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Wave solutions to the Landau-Ginzburg-Higgs equation and the modified KdV-Zakharov-Kuznetsov equation by the Riccati-Bernoulli sub-ODE method

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ABSTRACT

In this paper, exact traveling wave solutions of the (1+1)-dimensional Landau-Ginzburg-Higgs equation and the (3+1)-dimensional modified KdV-Zakharov-Kuznetsov equation, which are the partial differential equations for ion wave equations, are extracted using the Riccati-Bernoulli sub-ODE method. The obtained solutions are shown by hyperbolic and trigonometric functions, which can be transformed into kink waves and periodic waves in their physical nature.

KEYWORDS: Riccati-Bernoulli sub-ODE method, partial differential equations, (1+1)-dimensional Landau-Ginzburg-Higgs, (3+1)-dimensional modified KdV-Zakharov-Kuznetsov equation.

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1. INTRODUCTION

Partial differential equations are highly useful equations used in real-world situations in applied and engineering sciences. The most widely used scenarios are plasma physics, atmospheric pollutant dispersion, solid state physics, plasma waves, fluid mechanics, chemical kinematics, chemical physics, etc. Many researchers have employed a variety of methods to get exact solutions of partial differential equations, namely the simple equation method (Sanjun & Chankaew, 2022), the Runge-Kutta method (Hosseini et al., 2023), the modified simple equation method (Akter & Akbar, 2015), the tanh-coth method (Kumar & Pankaj, 2015), the tanh method (Babi & Mohyud-Din, 2014), the sin-cosine method (Raslan et al., 2017), the sine-Gordon expansion method (Iatkliang et al., 2023), the unified method (Abdel-Gawad et al., 2022), the ansatz method (Hosseini et al., 2023), the new generalized (G'/G)-expansion method (Akbar et al., 2018), the Exp-expansion method (He & Wu, 2006), the homotopy perturbation method (Roozi et al., 2011), the Jacobi elliptic function method (Ali, 2011 and Hosseini et al., 2023), the generalized method (Hosseini et al., 2023), etc.

The Landau-Ginzburg-Higgs equation (LGHE) is a typical nonlinear wave equation that was used to explain the drift cyclotron waves for a coherent ion-cyclotron wave in a radially inhomogeneous plasma and has the following form:

$$\frac{\partial^2 q}{\partial t^2}-\frac{\partial^2 q}{\partial x^2}-g^2 q+h^2 q^3=0,$$

where x and t are normalized space and time coordinates, respectively, and $q(x,t)$ is the ion-cyclotron wave is electrostatic potential (Barman et al., 2021).

The Zakharov-Kuznetsov (ZK) equation governs the behavior of weakly nonlinear ion-acoustic waves in a plasma comprising cold ions and hot isothermal electrons in the presence of a uniform magnetic field (Munro & Parkes, 1999). The modified (3+1)-dimensional KdV-ZK equation is shown as:

$$u_t + au^2 u_x + (u_{xx} + u_{yy} + u_{zz})_x = 0,$$

where $u = u(x, y, z, t)$ and a is a nonzero constant (Alam et al., 2015).

The Riccati-Bernoulli sub-ODE method was first proposed to construct exact traveling wave solutions, solitary wave solutions, and peaked wave solutions for nonlinear partial differential equations (Yang, 2015). The Riccati-Bernoulli sub-ODE method was used to investigate exact solutions to various equations, such as the perturbed nonlinear Schrodinger equation in 2016 (Shehata, 2016). The thin film equation, the dispersive long wave equation, the modified KdV-KP equation and the nonlinear ZK-MEW equation in 2020 (Alharbi & Almatrafi, 2020). Hassan & Abdelrahman, 2019) were solved by the Riccati-Bernoulli sub-ODE method to investigate the exact solutions of the Schrödinger equation and the 2D Ginzburg-Landau equation.

In this work, we use the traveling wave to transform the (1+1)-dimensional LGH equation and the (3+1)-dimension modified KdV-ZK equation into nonlinear ordinary differential equations. Then, using the Riccati-Bernoulli sub-ODE method, we have obtained the wave solutions in 3D graphs.

2. Algorithm of the Riccati-bernoulli sub-ODE method

In this section, we provide a straightforward approach for finding traveling wave solutions to nonlinear equations, namely the Riccati-Bernoulli sub-ODE method. Assume that the nonlinear partial differential equation with two independent variables x and t is represented by:

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt} \ldots) = 0, \quad (1)$$

where P is a polynomial in $u(x,t)$ and its partial derivatives in which the highest-order derivatives and nonlinear terms are involved. The following steps are the main procedure of the Riccati-Bernoulli sub-ODE method (Abdelrahman et al., 2019):

Step 1: Denoting the traveling wave solution of PDE (1) as:

$$u(x,t) = u(\xi), \ \xi = x - \lambda t, \quad (2)$$

where λ is speed of traveling wave, then Equation (1) reduces to a nonlinear ordinary differential equation ODE:

$$Q(u, u', u'', \ldots) = 0, \quad (3)$$

where Q is a polynomial of $u(\xi)$ and its derivatives, where the prime represents the derivative with respect to ξ.

Step 2: Assume the solution (2) of Equation (3) satisfies

$$u'(\xi) = au^{2-n} + bu + cu^n, \quad (4)$$

where a, b, c and n are constants to be determined. From Equation (4), we get

$$u''(\xi) = ab(3-n)u^{2-n} + a^2(2-n)u^{3-2n} + nc^2u^{2n-1} + bc(n+1)u^n + (2ac + b^2)u, \quad (5)$$

$$u'''(\xi) = \Big(ab(3-n)(2-n)u^{1-n} + a^2(2-n)\times(3-2n)u^{2-2n} + n(2n-1)c^2u^{2n-2} + bcn(n+1)u^{n-1} + (2ac + b^2)\Big)u'. \quad (6)$$

2.1 Classification of the solution

The solution of Equation (4) can be classified into the following cases.

Case 1: When $n = 1$, then

$$u(\xi) = \mu e^{(a+b+c)\xi}. \quad (7)$$

Case 2: When $n \neq 1, b = 0$ and $c = 0$ then

$$u(\xi) = \big(a(n-1)(\xi + \mu)\big)^{\frac{1}{n-1}}. \quad (8)$$

Case 3: When $n \neq 1, b \neq 0$ and $c = 0$ then

$$u(\xi) = \left(-\frac{a}{b} + \mu e^{b(n-1)\xi}\right)^{\frac{1}{n-1}}. \quad (9)$$

Case 4: When $n \neq 1, a \neq 0$ and $b^2 - 4ac < 0$, then

$$u(\xi) = \left(\frac{-b}{2a} + \frac{\sqrt{4ac-b^2}}{2a}\tan\left(\frac{(1-n)\sqrt{4ac-b^2}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}}, \quad (10)$$

and

$$u(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{4ac-b^2}}{2a}\cot\left(\frac{(1-n)\sqrt{4ac-b^2}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}}. \quad (11)$$

Case 5: When $n \neq 1, a \neq 0$ and $b^2 - 4ac > 0$, then

$$u(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}\coth\left(\frac{(1-n)\sqrt{b^2-4ac}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}}, \quad (12)$$

and

$$u(\xi) = \left(\frac{-b}{2a} - \frac{\sqrt{b^2-4ac}}{2a}\tanh\left(\frac{(1-n)\sqrt{b^2-4ac}}{2}(\xi+\mu)\right)\right)^{\frac{1}{1-n}}. \quad (13)$$

Case 6: When $n \neq 1, a \neq 0$ and $b^2 - 4ac = 0$, then

$$u(\xi) = \left(\frac{1}{a(n-1)(\xi+\mu)} - \frac{b}{2a}\right)^{\frac{1}{(1-n)}}. \quad (14)$$

For Equations (7) - (14), the parameter μ is an arbitrary constant.

Step 3: Substituting the derivatives of u into Equation (3) gives an algebraic equation for u. Then setting the coefficients of $u^i, (i = 0,1,2,\ldots)$, to be zero yields a set of algebraic equations for a, b, c, and λ. Solving these algebraic equations and substituting a, b, c, n, λ, and $\xi = x - \lambda t$ into Equations (7)-(14), we have traveling wave solutions of Equation (1).

3. RESULTS

Next, using the Riccati-Bernoulli sub-ode method explained above, we want to solve the Landau-Ginzburg-Higgs equation and the (3+1)-dimensional modified KdV-Zakharov equation as follows.

3.1 Results of the (1+1)-dimensional LGH equation

The (1+1)-dimensional Landau-Ginzburg-Higgs equation is

$$u_{tt} - u_{xx} - p^2 u + q^2 u^3 = 0, \quad (15)$$

where p and q are parameters. We will reduce it to an ODE using $u(\xi) = u(x,t)$ and the traveling wave variable $\xi = x - \lambda t$. The substitution of the transformation into equation (15) yields

$$(\lambda^2 - 1)u'' - p^2 u + q^2 u^3 = 0. \quad (16)$$

Substituting Equation (5) into Equation (16), the outcome is

$$(\lambda^2-1)\left(ab(3-n)u^{2-n} + a^2(2-n)u^{3-2n} + nc^2u^{2n-1} + bc(n+1)u^n + (2ac+b^2)u\right) - p^2u + q^2u^3 = 0. \quad (17)$$

Setting $n = 0$, Equation (17) is rewritten as

$$(\lambda^2-1)bc + \left((\lambda^2-1)(2ac+b^2) - p^2\right)u + (\lambda^2-1)3abu^2 + q^2u^3 = 0, \quad (18)$$

then, equating the coefficient of u^i to zero, where u^0, u^1, u^2 and u^3 yields:

$$(\lambda^2-1)bc = 0, \quad (19)$$

$$(\lambda^2-1)(2ac+b^2) - p^2 = 0, \quad (20)$$

$$(\lambda^2-1)3ab = 0, \quad (21)$$

$$(\lambda^2-1)(2a^2) + q^2 = 0. \quad (22)$$

Solving Eq. (19)-(22), we obtain

$$b = 0, \quad (23)$$

$$a = \pm\frac{q}{\sqrt{2(1-\lambda^2)}}, \quad (24)$$

$$c = \mp \frac{p^2}{q\sqrt{2\left(1-\lambda^2\right)}}, \tag{25}$$

$$ac = \frac{p^2}{2\left(\lambda^2 - 1\right)}. \tag{26}$$

The solution of Equation (15) can be classified into the following cases according to Section 2.1:

Case I: When $n=0$, $a \neq 0$ and $b^2 - 4ac > 0$, we get

$$u_{1,2}(x,t) = \pm \frac{p}{q} \coth\left(\frac{p}{\sqrt{2\left(1-\lambda^2\right)}} \left(x - \lambda t + \mu\right) \right), \tag{27}$$

and

$$u_{3,4}(x,t) = \pm \frac{p}{q} \tanh\left(\frac{p}{\sqrt{2\left(1-\lambda^2\right)}} \left(x - \lambda t + \mu\right) \right). \tag{28}$$

When λ and μ are arbitrary constants.

If equation (17) is set to $n=1$, we will get $q=0$, causing the equation of the (1+1)-dimensional Landau-Ginzburg-Higgs to have some terms missing, and therefore incomplete.

3.2 Results of the (3+1)-dimensional modified KdV-ZK equation

The (3+1)-dimensional modified KdV-ZK equation:

$$v_t + \beta v^2 v_x + \left(v_{xx} + v_{yy} + v_{zz}\right)_x = 0, \tag{29}$$

where β is a nonzero constant. Using

$v(\xi) = v(x + y + z - \lambda t)$ and the traveling wave variable $\xi = x + y + z - \lambda t$, (31) is transformed into the following ODE:

$$-\lambda v' + \beta v^2 v' + \left(3v''\right)' = 0. \tag{30}$$

Integrating Eq. (32) with zero constant, we get:

$$-\lambda v + \frac{1}{3}\beta v^3 + 3v'' = 0. \tag{31}$$

Substituting $v''(\xi) = ab(3-n)v^{2-n} + a^2(2-n)v^{3-2n} + nc^2 v^{2n-1} + bc(n+1)v^n + \left(2ac + b^2\right)v$, into Equation (31), and also the outcome is:

$$-\lambda v + \frac{1}{3}\beta v^3 + 3\Big(ab(3-n)v^{2-n} + a^2(2-n)v^{3-2n} + nc^2 v^{2n-1} + bc(n+1)v^n + \left(2ac + b^2\right)v\Big) = 0. \tag{32}$$

Setting $n=0$, Equation (32) is rewritten to:

$$3bc + \left(6ac + 3b^2 - \lambda\right)v + 9abv^2 + \left(\frac{1}{3}\beta + 6a^2\right)v^3 = 0, \tag{33}$$

then, equating the coefficient of v^i to zero, where v^0, v^1, v^2 and v^3 yields:

$$3bc = 0, \tag{34}$$

$$6ac - 3b^2 - \lambda = 0, \tag{35}$$

$$9ab = 0, \tag{36}$$

$$\frac{1}{3}\beta + 6a^2 = 0. \tag{37}$$

Solving Eq. (34)-(37), we obtain

$$b = 0, \tag{38}$$

$$a = \pm \frac{1}{3}\sqrt{-\frac{\beta}{2}}, \tag{39}$$

$$c = \pm \frac{\lambda}{\sqrt{-2\beta}}. \tag{40}$$

$$ac = \frac{\lambda}{6}. \tag{41}$$

The solution Equation (29) can be classified into the following cases according to Section 2.1:

Case I: When $n=0$, $a \neq 0$ and $b^2 - 4ac < 0$, we get

$$v_{1,2}(x,y,z,t) = \pm \sqrt{-\frac{3\lambda}{\beta}} \tan\left(\sqrt{\frac{\lambda}{6}} \left(x + y + z - \lambda t + \mu\right) \right), \tag{42}$$

and

$$v_{3,4}(x,y,z,t) = \pm\sqrt{-\frac{3\lambda}{\beta}}\cot\left(\sqrt{\frac{\lambda}{6}}(x+y+z-\lambda t+\mu)\right). \tag{43}$$

When λ and μ are arbitrary constants.

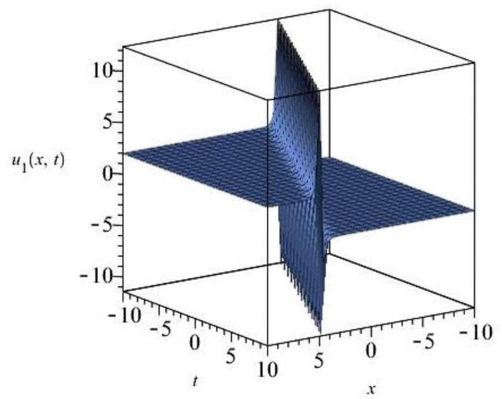
Similarly, if equation (32) is set to $n=1$, we will get $\beta = 0$, causing the equation of the (3+1)-dimensional modified KdV-ZK to have some terms missing, therefore incomplete.

4. Graphical representation of some obtained solution

In this section, we will discuss the physical interpretations from the graphical representations of the solution of the (1+1)-dimensional LGH equation and the (3+1)-dimensional modified KdV-ZK equation.

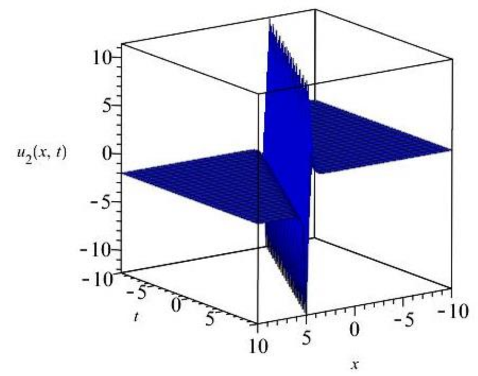
4.1 Graphical representation of the (1+1)-dimensional LGH equation

We present the three-dimensional plots of the exact solutions $u_{1,2}(x,t)$ in Case I shown in Figure 1 and Figure 2 which both plots represent the shape of a kink wave solution.



$$u_1(x,t) = \frac{p}{q}\coth\left(\frac{p}{\sqrt{2(1-\lambda^2)}}(x-\lambda t+\mu)\right).$$

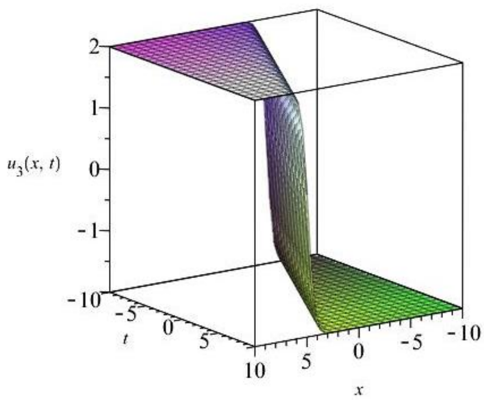
Figure 1 The solution $u_1(x,t)$ with $p=2, q=1$, $\lambda=0.5, \mu=0.1$ and $-10 \le x,t \le 10$.



$$u_2(x,t) = -\frac{p}{q}\coth\left(\frac{p}{\sqrt{2(1-\lambda^2)}}(x-\lambda t+\mu)\right).$$

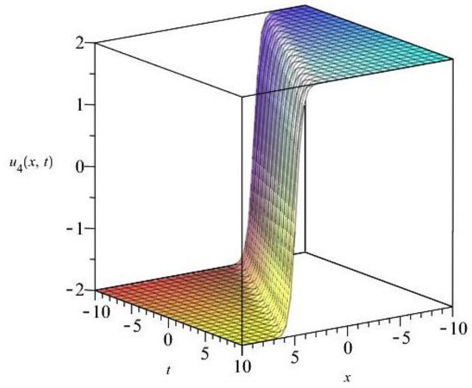
Figure 2 The solution $u_2(x,t)$ with $p=2, q=1$, $\lambda=0.5, \mu=0.1$ and $-10 \le x,t \le 10$.

The graphs of $u_{3,4}(x,t)$ in Case I, as shown in Figure 3 and Figure 4, are the shapes of kink waves that rise or descend from one asymptotic state to another.



$$u_3(x,t) = \frac{p}{q}\tanh\left(\frac{p}{\sqrt{2(1-\lambda^2)}}(x-\lambda t+\mu)\right).$$

Figure 3 The solution $u_3(x,t)$ with $p=2, q=1$, $\lambda=0.5, \mu=0.1$ and $-10 \le x,t \le 10$.

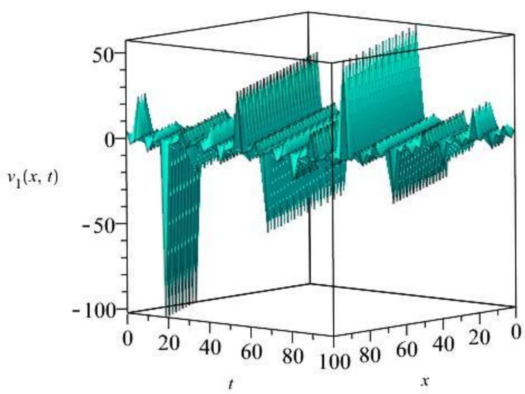


$$u_4(x,t) = -\frac{p}{q}\tanh\left(\frac{p}{\sqrt{2\left(1-\lambda^2\right)}}\left(x-\lambda t+\mu\right)\right),$$

Figure 4 The solution $u_4(x,t)$ with $p=2, q=1$, $\lambda=0.5, \mu=0$ and $-10 \le x,t \le 10$.

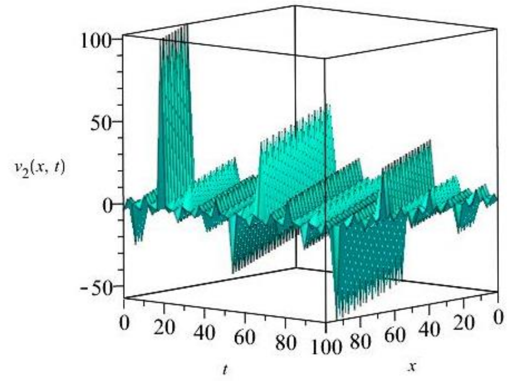
4.2 Graphical representation of the (3+1)-dimensional modified KdV-ZK equation

Next, we present the shape of solutions to the (3+1)-dimensional modified KdV-ZK equation. Solutions $v_{1,2}(x,y,z,t)$ correspond to Figures 5 and 6, and solutions $v_{3,4}(x,y,z,t)$ correspond to Figures 7 and 8. All of them produce a periodic traveling wave solution.



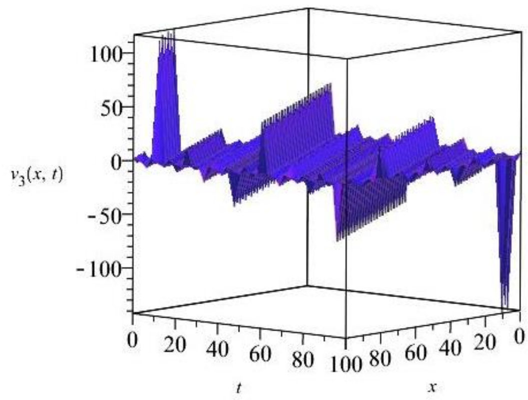
$$v_1(x,y,z,t) = \sqrt{-\frac{3\lambda}{\beta}}\tan\left(\sqrt{\frac{\lambda}{6}}\left(x+y+z-\lambda t+\mu\right)\right),$$

Figure 5 The solution $v_1(x,y,z,t)$ with $\beta=-3$ $\lambda=2,\ \mu=0.1$ and $0 \le x,t \le 100$ for $y=z=0$.



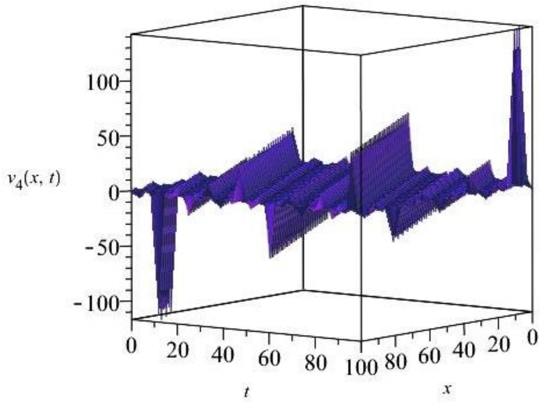
$$v_2(x,y,z,t) = -\sqrt{-\frac{3\lambda}{\beta}}\tan\left(\sqrt{\frac{\lambda}{6}}\left(x+y+z-\lambda t+\mu\right)\right),$$

Figure 6 The solution $v_2(x,y,z,t)$ with $\beta=-3$ $\lambda=2,\ \mu=0.1$ and $0 \le x,t \le 100$ for $y=z=0$.



$$v_3(x,y,z,t) = \sqrt{-\frac{3\lambda}{\beta}}\cot\left(\sqrt{\frac{\lambda}{6}}\left(x+y+z-\lambda t+\mu\right)\right).$$

Figure 7 The solution $v_3(x,y,z,t)$ with $\beta=-3$ $\lambda=2,\ \mu=10$ and $0 \le x,t \le 100$ for $y=z=0$.



$$v_4(x,y,z,t) = -\sqrt{-\frac{3\lambda}{\beta}}\cot\left(\sqrt{\frac{\lambda}{6}}\left(x+y+z-\lambda t+\mu\right)\right).$$

Figure 8 The solution $v_4(x,y,z,t)$ with $\beta=-3$ $\lambda=2,\ \mu=10$ and $0 \le x,t \le 100$ fo $y=z=0$.

5. Conclusion

In this work, using the Riccati-Bernoulli sub-ODE method, we investigated exact traveling wave solutions for the (1+1)-dimensional Landau-Ginzburg-Higgs equation and the (3+1)-dimensional modified KdV-Zakharov-Kuznetsov equation. The solutions are found in trigonometric and hyperbolic forms. In addition, we can solve nonlinear evolution equations in mathematical physics using this method.

The Riccati-Bernoulli sub-ODE method is easy to understand. This research also demonstrates that the proposed method is quite practical and appropriate for finding exact solutions to the (1+1)-dimensional Landau-Ginzburg-Higgs equation and the (3+1)-dimensional modified KdV-Zakharov-Kuznetsov equation. The performance of this method is reliable and effective, and it gives exact solitary wave solutions.

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