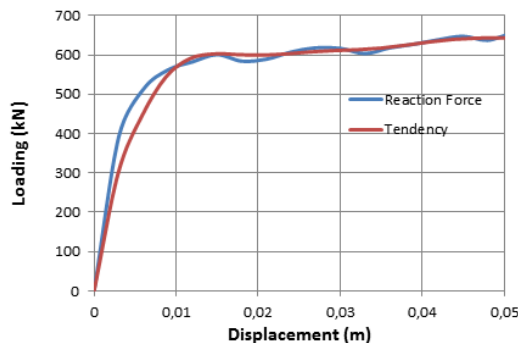


Figure 4 Load-displacement curve for the deterministic case

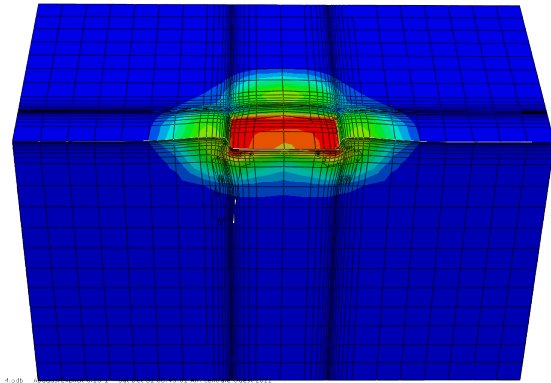


Figure 5 Failure Mechanism for the deterministic case

4.2 Stochastic Analyses

Several simulations were conducted for both independent and correlated random fields, and the corresponding bearing capacity of the foundation, together with its mean value and standard deviation, failure probability, and reliability index, were evaluated. Each simulation generated 500 sets of random numbers. Figure 6 shows an example of the load-displacement curves represented for a set of random fields realizations simulated for the first soil configuration, assuming that c and ϕ are independent random fields. The numerical convergence is presented in Figure 7, showing that the predicted normalized mean value of the bearing capacity with respect to the deterministic value remains practically constant for sample sizes larger than 400.

The normalized mean value of the bearing capacity is about 0.94, showing that taking into account spatial soil variability, the bearing capacity of the foundation is about 6% less than the one estimated considering deterministic shear strength parameters. Similar behavior was previously observed in 2-D cases (e.g., Fenton and Griffiths (2003)), and 3-D cases (e.g., Fenton and Griffiths (2005) and Chwała (2019)).

As an indication, Pieczynska-Kozłowska et al. (2015) found that a stable mean value (with the probability of deviation less than 5%) was achieved using 300 realizations for a square foundation where the soil is supposed to have an anisotropic behavior with vertical and horizontal scales of fluctuation $\delta_v = 0.5 \text{ m}$ and $\delta_h = 1 \text{ m}$, and relatively high values of coefficient of variation for c and ϕ assumed to be independent ($COV_c = 55\%$; $COV_\phi = 25\%$). Soubra et al. (2008) had shown that the predicted mean value and coefficient of variation remain practically constant for sample size larger than 100 when assuming strip foundation with breadth $B=2 \text{ m}$ subjected to a central vertical load and the soil is governed by an isotropic scale of fluctuation $\delta_v = \delta_h = 2 \text{ m}$.

By assuming that c and ϕ are modelled using independent random fields, the ultimate load lies within a range of [350; 1380] kN, resulting enough in itself to show the reliability analyses importance in a foundation bearing capacity assessment.

The COV of bearing capacity is about 0.23. Kawa and Pula (2020) have shown that the COV of bearing capacity of a square foundation

is sensitive to the horizontal scale of fluctuation and lies between 0.15 and 0.36 when the two fields of c and ϕ were assumed to be stochastically independent.

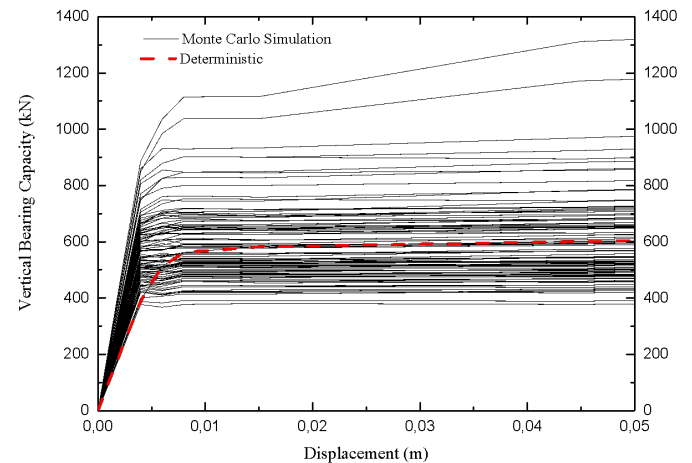


Figure 6 Examples of load-displacement curves for independent random fields

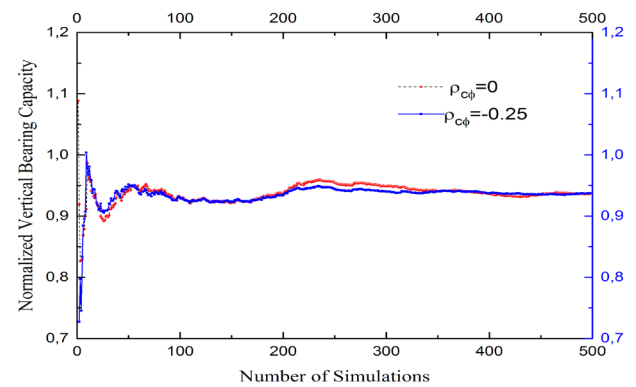


Figure 7 Numerical convergence speed of the bearing capacity: mean value of the bearing capacity versus sample size

To investigate the sensitivity of bearing capacity to the cross-correlation between random fields, a second configuration was considered, assuming a cross-correlation coefficient $\rho_{c,\phi} = -0.25$. Figure 8 shows an example of the load-displacement curves represented for a set of random fields realizations simulated in this configuration. The convergence speed is shown in Figure 7, confirming that the predicted mean value of the normalized vertical bearing capacity remains practically constant for sample sizes larger than 400.

By assuming that c and ϕ are modeled using correlated random fields, the ultimate load lies within a range of [380; 1130] kN for the 500 sets of random numbers generated in this study. It can be observed that the difference between the obtained extreme bearing capacities narrows when the shear strength parameters are assumed to be negatively correlated.

A sensitivity study of bearing capacity is conducted, with results shown in Figures 9 and 10, which present the mean value and standard deviation of bearing capacity, respectively, versus sample size. Even though the final mean of the bearing capacity is almost the same for the two configurations, the standard deviation of the bearing capacity when assuming correlated parameters is less than the predicted one when assuming that random fields are independent and is about 0.19.

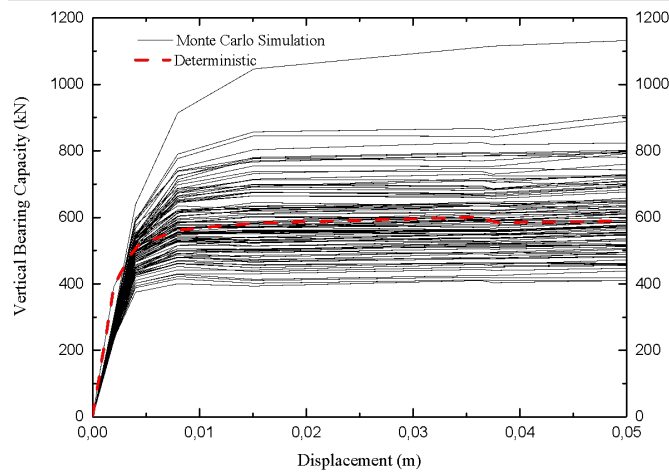


Figure 8 Examples of load-displacement curves assuming that cohesion and friction angle are negatively correlated fields

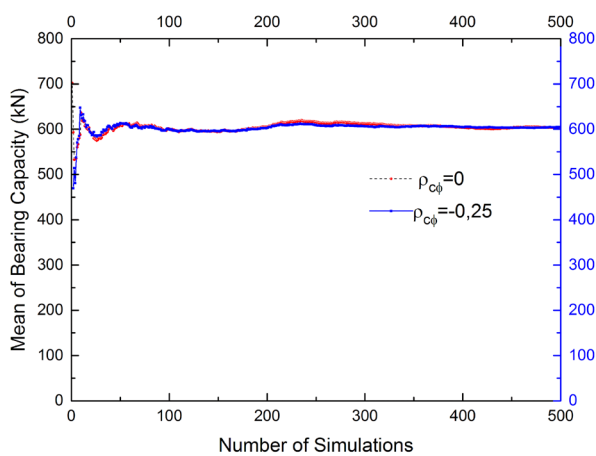


Figure 9 Numerical convergence of the bearing capacity mean value

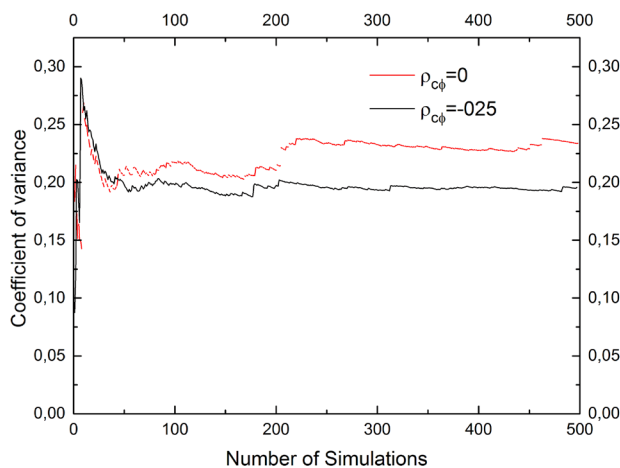


Figure 10 Numerical convergence of the bearing capacity standard deviation

4.3 Failure Mechanisms

In addition to the above-mentioned results, Figures 11 and 12 illustrate respectively the horizontal and vertical displacements associated to a configuration of the soil variations assuming independent shear strength parameters. These figures show that soil spatial variability has triggered an asymmetric failure mechanism which is different from that predicted by a traditional deterministic analysis. This change in the failure surface can significantly affect the foundation's ultimate bearing capacity and settlement.

Furthermore, it was noticed for this bearing capacity assessment

that consideration of the spatial variability of soil properties leads to totally different failure mechanism surfaces from one realization to another. Vertical displacements are often associated with equivalent horizontal displacements, as shown in Figure 11. No 'average' failure surface could be considered in this problem and the only method to solve it accurately is through the use of probabilistic approaches.

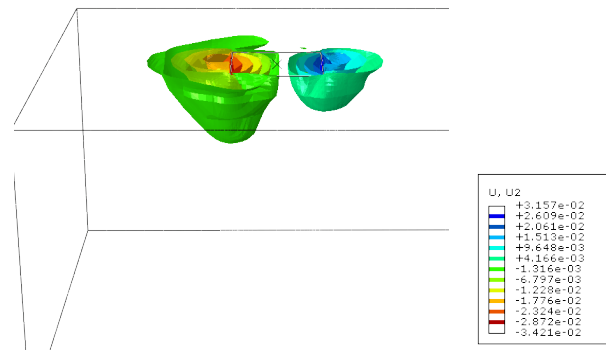


Figure 11 Horizontal displacement (U_2) field in the soil for one configuration assuming that c and ϕ are independent fields

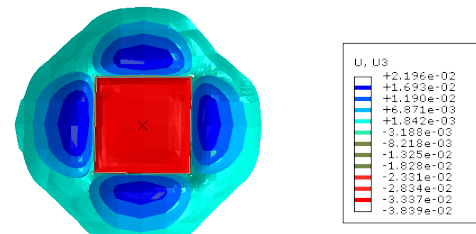


Figure 12 Vertical displacement (U_3) field in the soil for one configuration assuming that c and ϕ are independent fields

4.4 Reliability Analysis

Table 1 and Figure 13 summarize the obtained Hasofer-Lind reliability indices for different values of the applied vertical load Q and, hence, different safety levels of the foundation according to both configurations of soil variations. The reliability index decreases with the increase of the applied load (i.e., the decrease of the safety factor F_p).

The obtained results indicate that the random fields simulation of shear strength parameters provides a probability of exceeding the deterministic bearing capacity ($F_p = 1$) greater than 36% for both configurations. This means that the median bearing capacity accounting for spatial soil variability is always smaller than the deterministic bearing capacity, as noted by Cho and Park (2010) and Massih and Soubra (2008). A failure probability value greater than 1 % for $F_p = 3$ was obtained when assuming that the shear strength parameters are independent. However, when assuming that c and ϕ are correlated, the probability of failure drops to about 0.4 % for $F_p = 3$.

The comparison of the results of correlated random fields with those of independent ones shows that the reliability index corresponding to uncorrelated variables is smaller, except for $F_p = 1$, than the one of negatively correlated random fields. One can conclude that the hypothesis of independent shear strength parameters is conservative compared with that of negatively correlated parameters. This has also been noted by Imanzadeh et al. (2020). For instance, when the safety factor is 2.0, the reliability index increases by about 13% if the variables c and ϕ are assumed to be negatively correlated. This finding emphasizes the importance of good statistical characterization of soil parameters, together with 3-D simulations, for accurate identification of statistical characteristics.

EN 1990 recommends a target probability $p_t = 7.24 \times 10^{-5}$ for the Ultimate Limit State (ULS), corresponding to a target reliability index $\beta_t = 3.8$ for a 50-year reference period and common types of structures

in the consequence class CC2 (ordinary buildings and their foundations).

Results of this study would suggest to recommend for the considered statistical characterization of the random fields, a relatively high safety factor ($F_p = 4$) to achieve the target reliability index $\beta_t = 3.8$ when random fields are considered as negatively correlated. A higher safety factor is required, however, to achieve the target reliability index of $\beta_t = 3.8$ when assuming c and ϕ to be modeled using independent random fields.

Table 1 Reliability index and probability of failure values

F_p	Q (kPa)	$\rho_{c\phi} = 0$		$\rho_{c\phi} = -0.25$	
		β_{HL}	$P_f(\%)$	β_{HL}	$P_f(\%)$
4	161.25	3.40	$3.49 \cdot 10^{-2}$	3.83	$6.41 \cdot 10^{-3}$
3	215	2.98	$1.44 \cdot 10^{-1}$	3.37	$3.76 \cdot 10^{-2}$
2	322.5	2.15	1.6	2.44	$7.34 \cdot 10^{-1}$
1.25	516	1.32	9.3	1.51	6.65
1	645	0.34	36.7	0.34	36.7

Table 1 confirmed again that the hypothesis of independent parameters was found to be conservative in comparison to the one of negatively correlated random fields. It confirmed also that greater safety factors are required to achieve allowable failure probability for the specified statistical soil configuration.

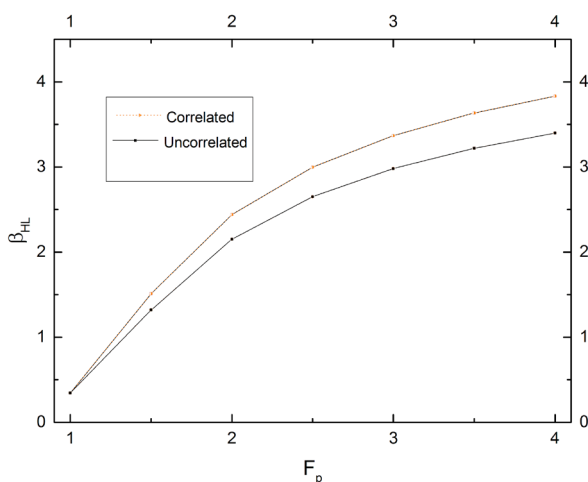


Figure 13 Reliability index versus safety factor

5. CONCLUSIONS

This paper contributes to the development of probabilistic foundation analyses by performing a 3-D mechanical-reliability coupling to assess the bearing capacity of a square shallow foundation subjected to a centered vertical load. To evaluate the effects of uncertainties on foundation performance, a probabilistic methodology that focuses on soil variability is proposed. For this purpose, the spatial variability of shear strength parameters is accounted for using 3-D stationary random fields. An explicit resolution method has been tested and used to overcome the required computing power in a 3-D model.

The reliability analysis permits the determination of approximations of the foundation reliability index and probability of failure for two statistical sets of shear strength parameters using the Monte Carlo simulation method.

It is worth noting that in this study, the variability in bearing capacity was solely due to soil spatial variability and not to statistical uncertainties arising from limited sampling. In cases in which these two uncertainty sources are not easily separable, it is still assumed that the adopted COV is due only to spatial variability.

Although at most building sites the soil properties at soil sample locations are known, this was not incorporated in the present study. These findings highlight the importance of the probabilistic foundation analyses and show the insufficiency of the conventional

approach to predict the foundation failure. It emphasizes the importance of conducting 3-D simulations for a good identification of statistical characteristics. The hypothesis of independent parameters was found to be conservative in comparison to the one of negatively correlated shear strength parameters.

It is worth noting that the results presented in this study are limited to the specific case considered. However, the methodology demonstrates a straightforward and effective way of quantifying and understanding uncertainty in the ultimate limit state design of 3D foundations in heterogeneous soil conditions.

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