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Enhanced Frameworks for Accurate ARL Estimation in Statistical Process Control Systems

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ABSTRACT

This study presents an enhanced framework for accurately estimating the Average Run Length (ARL) in Statistical Process Control (SPC) systems especially for control charts, which are essential for monitoring, maintaining, and improving process quality in industrial and economic applications. The ARL, a key performance metric in SPC, indicates the average number of samples taken before a control signal is triggered, with ARL_0 representing false alarms and ARL_1 indicating true detections. This paper reviews and compares four prominent ARL evaluation methods: Monte Carlo Simulation, Markov Chain Approach, Numerical Integral Equation, and Explicit Formulas. Each method's strengths and limitations are discussed in terms of accuracy, and flexibility. The findings highlight the need for method selection based on the complexity of monitored processes, particularly in autocorrelation and non-normality are prevalent. Furthermore, integrating these methods with AI-driven optimization techniques - such as machine learning algorithms for data analysis and adaptive control - offers promising avenues for enhancing the precision and responsiveness of process monitoring in dynamic and complex environments.

KEYWORDS: Statistical process control, Average run length, Markov chain approach, Numerical integral equation, Explicit formulas.

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In modern industrial and economic systems, process monitoring and quality control have become critical for ensuring stability, minimising costs, and maximising efficiency. The application of Statistical Quality Control (SQC) tools has expanded beyond manufacturing to areas such as healthcare, environmental science, and econometrics. Among these tools, control charts are fundamental instruments for detecting shifts in process behaviour over time.

Introduced by Shewhart (1931), the classical control chart focuses on detecting significant deviations by monitoring sample statistics against predetermined control limits. However, classical methods like the Shewhart chart often lack sensitivity to small or gradual process shifts. This limitation led to the development of more sophisticated techniques, such as the Cumulative Sum (CUSUM) introduced by Page (1954) and Exponentially Weighted Moving Average (EWMA) charts proposed by Roberts (1959), which enhance sensitivity and allow for earlier detection of shifts.

The performance of any control chart is commonly evaluated using the Average Run Length (ARL), which represents the expected number of samples taken before a signal indicates a potential process change. The ARL_0 refers to the expected run length when the process is in-control, and the ARL_1 refers to when the process is out-of-control. Optimising ARL values is crucial: a high ARL_0 minimises false alarms, while a low ARL_1 ensures prompt detection of actual process changes.

Given the importance of ARL, several analytical and numerical methods have been developed to evaluate it accurately, especially in econometric applications where data often exhibit autocorrelation, non-normality, and structural breaks.

The ARL is a probabilistic metric that quantifies the average number of observations needed before a control chart signals. Formally:

ARL_0: Expected number of samples before a false alarm when the process is in control.

ARL_1: Expected number of samples until a true signal when the process is out of control.

The choice of ARL evaluation method depends on factors such as the type of control chart, the probability distributions: discrete and continuous, and the nature of the monitored process (stationary vs. non-stationary).

Methods for Evaluating Average Run Length

1. Monte Carlo Simulation (MC)

Monte Carlo Simulation is one of the most flexible and widely used methods for ARL estimation, especially when analytical solutions are intractable.

Process: Simulate a large number of process runs (typically 10,000 or more) under both in-control and out-of-control conditions. For each simulation, count the number of observations until a control limit is breached.

Strengths: High flexibility, applicable to complex models including non-linear, non-normal, or dependent data.

Limitations: Computationally intensive; requires significant time for high-precision results.

Example: Sales et al. (2020) used the Monte Carlo method to evaluate ARL in Poisson mixed integer autoregressive processes for crime and network data. See details in the reference; Saengsura et. al. (2024), Talordphop et. al. (2025).

2. Markov Chain Approach (MCA)

The Markov Chain Approach proposed by Brook & Evans (1972) is a powerful technique that models the transition probabilities between different process states.

Table 1 Comparative Analysis of ARL Evaluation Methods

Method	Accuracy	Flexibility	Best Use Cases
MC	High (with samples)	Very High (all data types)	Complex models, unknown distributions
MCA	Medium to High	Medium (discrete states)	CUSUM/EWMA, simple discrete processes
NIE	Very High	Medium (continuous only)	Continuous distributions, time series models
Explicit	Very High	Low (specific assumptions)	Standard distributions, quick estimation

Process: Define discrete states (e.g., in-control, out-of-control), establish a transition probability matrix, and solve for steady-state probabilities.

Strengths: Efficient for processes that can be discretised into finite states; suitable for CUSUM and EWMA charts.

Limitations: May not converge for some high-dimensional or continuous-state processes; assumes Markovian properties.

Example: Lucas and Saccucci (1990) applied MCA to evaluate ARL for EWMA charts. Chananet et al. (2015) studied MCA to evaluate ARL on EWMA and CUSUM control charts based on the ZINB process.

3. Numerical Integral Equation (NIE) Method

The NIE method involves formulating ARL estimation as an integral equation, which is then solved numerically.

Process: Represent ARL as a Fredholm integral equation, discretise using numerical techniques (e.g., Simpson's Rule), and solve iteratively.

Strengths: High precision, especially for continuous control charts; applicable to time series with continuous distributions.

Limitations: Limited to continuous distributions; requires careful numerical implementation.

Example: Crowder (1987) used NIE for the EWMA chart, achieving highly accurate ARL estimates. For more information, see the following references: Bualuang & Peerajit (2023), Saesuntia et. al. (2023).

4. Explicit Formulas

Explicit formulas provide closed-form expressions for ARL based on assumptions about the data and control chart structure.

Process: Derive ARL using mathematical tools like probability theory, the central limit theorem, or Laplace transforms.

Strengths: Extremely fast computation; no need for simulation or iteration.

Limitations: Valid only for specific distributions (e.g., normal, exponential); lacks flexibility for complex or non-standard processes.

Example: Sukparungsee and Areepong (2017) derived explicit ARL formulas for EWMA charts in autoregressive processes. Refer to the following sources for more details: Phanyaem (2022), Petcharat (2022), Suriyakat & Petcharat (2022), Supharakonsakun & Areepong (2023), Areepong & Sukparungsee (2023), Areepong & Peerajit (2024), Neammai et. al. (2025).

In practice, the choice of method depends on the trade-off between accuracy, flexibility, and computational cost. Table 1 shows the comparative analysis of four methods. For example, in econometric models involving autocorrelated or heteroskedastic data, Monte Carlo or NIE methods may be preferred due to their robustness.

Evaluating Average Run Length (ARL) is central to assessing and improving the effectiveness of control charts in process monitoring. While traditional control charts suffice for simple, well-behaved data, modern applications - especially in econometrics - require more sophisticated evaluation techniques.

Monte Carlo simulations give unparalleled flexibility, although at the time consuming, whilst Markov Chain and NIE approaches deliver analytical rigour under specific constraints. Explicit formulas are essential for obtaining rapid and precise results under conventional assumptions.

As data complexity grows, integrating these methods with AI-driven optimisation and real-time analytics could further enhance their applicability in dynamic systems.

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