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สูตรสำเร็จสำหรับค่าความยาววิ่งเฉลี่ยของแผนภูมิควบคุมสำหรับกระบวนการ ARX(p)

Exact expression for average run length of control chart of ARX(p) procedure

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บทคัดย่อ

การควบคุมกระบวนการทางสถิติ (SPC) สามารถนิยามเป็นการใช้วิธีการทางสถิติและเทคนิคเพื่อปรับปรุงประสิทธิภาพกระบวนการผลิตและคุณภาพของผลิตภัณฑ์ หนึ่งในเครื่องมือที่มีประสิทธิภาพของ SPC คือ แผนภูมิควบคุม แผนภูมิควบคุมผลรวมสะสม (CUSUM) และ แผนภูมิควบคุมค่าเฉลี่ยเคลื่อนที่ถ่วงน้ำหนักแบบเลขชี้กำลัง (EWMA) เป็นแผนภูมิควบคุมที่นำไปประยุกต์ใช้กันอย่างแพร่หลาย เช่น ทางเศรษฐกิจ อุตสาหกรรม และวิศวกรรม สำหรับกระบวนการที่น่าสนใจ ค่าสังเกตมักเกี่ยวข้องกับเวลาซึ่งจะมีความสัมพันธ์กัน ค่าความยาววิ่งเฉลี่ย (ARL) ถูกนำมาใช้ในการวัดประสิทธิภาพการดำเนินงานของแผนภูมิควบคุม เป้าหมายหลักของงานวิจัยนี้คือเพื่อให้ได้สูตรสำเร็จสำหรับค่าความยาววิ่งเฉลี่ยของแผนภูมิควบคุมรวมสะสมและแผนภูมิควบคุมค่าเฉลี่ยเคลื่อนที่ถ่วงน้ำหนักแบบเลขชี้กำลังสำหรับกระบวนการ ARX(p) การเปรียบเทียบระหว่างแผนภูมิควบคุมพบว่าแผนภูมิควบคุมค่าเฉลี่ยเคลื่อนที่ถ่วงน้ำหนักแบบเลขชี้กำลังให้ผลดีกว่าเมื่อค่าเฉลี่ยของพารามิเตอร์เปลี่ยนแปลงเล็กน้อย

ABSTRACT

Statistical Process Control (SPC) can be defined as the use of statistical methods and technicals to improve process productivity and product quality. One of efficient tools of SPC are the control charts. The Cumulative Sum (CUSUM) control chart and Exponentially Weighted Moving Average (EWMA) control chart are widely used in several applications such as economic, engineering and industries. For many processes of interest, observations are closely space in time which will be correlated. The measurement of performance used in the average run length (ARL). The main goal of this paper is to derive analytical solutions for average run length of the Cumulative Sum and Exponential Weighted Moving Average control charts for ARX(p) processes with exponential white noise. The comparing of control charts found that EWMA control chart is better when mean shift is small.

คำสำคัญ: ความยาววิ่งเฉลี่ย แผนภูมิควบคุม กระบวนการ ARX(p)
Keyword: Average run length, Control chart, ARX (p) procedure.

1. INTRODUCTION

Statistical Process Control (SPC) can be generally defined as the use of statistical methods and technicals to improve process productivity and product quality. The important tool of SPC is the control chart. The control charts are used for determine when process deviated from it's in-control status. A control chart is a plot of the process or a transformation of the process, with control limits that represent the likely boundaries of data. Data points which fall beyond the control limits are said to be out-of-control. An out-of-control stated usually results in an investigation by the process engineer or operator to determine the cause of the change in the process mean. In several researches, the Exponential Weighted Moving Average (EWMA) control chart proposed by Page (Page, 1954) and the Cumulative SUM control chart introduced by Robert (Robert, 1959) have been proposed as good alternative to the Shewhart control chart for detecting small shift.

The EWMA as a statistic has a long history of varied use. Muth (1960) discusses how it was used in forecasting sales, inventory control, economics and agriculture. In his landmark paper, Roberts (1959) introduced use of the geometric moving average (another name for the EWMA) technique as a control chart method. In this procedure the entire history of observations is assigned a series of weights, with the weights decreasing with the age of the data as a geometric progression. The CUSUM chart is primary used for maintain (rather than improve) current control of a process (Duncan, 1965). The primary advantage of the CUSUM chart is that it will identified a sudden or persistent change in the process average more rapidly than a Shewhart control chart incorporating the initial Shewhart interpretation rule. Furthermore, it is often possible to pinpoint the exact sample where the change in the process occurred (Wetherill and Brown, 1991).

The main assumption of techniques in many traditional control charts are the observations which independent over time. If the variables in the process exhibit correlation over time, this assumption may be violated while the autocorrelation may affect with the rate of false alarm. The effect of autocorrelation in the control charts have been studied by Vasillopoulos and Stamboulis (Vasillopoulos and Stamboulis, 1978), Alwan and Robert (1988), Harris and Ross (1991) and Schmid and Schone (1997). The correlated with exponential distribution has been discussed by several authors, such as Grunwald et al. (2000).

The EWMA and CUSUM control chart have been frequently recommended for the monitoring of correlated observation. Alwan and Roberts (1988) stated that more than 70 percent of the studied processes are subjected to autocorrelation. The run length properties of CUSUM and EWMA charts are strongly affected by the presence of autocorrelation in data. Consequently, there has been considerable research in recent years on designing control charts, suitable for autocorrelated processes.

The measure of the control charts will be average run length (ARL), for in-control (ARL_0) and out-of-control (ARL_1). The method of comparing and contrasting control chart methods compared their shift detection capabilities through their average run length. Harris and Ross (1991) discussed the three way that $ARLs$ can be calculate: using Markov Chain Approach (MCA), solving a set of integral equation (IE), or using Monte Carlo simulation (MC). Superville and Adams (1994) discussed the individuals CUSUM and EWMA control chart, the ARL for evaluating the performance of the control charts were used by simulation study. VanBrackle and Reynolds (1997) present EWMA and CUSUM control charts for the process of mean when the observation were from an AR(1) process with additional random error, the ARL and steady state ARL of control charts were evaluated numerically using an integral equation approach and Markov Chain approach. Recently, Areepong and Novikov (2009) used the Nystrom method and presented the error term of numerical integral equations for approximating ARL_0 and ARL_1 for EWMA control chart. Areepong and Sukparungsee (2010) presented analytical derivation for the ARL of an EWMA control chart when observations are exponentially distributed. Mititelu et al. (2010) presented analytical expressions for ARL of EWMA and CUSUM control chart when observations have hyperexponential distribution using the Fredholm integral equations approach. Petcharat et al. (2012) derived closed-form expressions for the ARL of CUSUM control chart when observations are Pareto and Weibull distributed by approximating those distributions with hyperexponential distribution. In addition, Suriyakat et al.

(2012) presented the explicit formulas of ARL for EWMA control chart for monitoring an AR(1) processes. The explicit formulas of ARL for EWMA control chart based on ARMA(1,1) was proposed by Phanyaem et al. (2013).

In this paper, we derived analytically explicit formulas for ARL_0 and ARL_1 for EWMA and CUSUM control chart when observations are autoregressive with explanatory order p (ARX(p)) with exponential white noise. We used integral equation to derived exact expression for ARL_0 and ARL_1.The operations of the paper are follows: in section 2 introduced the Explicit formulas of ARL for ARX(p) process for control chart. The comparison of the results is addressed in section 3 and conclusions are presented in section 4

2. THE EXPLICIT FORMULAS OF ARL FOR ARX(p) PROCESS FOR CONTROL CHART

In this section, characteristics of CUSUM and EWMA control chart for ARX(p) processes which is much more effective than Shewhart control chart in detecting small and moderate-side sustained in the parameter of the probability distribution of the equal charateristics (for example, Montgomery (2009))

2.1 CUSUM control chart

Given Y_t be a sequence of the Autoregressive with explanatory variable: ARX (p) random processes. We assume that the CUSUM statistic is denote by C_t.

The definition of CUSUM statistics based on ARX(p) process is the following recursion:

$$C_t = \max\left(C_{t-1} + \varepsilon_t - a, 0\right); t = 1,2,... \tag{1}$$

Where C_t is the CUSUM statistics, ε_t are ARX(p) process with the exponential white noise. The value of C_0 is an initial value of CUSUM statistics, C_0 = u and a is non-zero constant.

The Autoregressive process with explanatory variable: ARX(p) process can be written as:

$$Y_t = \beta X_t + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + ... + \phi_p Y_{t-p} + \varepsilon_t \tag{2}$$

Where ε_t is to be a white noise processes assumed with exponential distribution. The initial value is normally to be the process mean, an autoregressive coefficient $-1 < \phi_i < 1; i = 1,2,..., p$. It is assumed that the initial value of ARX(p) process $Y_{t-1}, Y_{t-2},..., Y_{t-p} = 1$ and $X_t = 1$ are explanatory variables.

In this paper, the case of positive change in distribution which crossing the upper control limit raises alarm is mainly discussed. Given $\varepsilon_t, t = 1,2,...$ is a sequence of independent identically distribution random variables with exponential parameter (α). It is normally assumed that under in control state, the parameter has known in-control value $(\alpha = \alpha_0)$. The parameter α could be changed to out-of-control value $(\alpha = \alpha_1)$, when $(\theta = \infty)$, is the change-point time.

The stopping time of CUSUM control chart given by

$$\tau_h = \inf(\, t > 0 : C_t > h),\ h > u \tag{3}$$

Where τ_h is a stopping time

h is a constant parameter known as upper control Limit (UCL).

The most two characteristics control chart are ARL_0 and ARL_1 as following:

$$ARL_0 = E_\infty(\tau_h) \tag{4}$$

$$ARL_1 = E_\theta(\tau_h - \theta + 1 \mid \tau_h \geq \theta) \tag{5}$$

where

$E_\infty(.)$ is the expectation corresponding to the target value and is assumed to be large enough.

$E_\theta(.)$ is the expectation under the assumption that change-point occurs at time $\theta = 1$.

The notations P_c denote the probability measure and E_c denote the expression corresponding chart after it is reset at $u \in [0,h]$. Let $H(u) = E(\tau_h)$ be the ARL of CUSUM control chart after it is reset at $u \in [0,h]$. The solution of integral equation is as following

$$H(u) = 1 + E_c\left[\{0 < C_1 < b\} H(C_1)\right] + P_c\{C_1 = 0\} H(0)\,. \tag{6}$$

Therefore, the integral equation of CUSUM control chart is

$$H(u) = 1 + \alpha e^{\alpha(u-a+\beta X_t+\phi_1 Y_{t-1}+\phi_2 Y_{t-2}+\ldots+\phi_p Y_{t-p})} \int_0^h H(w) e^{-\alpha w} dw + (1 - e^{-\alpha(a-u-\beta X_t-\phi_1 Y_{t-1}-\phi_2 Y_{t-2}-\ldots-\phi_p Y_{t-p})})\ H(0)\,. \tag{7}$$

Let $k = \int_0^h H(w) e^{-\alpha w} dw.$ Consequently, $H(u)$ can be rewritten as

$$H(u) = 1 + \alpha e^{\alpha(u-a+\beta X_t+\phi_1 Y_{t-1}+\phi_2 Y_{t-2}+\ldots+\phi_p Y_{t-p})}\, k + (1 - e^{-\alpha(a-u-\beta X_t-\phi_1 Y_{t-1}-\phi_2 Y_{t-2}-\ldots-\phi_p Y_{t-p})})\ H(0)\,. \tag{8}$$

Consequently,

$$H(u) = 1 + \alpha k + e^{\alpha(a-\beta X_t-\phi_1 Y_{t-1}-\phi_2 Y_{t-2}-\ldots-\phi_p Y_{t-p})} - e^{\alpha u}$$

To find a constant k as following form

$$k = \int_0^h H(w) e^{-\alpha w} dw$$

$$= \frac{e^{\alpha h}}{\alpha}\left(1 - e^{-\alpha h}\right)\left(1 + e^{\alpha\left(a-\beta X_t-\phi_1 Y_{t-1}-\phi_2 Y_{t-2}-\ldots-\phi_p Y_{t-p}\right)}\right) - h e^{\alpha h}.$$

Thus, we get the explicit solution for ARL of CUSUM control chart as follow

$$H(u) = e^{\alpha h}\left(1 + e^{\alpha\left(a-\beta X_t-\phi_1 Y_{t-1}-\phi_2 Y_{t-2}-\ldots-\phi_p Y_{t-p}\right)} - \alpha h\right) - e^{\alpha u}.$$

Since the process is in-control state with exponential parameter $\alpha = \alpha_0$, we obtain the explicit solution for ARL_0 as follows

$$ARL_0 = e^{\alpha h_0}\left(1+e^{\alpha_0\left(a-\beta X_t-\phi_1 Y_{t-1}-\phi_2 Y_{t-2}-...-\phi_p Y_{t-p}\right)}-\alpha_0 h\right)-e^{\alpha_0 u}.$$

Since the process is out-of-control state with exponential parameter $\alpha=\alpha_1$, The explicit solution for ARL_1 can be written as follows

$$ARL_1 = e^{\alpha_1 h}\left(1+e^{\alpha_1\left(a-\beta X_t-\phi_1 Y_{t-1}-\phi_2 Y_{t-2}-...-\phi_p Y_{t-p}\right)}-\alpha_1 h\right)-e^{\alpha_1 u}.$$

Where $-1<\phi_i<1; i=1,2,...,p$ are Autoregressive coefficients, β is a coefficient, X_t is explanatory variable, h is a upper control limit.

2.2 EWMA control chart

Given Y_t be a sequence of the Autoregressive with explanatory variable: ARX(p) random processes. We assume that the EWMA statistic is denote by Z_t.

The definition of EWMA statistics based on ARX(p) process is the following recursion:

$$Z_t = (1-\lambda)Z_{t-1} + \lambda Y_t, \quad t=1,2,... \tag{9}$$

Where Y_t are sequence of ARX(p) process with exponential white noise, λ is an exponential smoothing parameter with $0<\lambda<1$ and initial value $Z_0 = u$.

Therefore, the EWMA statistic Z_t can be written in this form

$$Z_t = (1-\lambda)Z_{t-1} + \lambda(\beta X_t + \phi_1 Y_{t-1} + \phi_1 Y_{t-2} + ... + \phi_p Y_{t-p}).$$

$$Z_t = (1-\lambda)Z_{t-1} + \lambda\beta X_t + \lambda\phi_1 Y_{t-1} + \lambda\phi_1 Y_{t-2} + ... + \lambda\phi_p Y_{t-p}$$

The stopping time of EWMA control chart is defined as follows

$$\tau_b = \inf(t>0: Z_t > b),\ b>u \tag{10}$$

where τ_b is a stopping time

b is a constant parameter known as upper control Limit (UCL).

Assume that process is in-control at time t if the EWMA statistics Z_t is in range and the process is out-of-control if $Z_t > b$. Let $L(u)$ denote the ARL for ARX(p) process with an initial value $(Z_0 = u)$. Now, we define the function $L(u)$ as follows

$$ARL = L(u) = E_\infty(\tau) \geq T\ , Z_0 = u. \tag{11}$$

If Y_1 gives an in-control state for Z_t, then

$$0 < (1-\lambda)u + \lambda\beta X_t + \lambda\phi_1 Y_{t-1} + \lambda\phi_1 Y_{t-2} + ... + \lambda\phi_p Y_{t-p} < b$$

$$L(Z_t) = L((1-\lambda)u + \lambda\beta X_t + \lambda\phi_1 Y_{t-1} + \lambda\phi_1 Y_{t-2} + ... + \lambda\phi_p Y_{t-p}),$$

Consider function $L(u) = \int L(Z_1) f(\zeta_t) d\zeta_t$,

therefore

$$L(u) = \int L((1-\lambda)u + \lambda\beta X_t + \lambda\phi_1 Y_{t-1} + \lambda\phi_1 Y_{t-2} + ... + \lambda\phi_p Y_{t-p}) f(w)dw. \tag{12}$$

Changing variable in integration will be

$$L(u) = \int_0^b L((1-\lambda)u + \lambda\beta X_t + \lambda\phi_1 Y_{t-1} + \lambda\phi_1 Y_{t-2} + ... + \lambda\phi_p Y_{t-p}) f(w)dw.$$

$$L(u)=1+\frac{1}{\lambda\alpha}\int_0^b L(w)e^{-\frac{w}{\lambda\alpha}}e^{\left(\frac{(1-\lambda)u}{\alpha\lambda}+\frac{\beta X_t+\phi_1 Y_{t-1}+\phi_1 Y_{t-2}+\ldots+\phi_p Y_{t-p}}{\alpha}\right)}dw .$$

Let $C(u)=e^{\left(\frac{(1-\lambda)u}{\lambda\alpha}+\frac{\beta X_t+\phi_1 Y_{t-1}+\phi_1 Y_{t-2}+\ldots+\phi_p Y_{t-p}}{\alpha}\right)}$.

Consequently,

$$L(u)=1+\frac{C(u)}{\lambda\alpha}\int_0^b L(w)e^{-\frac{w}{\lambda\alpha}}dw,\quad 0\le u\le b .$$

Let $k=\int_0^b L(w)e^{-\frac{w}{\lambda\alpha}}dw$, so we have $L(u)=1+\frac{C(u)}{\lambda\alpha}k.$

$$L(u)=1+\frac{1}{\lambda\alpha}e^{\left(\frac{(1-\lambda)u}{\lambda\alpha}+\frac{\beta X_t+\phi_1 Y_{t-1}+\phi_1 Y_{t-2}+\ldots+\phi_p Y_{t-p}}{\alpha}\right)}k .$$

solving a constant $k=\int_0^b L(w)e^{-\frac{w}{\lambda\alpha}}dw.$

$$k=\frac{\lambda\alpha(1-e^{-\frac{b}{\lambda\alpha}})}{1-e^{\left(\frac{\beta X_t+\phi_1 Y_{t-1}+\phi_1 Y_{t-2}+\ldots+\phi_p Y_{t-p}}{\alpha}\right)}\frac{1}{\lambda}\left(1-e^{-\frac{b}{\alpha}}\right)} .$$

Substitute constant into function $L(u)$, Thus, we get the explicit solution for ARL of EWMA control chart as follow

$$L(u)=1-\frac{\lambda e^{\left(\frac{(1-\lambda)u}{\lambda\alpha}\right)}(e^{\left(-\frac{b}{\lambda\alpha}\right)}-1)}{\lambda e^{-\left(\frac{\beta X_t+\phi_1 Y_{t-1}+\phi_1 Y_{t-2}+\ldots+\phi_p Y_{t-p}}{\alpha}\right)}+\left(e^{-\frac{b}{\alpha}}-1\right)} .$$

Since the process is in-control state with exponential parameter $\alpha=\alpha_0$, we obtain the explicit solution for ARL_0 as follows

$$ARL_0=1-\frac{\lambda e^{\left(\frac{(1-\lambda)u}{\lambda\alpha_0}\right)}(e^{\left(-\frac{b}{\lambda\alpha_0}\right)}-1)}{\lambda e^{-\left(\frac{\beta X_t+\phi_1 Y_{t-1}+\phi_2 Y_{t-2}+\ldots+\phi_p Y_{t-p}}{\alpha_0}\right)}+\left(e^{-\frac{b}{\alpha_0}}-1\right)}$$

Since the process is out-of-control state with exponential parameter $\alpha=\alpha_1$, The explicit solution for ARL_1 can be written as follows

$$ARL_1=1-\frac{\lambda e^{\left(\frac{(1-\lambda)u}{\lambda\alpha_1}\right)}(e^{\left(-\frac{b}{\lambda\alpha_1}\right)}-1)}{\lambda e^{-\left(\frac{\beta X_t+\phi_1 Y_{t-1}+\phi_2 Y_{t-2}+\ldots+\phi_p Y_{t-p}}{\alpha_1}\right)}+\left(e^{-\frac{b}{\alpha_1}}-1\right)}$$

Where $-1<\phi_i<1;i=1,2,\ldots,p$ are Autoregressive coefficients, β is a coefficient, X_t is explanatory variable, b is a upper control limit. It is assumed that the initial value of ARX(p) process $Y_{t-1},Y_{t-2},\ldots,Y_{t-p}=1$ and $X_t=1$

3. COMPARISON OF PERFORMANCE BETWEEN CUSUM AND EWMA CONTROL CHARTS.

A comparison of performance between CUSUM and EWMA control charts are discussed in this section. Usually, the comparison of performance of control charts are made by designing the common ARL_0 and comparing the ARL_1 of control charts by given α, where $\alpha = 1.00$, 1.01,...,5. We use the explicit formulas obtained previously to evaluate ARL_0 and ARL_1 for CUSUM and EWMA control charts. The performance of control chart is considered by smaller of ARL_1.

In Table 1, the values of parameter for CUSUM and EWMA control charts were established by setting $ARL_0 = 370$, $\alpha_0 = 1.00$, $\beta = 0.1$ and $\phi_1 = 0.1$. We compare the results of ARL_1 for ARX(1) process between CUSUM and EWMA control charts.

Table 1 Comparison of ARL_1 for ARX(1) between CUSUM and EWMA control charts, given $ARL_1 = 370$, $u = 0$, $\beta = 0.1$ and $\phi_1 = 0.1$

α	CUSUM			EWMA		
	$a = 2.0$ $h = 5.441$	$a = 2.5$ $h = 3.97$	$a = 3.0$ $h = 3.265$	$\lambda = 0.01$ $b = 0.00820572$	$\lambda = 0.03$ $b = 0.0248217$	$\lambda = 0.05$ $b = 0.0417181$
1.00	370.071	370.113	370.225	370.212	370.074	370.056
1.01	339.028	346.371	347.839	50.6743	51.1992	51.7389
1.02	311.144	324.577	327.203	27.7089	28.0113	28.322
1.03	286.056	304.543	308.154	19.3014	19.5132	19.7307
1.04	263.445	286.100	290.548	14.9403	15.1031	15.2704
1.05	243.033	269.099	274.253	12.2707	12.4029	12.5388
1.06	224.576	253.406	259.153	10.4682	10.5795	10.6938
1.07	207.861	238.902	245.143	9.16916	9.26522	9.36389
1.08	192.701	225.480	232.129	8.18842	8.27291	8.3597
1.09	178.93	213.044	220.026	7.42167	7.49708	7.57453
1.10	166.402	201.507	208.758	6.8057	6.87378	6.9437
1.20	87.2578	121.752	129.529	4.0018	4.03628	4.07169
1.30	52.0945	79.8123	86.5784	3.05122	3.07427	3.09794
1.40	34.6177	55.8234	61.3861	2.57015	2.58745	2.60521
1.50	25.0572	41.1364	45.6416	2.27828	2.29212	2.30632
2.00	10.2254	14.8188	16.5128	1.67934	1.68624	1.69332
3.00	5.18473	5.87050	6.28861	1.36129	1.36473	1.36825
4.00	3.70802	3.81488	3.97075	1.24852	1.25081	1.25315
5.00	2.99359	2.96005	3.03025	1.18995	1.19166	1.19342

In Table 2, the values of parameter for CUSUM and EWMA control charts were established by setting $ARL_0 = 370, \alpha_0 = 1.00, \beta = 0.1, \phi_1 = 0.1, \phi_2 = 0.1$. We compare the results of ARL_1 for ARX(2) process between CUSUM and EWMA control charts.

Table 2 Comparison of ARL_1 for ARX(2) between CUSUM and EWMA control charts, given $ARL_0 = 370$, $u = 0$, $\beta = 0.1$, $\phi_1 = 0.1$ and $\phi_2 = 0.1$

α	CUSUM			EWMA		
	$a=2$ $h=4.926$	$a=2.5$ $h=4.415$	$a=3$ $h=3.154$	$\lambda=0.01$ $b=0.00820572$	$\lambda=0.03$ $b=0.0224315$	$\lambda=0.05$ $b=0.03767$
1.00	370.219	370.132	370.109	370.126	370.407	370.581
1.01	342.842	345.916	347.882	48.6687	49.1225	49.584
1.02	317.993	323.719	327.383	26.5460	26.8083	27.0669
1.03	295.401	303.344	308.453	18.4785	18.6582	18.8417
1.04	274.829	284.614	290.948	14.3005	14.4382	14.579
1.05	256.067	267.373	274.74	11.7452	11.8569	11.9711
1.06	238.93	251.481	259.715	10.0209	10.1147	10.2107
1.07	223.254	236.812	245.768	8.77872	8.85967	8.94246
1.08	208.895	223.256	232.807	7.84121	7.91235	7.98512
1.09	195.722	210.712	220.749	7.10844	7.17189	7.23679
1.10	183.622	199.09	209.517	6.51989	6.57714	6.63571
1.20	103.654	119.218	130.402	3.84260	3.87148	3.90104
1.30	64.9051	77.6594	87.3672	2.93607	2.95534	2.97505
1.40	44.1678	54.0927	62.0533	2.47785	2.49228	2.50704
1.50	32.124	39.7638	46.1948	2.20018	2.2117	2.22349
2.00	12.0292	14.3464	16.7394	1.63225	1.63797	1.64381
3.00	5.38253	5.76997	6.35134	1.33319	1.33602	1.33891
4.00	3.70268	3.78322	3.99658	1.22811	1.22999	1.23191
5.00	2.94832	2.94906	3.04323	1.17385	1.17525	1.17669

In Table 3, the values of parameter for CUSUM and EWMA control charts were established by setting $ARL_0 = 370, \alpha_0 = 1.00, \beta = -0.1, \phi_1 = -0.1$, $\phi_2 = -0.1$. We compare the results of ARL_1 for ARX(2) process between CUSUM and EWMA control charts.

Table 3 Comparison of ARL_1 for ARX(2) between CUSUM and EWMA control charts, given $ARL_0 = 370$, $u = 0$, $\beta = -0.1$, $\phi_1 = -0.1$ and $\phi_2 = -0.1$

α	CUSUM			EWMA		
	$a = 2.0$ $h = 3.97$	$a = 2.5$ $h = 3.265$	$a = 3.0$ $h = 2.681$	λ=0.01 $b = 0.01357013$	λ=0.03 $b = 0.0412753$	λ=0.05 $b = 0.0697693$
1.00	370.113	370.225	370.294	370.122	370.441	370.16
1.01	346.371	347.893	348.554	66.6166	68.0579	69.552
1.02	324.577	327.203	328.476	37.1531	38.0179	38.924
1.03	304.543	308.154	309.908	26.017	26.6316	27.2779
1.04	286.1	290.548	292.713	20.1639	20.6399	21.1413
1.05	269.099	274.253	276.771	16.5548	16.9428	17.3521
1.06	253.406	259.153	261.971	14.1064	14.4338	14.7793
1.07	238.902	245.143	248.215	12.3362	12.6192	12.918
1.08	225.48	232.129	235.415	10.9965	11.2457	11.5087
1.09	213.044	220.026	223.491	9.94711	10.1696	10.4045
1.10	201.507	208.758	212.37	9.10284	9.30376	9.51593
1.20	121.752	129.529	133.556	5.24465	5.3461	5.45315
1.30	79.8123	86.5784	90.2068	3.92982	3.9973	4.06843
1.40	55.8234	61.3861	64.465	3.26214	3.31253	3.3656
1.50	41.1364	45.6416	48.2078	2.85571	2.89585	2.93807
2.00	14.8188	16.5128	17.5946	2.01442	20.3414	2.05481
3.00	5.8705	6.28861	6.60229	1.55649	1.56617	1.57629
4.00	3.81488	3.97075	4.10502	1.3892	1.3956	1.40229
5.00	2.96005	3.03025	3.10039	1.30057	1.30535	1.31033

The table1-table 3 present the comparing result from the CUSUM and EWMA control charts shows that for the case of one-sided shift, it has been shown that the EWMA control chart performs better than CUSUM control chart when the process has small change parameter.

4. CONCLUSIONS

We compared the effectiveness of CUSUM and EWMA control charts for detecting small changes in mean of process. The comparison of the control charts based on the ARL_0 and ARL_1 criteria. The results of comparison have shown that the performance of EWMA control chart is superior to CUSUM control chart for detecting small change in mean of process.

REFERENCES

Alwan, L. C. and Roberts, H. V. (1988). Time-series modeling for statistical process control. Journal of Business & Economic Statistics 6(1): 87-95.

Areepong, Y. and Novikov, A. A. (2009). An integral equation approach for analysis of control chart, Ph.D. Thesis, University of Technology. Sydney, Australia: pp. 96.

Areepong, Y. and Sukparungsee, S. (2010). An Integral Equation Approach to EWMA Chart for Detecting a Change in Lognormal Distribution. Thailand Statistician 8(1): 47-61.

Grunwald S., Barak P., McSweeney K. and Lowery, B. (2000). Soil landscape models at different scales portrayed in Virtual Reality Modeling Language (VRML). Soil Science An Interdisciplinary Approach to Soils Research 165(8): 598-615.

Duncan, A. J. (1965). Quality Control and Industrial Statistics. Homewood, Illinois: Richard D. Irwin Inc. pp. 178-192

Harris, T. J. and Ross, W. H. (1991). Statistical process control procedures for correlated observations. Canadian Journal of Chemical Engineering 69(1): 48-57.

Mititelu, G., Areepong, Y., Sukparungsee S. and Novikov, A. (2010). Explicit analytical solutions for the average run length of CUSUM and EWMA charts. East-West Journal of Mathematics 1: 253-265.

Montgomery, D.C. (2009). Introduction to Statistical Quality Control, (6^{th}edition). Hoboken, NJ: John Wiley and Sons Inc. pp: 264-281.

Muth, J. F. (1960). Optimal Properties of Exponentially Weighted Forecasts. Journal of the American Statistical Association 55(290): 299-306.

Page, E. S. (1954). Continuous inspection schemes. Biometrics 41(1): 100-114.

Phanyaem, S., Areepong, Y., Sukparungsee, S. and Mititelu, G. (2013). Explicit Formulas of Average Run Length for ARMA(1,1). International Journal of Applied Mathematics and Statistics 43(13): 392-405.

Petcharat, K., Areepong, Y. and Sukparungsee S. (2013). Exact Formulas of Average Run Length of EWMA chart for MA(q) processes. Far East Journal of Mathematic Science 78(2): 291-300.

Robert, S.W. (1959). Control chart tests based on geometric moving average. Technometrics 42(1): 239-250.

Schmid, W. and Schone, A. (1997). Some properties of the EWMA control chart in the presence of autocorrelation. Annals of Statistics 25(3): 1277-1283.

Superville, C. R., and Adams, B. M. (1994). An Evaluation of Forecast-Based Quality Control Schemes. Communications in Statistics: Simulation and Computation 23(3): 645-661.

Suriyakat, W., Areepong, Y., Sukparungsee S. and Mititelu, G. (2012). On EWMA Procedure for an AR(1) Observations with Exponential White Noise. International Journal of Pure and Applied Mathematics 77(1): 73-83.

VanBrackle, L. N. and Reynolds, M. R. Jr. (1997). EWMA and CUSUM control charts in the presence of correlation. Communications in Statistics: Simulation and Computation 26(3): 979-1008.

Vasillopoulos, A. V. and Stamboulis, A. P. (1978). Modification of Control Chart Limits in the Presence of Data Correlation. Journal of Quality Technology 10(1): 20-30.

Wetherill, G. B., and Brown, D. W. (1991). Statistical Process Control: Theory and Practice. London: Chapman and Hall. pp. 134-148.

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