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Exact solutions of the fractional Landau-Ginzburg-Higgs equation and the (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation using the simple equation method

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ABSTRACT

In this article, we analyze the fractional Landau-Ginzburg-Higgs equation and the (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation, two ion wave equations, using the simple equation approach and the Bernoulli equation. The traveling wave solutions are demonstrated by the exponential and hyperbolic functions, which can be interpreted as kink waves. Their graphical representations are three-dimensional graphs, and contour graphs are shown using appropriate parameter values. Additionally, the results demonstrated that the technique employed in this study is a powerful analytical tool for obtaining exact traveling wave solutions to nonlinear models encountered in a variety of scientific and engineering fields.

KEYWORDS: simple equation method, fractional partial differential equations, (3+1)-dimensional space-time fractional modified KdV-Zakharov Kuznetsov equation, fractional Landau-Ginzburg-Higgs equation

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1. INTRODUCTION

Many phenomena of nature are associated with the ideas of physical science: shallow water waves (Islam et al., 2023), nonlinear optics (Jhangeer et al., 2021), plasma physics (Salas et al., 2022), fluid dynamics (Shah et al., 2022), quantum mechanics (Alabedalhadi, 2022), biology (Simpson, 1963), hydrodynamics (Zeng & Liu, 2022), and acoustics (Li et al., 2022). The present work concerns the representation of these phenomena by means of fractional differential equations (FDEs), which are used to simulate various physical phenomena and processes for solving linear and nonlinear problems (Çenesiz et al., 2017).

Many researchers have used diverse methods to get exact solutions of fractional differential equations, such as the Kudryashov method (Thadee et al., 2022), the modified Kudryshov method (Kumar et al., 2018), the riccati-sub equation method (Khodadad et al., 2017), the exp-function method (Islam et al., 2019), the first integral method (Eslami et al.,2014), the (G'/G) - expansion method (Baishya & Rangarajan, 2018), the new extended direct algebraic method (Rezazadeh et al., 2020), the extended auxiliary equation method (Sabi'u et al., 2023), the auxiliary equation method (Rezazadeh et al., 2020), and the Sine-Cosine method (Sabi'u et al., 2019), etc.

In the present work, we take into consideration the fractional Landau-Ginzburg-Higgs equation (Guner et al., 2017),

$$D_t^{2\alpha} u - D_x^{2\alpha} u - m^2 u + n^2 u^3 = 0, \quad 0 < \alpha \le 1, \tag{1.1}$$

where $u = u(x,t)$, m and n are parameters. Furthermore, we analyze the (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation (Sahoo & Ray, 2015),

$$D_t^{\alpha}u + \delta u^2 D_x^{\alpha}u + D_x^{3\alpha}u + D_x^{\alpha}D_y^{2\alpha}u + D_x^{\alpha}D_z^{2\alpha}u = 0,$$
$$0 < \alpha \leq 1, \quad (1.2)$$

where δ is an arbitrary constant and $D_t^{\alpha}u$ denotes Jumarie's modified Riemann-Liouville derivatives of u, where $u = u(x,y,z,t)$.

The fractional variational iteration transform method was used to analyze the analytical solution of the fractional Landau-Ginzburg-Higgs equation (Guner et al., 2017). The exact solution of the (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation was established by the improved generalized tanh-coth method in 2019 (Torvattanabun et al., 2019), the improved fractional sub-equation method in 2015 (Sahoo & Ray, 2015), the fractional variable method in 2017 (Çenesiz et al., 2017), and the unified method in 2019 (Osman al., 2019), for which the fractional Landau-Ginzburg-Higgs equation and the (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation have not yet been analyzed using the simple equation method.

In this article, we have used Jumarie's modified Riemann-Liouville derivative and the simple equation method with the Bernoulli equation to solve the fractional Landau-Ginzburg-Higgs equation and the (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation. We have displayed the analytical solutions and the wave effects in a three-dimensional graph.

2. DESCRIPTIONS OF MODIFIED RIEMANN-LIOUVILLE DERIVATIVE AND THE PROPOSED METHOD

Jumarie's modified Riemann-Liouville derivative and the properties of the modified Riemann-Liouville derivative (Sahoo & Ray, 2015) of order α are defined by the expression

$$f^{(\alpha)}(x) = \lim_{h\to 0}\left(\frac{\sum_{k=0}^{\infty}(-1)^k \binom{\alpha}{k} f\left(x+(\alpha-k)h\right)}{h^{\alpha}}\right),$$
$$0 < \alpha \leq 1, \quad (2.1)$$

which can be written as

$$D_x^{\alpha}f(x) = \begin{cases} \dfrac{1}{\Gamma(-\alpha)}\displaystyle\int_0^x (x-\xi)^{(-\alpha-1)}[f(\xi)-f(0)]d\xi \\ \qquad \text{if } \alpha < 0, \\ \dfrac{1}{\Gamma(1-\alpha)}\dfrac{d}{dx}\displaystyle\int_0^x (x-\xi)^{(-\alpha)}[f(\xi)-f(0)]d\xi \\ \qquad \text{if } 0 < \alpha < 1, \\ [f^{(\alpha-n)}(x)]^{(n)} \quad \text{if } n \leq \alpha < n+1,\ n \geq 1, \end{cases}$$
$$(2.2)$$

some properties of the modified Riemann-Liouville derivative as follows:

$$D_x^{\alpha}x^{\gamma} = \frac{\Gamma(\gamma+1)x^{\gamma-\alpha}}{\Gamma(\gamma+1-\alpha)}, \quad \gamma > 0, \quad x > 0,$$
$$D_x^{\alpha}(f(x)g(x)) = g(x)D_x^{\alpha}f(x) + f(x)D_x^{\alpha}g(x), \quad (2.3)$$
$$D_x^{\alpha}f(g(x)) = f_g'[g(x)]D_x^{\alpha}g(x) = D_g^{\alpha}f[g(x)](g'(x))^{\alpha}.$$

The aforementioned characteristics are crucial to the simple equation method (Phoosree & Thadee, 2022).

3. DESCRIPTIONS OF SIMPLE EQUATION METHOD

In this part, we will outline a straightforward technique for getting the traveling wave solution to fractional PDEs, known as the simple equation method. Assume that x, y and t are independent variables in a nonlinear partial differential equation that is given by

$$G\left(u, D_x^{\alpha}u, D_y^{\alpha}u, D_t^{\alpha}u, D_x^{2\alpha}u, D_y^{\alpha}D_x^{\alpha}u, D_t^{\alpha}D_x^{\alpha}u, \ldots\right) = 0,$$
$$0 < \alpha \leq 1, \quad (3.1)$$

where $u(x,y,t)$ is an unknown function and G is a polynomial of $u(x,y,t)$ and its derivatives. The main steps of the SE method (Sanjun & Chankaew, 2022) are as follows:

Step 1. Wave transformation

We suppose that $u(x, y, t) = u(\xi)$,

$$\xi = \frac{kx^{\alpha}}{\Gamma(\alpha+1)} + \frac{ly^{\alpha}}{\Gamma(\alpha+1)} - \frac{\omega t^{\alpha}}{\Gamma(\alpha+1)}, \tag{3.2}$$

where constants k and l are non-zero constants, and ω is the speed of the traveling wave. We are capable of transforming Eq. (3.1) into an ordinary differential equation (ODE) for $u = u(\xi)$ using the traveling wave transformation Eq. (3.2) as follows:

$$Q(u, \frac{du}{d\xi}, \frac{d^2u}{d\xi^2}, \frac{d^3u}{d\xi^3}, \ldots) = 0, \tag{3.3}$$

where Q is a polynomial of $u(\xi)$ and its derivatives, in which the prime indicates the derivative with respect to ξ.

Step 2. Solution Assumption

Assume that Eq. (3.3) has the following formal solution:

$$u(\xi) = \sum_{i=0}^{N} a_i F^i(\xi), \tag{3.4}$$

where $a_i (i = 0, 1, 2, \ldots, N)$ is a constant that will be determined later. The function $F(\xi)$ satisfies the simple equations (ordinary differential equations). The Bernoulli equation, a well-known nonlinear ordinary differential equation, will be used in this study. Their solutions can be expressed as simple functions. The Bernoulli equation,

$$F'(\xi) = cF(\xi) + dF^2(\xi). \tag{3.5}$$

Where c and d are non-zero constants.

Step 3. balancing the integer N

The balance number N can be determined by balancing the highest-order derivative and nonlinear terms in Eq (3.3).

Step 4. Solution attainment

We get the general solutions of Eq. (3.5) as follows:

Case I: if $c > 0$ and $d < 0$, we get

$$F(\xi) = \frac{ce^{c(\xi+\xi_0)}}{1 - de^{c(\xi+\xi_0)}}, \tag{3.6}$$

where ξ_0 is the constant of the integration.

Case II: if $c < 0$ and $d > 0$, we get

$$F(\xi) = -\frac{ce^{c(\xi+\xi_0)}}{1 + de^{c(\xi+\xi_0)}}, \tag{3.7}$$

where ξ_0 is the constant of the integration.

4. APPLICATIONS

Now, we aim to solve the fractional Landau-Ginzburg-Higgs equation (1.1) and the (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation (1.2) by applying the simple equation method described above.

4.1 Solutions of the fractional Landau-Ginzburg-Higgs equation

The fractional Landau-Ginzburg-Higgs equation is

$$D_t^{\alpha}\left(D_t^{\alpha}u\right) - D_x^{\alpha}\left(D_x^{\alpha}u\right) - m^2u + n^2u^3 = 0, \quad 0 < \alpha \le 1, \tag{4.1.1}$$

where m and n are parameters. We will reduce it to an ODE using the traveling wave variable $\xi = \frac{kx^{\alpha}}{\Gamma(\alpha+1)} - \frac{\omega t^{\alpha}}{\Gamma(\alpha+1)}$. The substitution of the transformation into equation (4.1.1) leads to:

$$(\omega^2 - k^2)u'' - m^2u + n^2u^3 = 0. \tag{4.1.2}$$

Next, we balanced the highest-order derivative terms and the highest nonlinear terms of Eq.

(4.1.2). Then $N=1$. We have the solution to Eq. (4.1.2) as follows:

$$u(\xi)=\sum_{i=0}^{1} a_i F^i(\xi)=a_0+a_1 F, \qquad (4.1.3)$$

where F satisfies Eq. (3.5). Therefore, the expressions for u'' and u^3 are expressed as:

$$\begin{aligned} u'' &= a_1 c^2 F + 3a_1 cdF^2 + 2a_1 d^2 F^3, \\ u^3 &= a_0^3 + 3a_0^2 a_1 F + 3a_0 a_1^2 F^2 + a_1^3 F^3. \end{aligned} \qquad (4.1.4)$$

Substituting Eqs. (4.1.3) and (4.1.4) into Eq. (4.1.2), then equating the coefficient of F^i to zero, where $i=0,1,2,3,$ yields

$$\begin{aligned} &F^0(\xi):\ n^2 a_0^3 - m^2 a_0 = 0, \\ &F^1(\xi):\ (\omega^2-k^2)a_1 c^2 - m^2 a_1 + 3n^2 a_0^2 a_1 = 0, \\ &F^2(\xi):\ 3(\omega^2-k^2)a_1 cd + 3n^2 a_0 a_1^2 = 0, \\ &F^3(\xi):\ 2(\omega^2-k^2)a_1 d^2 + n^2 a_1^3 = 0. \end{aligned} \qquad (4.1.5)$$

Solving this system of algebraic equations, we obtain

$$a_0=\pm\frac{m}{n},\ a_1=\pm\frac{d}{n}\sqrt{-2(\omega^2-k^2)}$$

$$\text{and } \omega=\pm\sqrt{\frac{k^2c^2-2m^2}{c^2}}, \qquad (4.1.6)$$

where $-2(\omega^2-k^2)\geq 0$ and $\frac{k^2c^2-2m^2}{c^2}>0.$

The following two exact solutions are obtained by substituting Eq. (4.1.6) into Eq. (4.1.3). Then, we utilize the general solutions to the Bernoulli equations (3.6) and (3.7). We get four exact solutions of (4.1.1) written in terms to the exponential function.

Case I: if $c>0$ and $d<0$, we get

$$u_{1,2}(x,t)=\pm\frac{m}{n}\left(1+2d\left(\frac{e^{c(\xi+\xi_0)}}{1-de^{c(\xi+\xi_0)}}\right)\right), \qquad (4.1.7)$$

where $\xi=\frac{kx^\alpha}{\Gamma(\alpha+1)}-\frac{\omega t^\alpha}{\Gamma(\alpha+1)}$ and ξ_0 is a constant of the integration.

Case II: if $c<0$ and $d>0$, we get

$$u_{3,4}(x,t)=\pm\frac{m}{n}\left(1-2d\left(\frac{e^{c(\xi+\xi_0)}}{1+de^{c(\xi+\xi_0)}}\right)\right), \qquad (4.1.8)$$

where $\xi=\frac{kx^\alpha}{\Gamma(\alpha+1)}-\frac{\omega t^\alpha}{\Gamma(\alpha+1)}$ and ξ_0 is a constant of the integration.

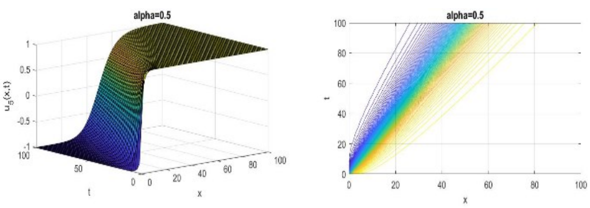
After that, we substitute $c=\pm 2$ and $d=\mp 1$ to get the following precise solutions in hyperbolic form:

$$u_{5,6}(x,t)=\pm\frac{m}{n}\tanh(\xi+\xi_0), \qquad (4.1.9)$$

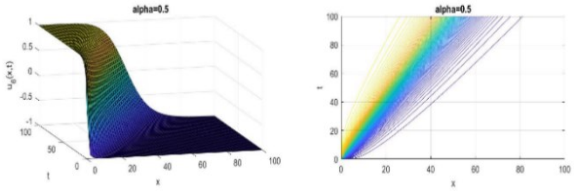
where $\xi=\frac{kx^\alpha}{\Gamma(\alpha+1)}-\frac{\omega t^\alpha}{\Gamma(\alpha+1)}$ and ξ_0 is a constant of the integration.

By using the simple equation method, we get the wave solution, which is expressed in terms of fractions of exponential functions, and then the wave solution is expressed in terms of hyperbolic functions when we set constants c and d.

In Figures 1 and 2, we present the three-dimensional and contour plots of some exact solutions, which are $u_5(x,t)$ and $u_6(x,t)$, expressed in Eq. (4.1.9). Employing $m=1, n=1, \alpha=0.5$ and $0\leq x,t\leq 100$, we obtain the plots of the selected exact traveling wave solution, which represents the kink wave solutions.



Figures 1: kink wave solution of $u_5(x,t)$ in 3D and contour



Figures 2. kink wave solution of $u_6(x,t)$ in 3D and contour

4.2 Solutions of the (3+1) - dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation

The (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation is

$$D_t^{\alpha} v + \delta v^2 D_x^{\alpha} v + D_x^{\alpha}\left(D_x^{\alpha}\left(D_x^{\alpha} v\right)\right) + D_x^{\alpha}\left(D_y^{\alpha}\left(D_y^{\alpha} v\right)\right) + D_x^{\alpha}\left(D_z^{\alpha}\left(D_z^{\alpha} v\right)\right) = 0, \qquad 0 < \alpha \leq 1, \tag{4.2.1}$$

where δ is a parameter. We will reduce it to an ODE using the traveling wave variable

$$\xi = \frac{kx^{\alpha}}{\Gamma(\alpha+1)} + \frac{ly^{\alpha}}{\Gamma(\alpha+1)} + \frac{mz^{\alpha}}{\Gamma(\alpha+1)} - \frac{\omega t^{\alpha}}{\Gamma(\alpha+1)}.$$

The substitution of the transformation into Eq. (4.2.1) leads to:

$$(k^3 + kl^2 + km^2)v''' - \omega v' + \delta k v^2 v' = 0. \tag{4.2.2}$$

Integrating Eq. (4.2.2) with respect to ξ taking the constant of integration to be zero, yields

$$k(k^2 + l^2 + m^2)v'' - \omega v + \delta k \frac{v^3}{3} = 0, \tag{4.2.3}$$

we balanced the highest-order derivative terms and the highest nonlinear terms of Eq. (4.2.3). Then, $N = 1$. We have the solution form of Eq. (4.2.3) as follows:

$$v(\xi) = \sum_{i=0}^{1} a_i F^i(\xi) = a_0 + a_1 F, \tag{4.2.4}$$

where F satisfies Eq. (3.5). Therefore, the following expressions for v'' and v^3 are:

$$v'' = a_1 c^2 F + 3a_1 cdF^2 + 2a_1 d^2 F^3,$$
$$v^3 = a_0^3 + 3a_0^2 a_1 F + 3a_0 a_1^2 F^2 + a_1^3 F^3. \tag{4.2.5}$$

Substituting Eqs. (4.2.4) and (4.2.5) into Eq. (4.2.3) and then equating the coefficient of F^i to zero, where $i = 0, 1, 2, 3,$ yields

$$F^0(\xi): -3\omega a_0 + \delta k a_0^3 = 0,$$
$$F^1(\xi): -\omega a_1 + \delta k a_0^2 a_1 + k(k^2 + l^2 + m^2)a_1 c^2 = 0,$$
$$F^2(\xi): \delta k a_0 a_0^2 + 3a_1 cdk(k^2 + l^2 + m^2) = 0,$$
$$F^3(\xi): \delta k a_1^3 + 6a_1 d^2 k(k^2 + l^2 + m^2) = 0. \tag{4.2.6}$$

Solving this system of algebraic equations, we obtain

$$a_0 = \pm\sqrt{\frac{3\omega}{\delta k}}, \quad a_1 = \pm\frac{2d}{c}\sqrt{\frac{3\omega}{\delta k}} \text{ and}$$
$$\omega = -\frac{k(k^2 + l^2 + m^2)c^2}{2}, \tag{4.2.7}$$

where $\frac{3\omega}{\delta k} > 0$.

The following two exact solutions are obtained by substituting Eq. (4.2.7) into Eq. (4.2.4). Then, we utilize the general solutions to the Bernoulli equations (3.6) and (3.7). We get four exact solutions of Eq. (4.2.1) from the exponential term.

Case I: if $c > 0$ and $d < 0$, we get

$$v_{1,2}(x,y,z,t) = \pm c\sqrt{\frac{-6\beta}{\delta}}\left(\frac{1}{2} + d\left(\frac{e^{c(\xi+\xi_0)}}{1 - de^{c(\xi+\xi_0)}}\right)\right), \tag{4.2.8}$$

where $\xi = \frac{kx^{\alpha}}{\Gamma(\alpha+1)} + \frac{ly^{\alpha}}{\Gamma(\alpha+1)} + \frac{mz^{\alpha}}{\Gamma(\alpha+1)} - \frac{\omega t^{\alpha}}{\Gamma(\alpha+1)},$

$\beta = k^2 + l^2 + m^2$ and ξ_0 is a constant of the integration.

Case II: if $c < 0$ and $d > 0$, we get

$$v_{3,4}(x,y,z,t) = \pm c\sqrt{\frac{-6\beta}{\delta}}\left(\frac{1}{2} - d\left(\frac{e^{c(\xi+\xi_0)}}{1+de^{c(\xi+\xi_0)}}\right)\right), \tag{4.2.9}$$

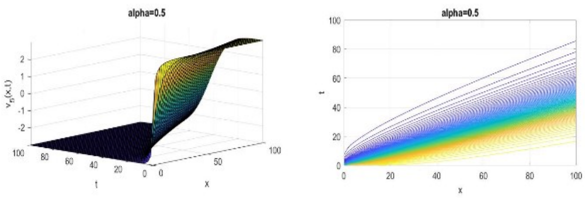
where $\xi = \frac{kx^\alpha}{\Gamma(\alpha+1)} + \frac{ly^\alpha}{\Gamma(\alpha+1)} + \frac{mz^\alpha}{\Gamma(\alpha+1)} - \frac{\omega t^\alpha}{\Gamma(\alpha+1)}$, $\beta = k^2 + l^2 + m^2$ and ξ_0 is a constant of the integration.

After that, we set $c = \pm 1$ and $d = \mp 1$ into Eqs. (4.2.8) and (4.2.9), and we get the following precise solutions in hyperbolic form:

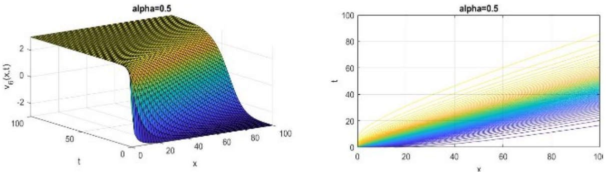
$$v_{5,6}(x,y,z,t) = \pm\frac{1}{2}\sqrt{\frac{-6\beta}{\delta}}\tanh\left(\frac{\xi+\xi_0}{2}\right), \tag{4.2.10}$$

where $\xi = \frac{kx^\alpha}{\Gamma(\alpha+1)} + \frac{ly^\alpha}{\Gamma(\alpha+1)} + \frac{mz^\alpha}{\Gamma(\alpha+1)} - \frac{\omega t^\alpha}{\Gamma(\alpha+1)}$, $\beta = k^2 + l^2 + m^2$ and ξ_0 is a constant of the integration.

Next, we represent the shape of solutions to the (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation by setting some parameters $\delta = -1$, $k = 1$, $m = 1$, $l = 1$ and $\alpha = 0.5$ in the interval $0 \le x,t \le 100$, and as displayed in Figures 3 and 4, it produces a kink wave solution.



Figures 3. kink wave solution of $v_5(x,y,z,t)$ in 3D and contour



Figures 4. kink wave solution of $v_6(x,y,z,t)$ in 3D and contour

5. CONCLUSIONS

In this work, we have investigated exact traveling wave solutions for nonlinear fractional evolution equations, namely, the fractional Landau-Ginzburg-Higgs equation and the (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation, via the simple equation method, and the solutions are found in exponential and hyperbolic forms. This method allows us to solve nonlinear fractional evolution equations in mathematical physics.

The simple equation method has many advantages: it is straight-forward and concise. Furthermore, this study shows that the proposed method is quite efficient and practically well suited for finding exact solutions to the fractional Landau-Ginzburg-Higgs equation and the (3+1)-dimensional space-time fractional modified KdV-Zakharov-Kuznetsov equation. Therefore, the simple equation method is reliable for constructing exact traveling wave solutions to nonlinear models encountered in a variety of scientific and engineering fields.

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