JASET KMUTNB Academic Article

Journal of Applied Science and Emerging Technology (JASET)
Published by Faculty of Applied Science, KMUTNB, Thailand
doi: 10.14416/JASET.KMUTNB.2024.01.004
Vol. 23, No. 1 [2024]: e252376

Global Stability and Sensitivity Analysis of a Conjunctivitis Epidemic Model in Two Populations with Patient Isolation, Treatment and Insecticide Control

Surapol Naowarat[1] and Kanyakon Onruk[1*]

[1]Department of Mathematics, Faculty of Science and Technology Suratthani Rajabhat University, Thailand

ABSTRACT

Conjunctivitis (pink eye) is a conjunctive infection. It is caused by viruses, bacteria, or allergies. In this study, we present and analyze a conjunctivitis model that includes the consequences of patient isolation, treatment control, and insecticide control. This SIR-SI (susceptible, infected, recovery - susceptible, infected) model describes disease transmission between humans and fly populations. The proposed model is examined using the standard method. The next generation matrix approach is used to calculate the basic reproductive number. The Lyapunov function is applied to determine the model's stability. The disease-free equilibrium point (DFE) is global asymptotically stable if $R_0 < 1$, whereas the endemic equilibrium point (EE) is global asymptotically stable if $R_0 > 1$. In addition, a sensitivity analysis of the model is performed to determine the significance of model parameters on disease transmission. According to DFE's sensitivity analysis, the effectiveness of patient isolation or human-fly contact prevention is the most sensitive parameter. The numerical results are used to support the theoretical findings. It has been determined that when the effectiveness of patient isolation and recovery of infected humans who visit the doctor increases, the number of infected humans decreases. Furthermore, when the effectiveness of insecticides to protect flies increases, the number of infected flies also decreases.

KEYWORDS: global stability, sensitivity analysis, conjunctivitis, patient isolation, treatment, insecticide control

*Corresponding Author: kanyakorn.onr@sru.ac.th

Received: 13/04/2023; Revised: 10/02/2024; Accepted: 14/02/2024

1. INTRODUCTION

Conjunctivitis (pinkeye) is an infection or inflammation of the conjunctiva (the thin, clear tissue that lies over the white part of the eye) and the inner side of the eyelid. It may be caused by viruses, some bacteria, and an allergic reaction like pollen, smoke, dust, or toxic substances. In this paper, we focus on the conjunctivitis caused by the virus. The symptoms of conjunctivitis are characterized by the white part of the eye becoming red, itchy eyes, blurred vision, and an increased amount of tears. It is transmitted by direct contact with infected people by touching their hands to an infected eye or by contact with infectious tears or susceptible individuals which their eyes contacted with carrier flies. The incubation period of susceptible individuals who get infected with the virus is about 1-2 days, and the period of infection is about 14 days (Alkhudhari et al., 2014). Mathematical modeling plays an important role in terms of understanding the underlying mechanisms that influence the spread of the disease and is used as a crucial instrument for effective prevention and intervention strategies against conjunctivitis. In 2014 (Unyong & Naowarat, 2014), the authors proposed an SEIR model of conjunctivitis that considers a nonlinear incidence term. The local stability of the model was carried out. It was concluded that a decrease in the number of infected humans was dependent on the increase in the number of infected individuals. In 2014 (Suksawat & Naowarat, 2014), the authors proposed and analyzed the conjunctivitis model, incorporating rainfall. It is concluded that the rainfall has an effect on the transmission of disease. In 2015 (Sangthongjeen et al., 2015), the authors proposed a modified model of conjunctivitis that incorporated the effects of educational campaigns. To obtain better results, a combination of the perturbation iteration method and

Taylor series expansion was applied. In 2019 (Viriyapong & Khedwan, 2019), the authors proposed and analyzed the extended model by adding the effect of patients' isolation and the role of recovered individuals, who are not educated enough by the doctor to protect themselves. In 2019 (Nana-Kyer et al., 2019), the authors proposed a stochastic optimal control model for AHC. The maximum principle was applied to derive the necessary conditions for the existence of optimal control. In 2019 (Uchenna et al., 2019), the authors proposed and analyzed the model of AHC both analytically and numerically. Furthermore, the modified model was extended as an optimal control problem, taking into account the effects of proper sanitation and the training of the educators. The maximum principle was applied to obtain the necessary conditions for the existence of optimal control. The results showed agreement between analytical and numerical solutions. In addition, if sanitation, which includes the serenity of the school environment, conduciveness of the classrooms, and personal hygiene, is observed in and outside the school and the education of the caregivers (teachers, menders, parents, and even pupils) is an articulated property, the number of infected students will decrease drastically over time. In 2006 (Chowell et al., 2006), the authors suggested that the patient's isolation is an intervention to prevent the transmission of this disease. From the reviewed literature, most researchers proposed conjunctivitis models that consist of only the human population. In 2017 (Thongtha et al., 2017), the authors proposed a new model that considers human-human and fly-human transmission, while human-human transmission is defined in the context of infection by hyper infection vibrio. In 2021 (Thongtha & Modnak, 2021), the authors studied the transmission of cholera models that incorporate data on public health interventions. They were interested in the spread of cholera from vibrios and fly transmission to human populations. In this study, the fly population is divided into two groups: susceptible flies and infected flies. In addition, flies are the vector of conjunctivitis. Therefore, in this study, we will include the fly population in our model to investigate conjunctivitis transmission.

The objective of this paper is to propose and analyze an SIR-SI model of conjunctivitis in two populations with the effects of patients' isolation, treatment control, and insecticide control. In this paper, a mathematical model is formulated for the dynamics model of conjunctivitis in Section 2. The model properties, both positivity and boundedness of solutions, are derived in Section 3. Model analysis, which consists of equilibrium points, basic reproductive numbers, stability analysis, and sensitivity analysis, is performed in Section 4. In Section 5, the numerical results are presented to support the theoretical results. Finally, the conclusions are summarized in Section 6.

2. MODEL FORMULATION

In this section, a human and fly (SIR-SI) model of conjunctivitis transmission is formulated by taking into account the patient's isolation, treatment control, and insecticide control strategies. The human population and the fly population are denoted by N_h and N_f, respectively. The schematic diagram of this model is shown in Figure 1. The human population is divided into three compartments, namely susceptible human (S_h), infected human (I_h), and recovered human (R_h). The fly population is also classified into two compartments, namely susceptible flies (S_f), and infected flies (I_f). For the proposed model, there are some assumptions: all newborn flies are susceptible, and they will become infected flies if they are contacted with the discharges or gums of an infected human.

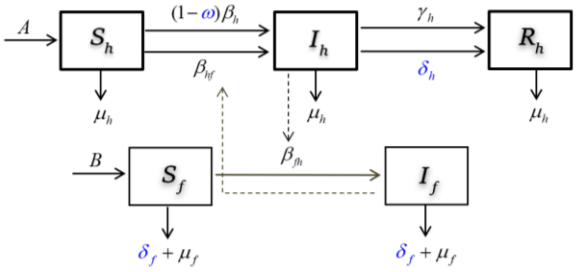


Figure 1 A schematic diagram of conjunctivitis model.

The system of ordinary differential equations is as follows:

$$\frac{dS_h}{dt} = A-\left((1-\omega)\beta_h I_h + \beta_{hf} I_f\right)S_h - \mu_h S_h, \quad (1)$$

$$\frac{dI_h}{dt} = \left((1-\omega)\beta_h I_h + \beta_{hf} I_f\right)S_h - (\gamma_h + \delta_h + \mu_h)I_h, \quad (2)$$

$$\frac{dR_h}{dt} = (\gamma_h + \delta_h)I_h - \mu_h R_h, \quad (3)$$

$$\frac{dS_f}{dt} = B - \beta_{fh} S_f I_h - \left(\delta_f + \mu_f\right)S_f, \quad (4)$$

$$\frac{dI_f}{dt} = \beta_{fh} S_f I_h - \left(\delta_f + \mu_f\right)I_f, \quad (5)$$

with $N_h = S_h + I_h + R_h$ and $N_f = S_f + I_f$,

where A is the recruitment rate of the human population, ω is the effectiveness of patient isolation, β_h is the transmission rate of conjunctivitis from infected human to susceptible human, β_{hf} is the transmission rate of conjunctivitis from infected flies to susceptible human, β_{fh} is the transmission rate of conjunctivitis from infected human to susceptible flies, μ_h is the natural death rate of human population, γ_h is the recovery rate of an infected human who does not visits a doctor, δ_h is the recovery rate of an infected human who visits a doctor, B is the recruitment rate of the fly population, δ_f is the effectiveness of insecticides in protecting flies, and μ_f is the natural death rate of fly population.

3. MODEL PROPERTIES

In this section, we find the fundamental properties of the system, which are essential in the proof of the following section.

3.1 Positivity of Solutions

The associated parameters of the system (1) – (5) with respect to the initial conditions are non-negative for all $t > 0$ and we prove this in the following procedures to get the results.

Lemma 1. If $S_h(0), I_h(0), R_h(0), S_f(0), I_f(0)$ and all associated parameters of the system are positive, then solutions $S_h(t), I_h(t), R_h(t), S_f(t), I_f(t)$ are positive for all time $t > 0$.

Proof If $S_h(0) \geq 0,\ I_h(0) \geq 0,\ R_h(0) \geq 0,\ S_f(0) \geq 0,\ I_f(0) \geq 0$, it follows by the first equation, we have

$$\frac{dS_h}{dt} + (\phi_1 + \mu_h)S_h = A, \text{ where } \phi_1 = (1-\omega)\beta_h I_h + \beta_{hf} I_f$$

It can be re-written as:

$$\frac{dS_h}{dt} e^{\int_0^t (\phi_1+\mu_h)dt} + S_h(\phi_1 + \mu_h)e^{\int_0^t (\phi_1+\mu_h)dt} = Ae^{\int_0^t (\phi_1+\mu_h)dt},$$

$$\frac{d}{dt}\left(S_h e^{\int_0^t (\phi_1+\mu_h)dt}\right) = Ae^{\int_0^t (\phi_1+\mu_h)dt}.$$

Therefore, $S_h e^{\int_0^t (\phi_1+\mu_h)dt} + S_h(0) = \int_0^t \left(Ae^{\int_0^t (\phi_1+\mu_h)dt}\right)dt$.

Consequently,

$$S_h(t) = S_h(0)e^{-\int_0^t (\phi_1+\mu_h)dt} + e^{-\int_0^t (\phi_1+\mu_h)dt}\int_0^t \left(Ae^{\int_0^t (\phi_1+\mu_h)dt}\right)dt > 0.$$

For the state variables $I_h(t), R_h(t), S_f(t)$ and $I_f(t)$, we can apply the same method to get $I_h(t) \geq 0,\ R_h(t) \geq 0,\ S_f(t) > 0,\ I_f(t) \geq 0$ for $t > 0$.

3.2 Boundness of Solutions

In this section, the boundary of the solutions of a system of equations is determined. The total human population in this model is $N_h = S_h + I_h + R_h$.

Therefore, we have

$$\frac{dN_h}{dt} = A - \mu_h N_h.$$

Hence, $N_h(t) = \frac{A}{\mu_h} - \left(\frac{A}{\mu_h} - N_h(0)\right)e^{-\mu_h t}$, where $N_h(0)$ denotes the initial values of the total human population and $N_h(0) \leq \frac{A}{\mu_h}$. As $t \to \infty$ then $N_h(t) \to \frac{A}{\mu_h}$, implies that $N_h(t) \leq \frac{A}{\mu_h}$, and $N_f = S_f + I_f$,

$$\frac{dN_f}{dt} = B - (\delta_f + \mu_f)N_f.$$

We obtain $N_f = \frac{B}{\delta_f + \mu_f} - \left(\frac{B}{\delta_f + \mu_f} - N_f(0)\right)e^{-\mu_f t}$, where $N_f(0)$ denotes the initial values of the total fly population and $N_f(0) \leq \frac{B}{\delta_f + \mu_f}$. As $t \to \infty$ then $N_f \to \frac{B}{\delta_f + \mu_f}$ implies that $N_f \leq \frac{B}{\delta_f + \mu_f}$. Therefore, all solutions of this model are bounded and enter the region Ω

$$\left\{\left(S_h, I_h, R_h, S_f, I_f\right) \in \Re_+^5 : 0 < N_h \leq \frac{A}{\mu_h}, 0 < N_f \leq \frac{B}{\delta_f + \mu_f}\right\}. \quad (6)$$

Hence, Ω is the positively invariance. This means that every solution of this model remains in the region for all $t>0$.

4. MODEL ANALYSIS

4.1 Equilibrium points

By applying the standard method to analyze the present model, it is found that there are two equilibrium points, as follows:

1) Disease-free equilibrium (DFE) denoted by

$$E_0=\left(\frac{A}{\mu_h},\ 0,\ 0,\ \frac{B}{\delta_f+\mu_f},\ 0\right)$$

2) Endemic equilibrium (EE) denoted by

$$E_1=\left(S_h^*,\ I_h^*,\ R_h^*,\ S_f^*,\ I_f^*\right),$$

$$S_h^*=\frac{A-\left(\delta_h+\gamma_h+\mu_h\right)I_h^*}{\mu_h},\quad I_h^*=\frac{-a_1+\sqrt{a_1^2+4a_2}}{2},$$

$$R_h^*=\frac{\left(\delta_h+\gamma_h\right)I_h^*}{\mu_h},\qquad S_f^*=\frac{B}{\beta_{fh}I_h^*+\delta_f+\mu_f},$$

$$I_f^*=\frac{B\beta_{fh}I_h^*}{\left(\delta_f+\mu_f\right)\left(\beta_{fh}I_h^*+\delta_f+\mu_f\right)},$$

where $a_1=\dfrac{m_1-Am_2}{\left(\delta_h+\gamma_h+\mu_h\right)m_2}$,

$$a_2=\frac{\mu_h\left(\gamma_h+\delta_h+\mu_h\right)\left(\gamma_h+\delta_h\right)\left(R_0-1\right)}{\left(1-\omega\right)\beta_h\beta_{fh}},$$

$$m_1=\left(1-\omega\right)\beta_h\left(\gamma_h+\delta_h\right)^2+B\beta_{hf}\beta_{fh}>0,$$

$$m_2=\left(1-\omega\right)\left(\delta_f+\mu_f\right)\beta_{fh}\beta_h>0,$$

$$m_3=\left(1-\omega\right)\beta_h\left(\delta_f+\mu_f\right)^2+B\beta_{hf}\beta_{fh}>0,$$

$$R_0=\frac{A\left[\left(1-\omega\right)\beta_h\left(\delta_f+\mu_f\right)^2+\beta_{hf}\beta_{fh}B\right]}{\mu_h\left(\delta_h+\gamma_h+\mu_h\right)\left(\delta_f+\mu_f\right)^2}.$$

From I_h^*, it was found that (i) if $m_1>Am_2$ and $R_0<1$, then $a_1>0$, $a_2<0$ and there is no positive equilibrium point. (ii) If $m_1<Am_2$ and $R_0>1$, then $a_1<0$, $a_2>0$ and the system has the positive equilibrium point (E_1). (iii) If $m_1>Am_2$, $R_0>1$ and $\sqrt{a_1^2+4a_2}>a_1$, then the system has one positive equilibrium point (E_1). The basic reproductive number R_0, obtained from the equilibrium analysis, is computed by using the next generation matrix method, which has been shown in the next topic.

4.2 Basic reproductive number

The basic reproductive number (R_0) is the average number of secondary infections induced by an infected individual introduced into the total susceptible population. It is one of the threshold conditions in an epidemic that allow us to predict whether the disease will be died out or persist (Heffernan et al., 2005 and Diekmann et al., 2019). We calculate our basic reproductive number using the next generation method and use the spectral radius (Van den Driessche & Watmough, 2002). By applying this approach, it is given by $R_0=\rho\left(FV^{-1}\right)$ where $\rho(A)$ denotes the spectral radius of a matrix A (the largest eigenvalues of A), where $F(X)$ is the non-negative matrix of new infection terms and $V(X)$ is the non-singular matrix of the remaining transfer terms. The system (1) – (5) is rewritten in a matrix form:

$$\frac{dX}{dt}=F(X)-V(X),\ X=(S_h,\ I_h,\ R_h,\ S_f,\ I_f)^T \qquad (7)$$

$$F(X)=\begin{bmatrix}0\\ \left(1-\omega\right)\beta_hS_hI_h+\beta_{hf}S_hI_f\\ 0\\ 0\\ 0\end{bmatrix},$$

$$V(X)=\begin{bmatrix}-A+\left(1-\omega\right)\beta_hS_hI_h+\beta_{hf}S_hI_f+\mu_hS_h\\ \left(\delta_h+\gamma_h+\mu_h\right)I_h\\ -\left(\delta_h+\gamma_h\right)I_h+\mu_hR_h\\ -B+\beta_{fh}S_fI_h+\left(\delta_f+\mu_f\right)S_f\\ -\beta_{fh}S_fI_h+\left(\delta_f+\mu_f\right)I_f\end{bmatrix}.$$

By finding the Jacobian matrix of F and V at E_0, respectively, the following matrices are obtained

$$F=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ 0 & \dfrac{\left(1-\omega\right)\beta_hA}{\mu_h} & 0 & 0 & \dfrac{\beta_{hf}A}{\mu_h}\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}_{E_0},$$

$$V=\begin{bmatrix}\mu_h & \dfrac{\left(1-\omega\right)\beta_hA}{\mu_h} & 0 & 0 & \dfrac{\beta_{hf}A}{\mu_h}\\ 0 & \delta_h+\gamma_h+\mu_h & 0 & 0 & 0\\ 0 & -\left(\delta_h+\gamma_h\right) & \mu_h & 0 & 0\\ 0 & \dfrac{\beta_{fh}B}{\delta_f+\mu_f} & 0 & \delta_f+\mu_f & 0\\ 0 & -\dfrac{\beta_{fh}B}{\delta_f+\mu_f} & 0 & 0 & \delta_f+\mu_f\end{bmatrix}_{E_0}.$$

Moreover, FV^{-1} is obtained as

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{(1-\omega)\beta_h A\left(\delta_f+\mu_f\right)^2+\beta_{hf}\beta_{fh}AB}{\mu_h\left(\delta_h+\gamma_h+\mu_h\right)\left(\delta_f+\mu_f\right)^2} & 0 & 0 & \dfrac{(1-\omega)\beta_h A}{\mu_h\left(\delta_f+\mu_f\right)} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

So, the basic reproductive number is obtained as

$$R_0=\frac{A\left[(1-\omega)\beta_h\left(\delta_f+\mu_f\right)^2+\beta_{hf}\beta_{fh}B\right]}{\mu_h\left(\delta_h+\gamma_h+\mu_h\right)\left(\delta_f+\mu_f\right)^2}. \quad (8)$$

From the basic reproductive number, it was found that the parameters of interest, such as the effectiveness of patient separation (ω), the recovery rate of infected people who see a doctor (δ_h), and the effectiveness of insecticides in protecting flies (δ_f), affect R_0. Therefore, the condition of the epidemic of the conjunctivitis infection is that if $R_0<1$, then the epidemic of conjunctivitis decreases. However, if $R_0>1$, the epidemic of conjunctivitis increases.

4.3 Global stability of disease-free equilibrium point

We prove the global stability of the disease-free equilibrium point (E_0) by using the Lyapunov theorem (see, e.g., Alkhudhari et al., 2014 and Van den Driessche & Watmough, 2002).

Theorem 1. If $R_0\le 1$, then the disease-free equilibrium point (E_0) is globally asymptotically stable on Ω.

Proof Let $V(t)=S_h^*\left(\frac{S_h}{S_h^*}-S_h^*\ln\left(\frac{S_h}{S_h^*}\right)\right)+I_h$

$$+\frac{\beta_{fh}A}{\left(\delta_f+\mu_f\right)\mu_h}S_f^*\left(\frac{S_f}{S_f^*}-S_f^*\ln\left(\frac{S_f}{S_f^*}\right)\right)+\frac{\beta_{fh}A}{\mu_h\left(\delta_f+\mu_f\right)}I_f. \quad (9)$$

Next, for the derivative of V with respect to time t, we have

$$\begin{aligned}\frac{dV}{dt}&=\frac{dS_h}{dt}-\frac{S_h^*}{S_h}\frac{dS_h}{dt}+\frac{dI_h}{dt}+\frac{\beta_{hf}A}{\left(\delta_f+\mu_f\right)\mu_h}\left(\frac{dS_f}{dt}-\frac{S_f^*}{S_f}\frac{dS_f}{dt}\right)\\&\quad+\frac{\beta_{hf}A}{\left(\delta_f+\mu_f\right)\mu_h}\frac{dI_f}{dt}\\&=\left(1-\frac{S_h^*}{S_h}\right)\frac{dS_h}{dt}+\frac{dI_h}{dt}+\frac{\beta_{hf}A}{\left(\delta_f+\mu_f\right)\mu_h}\left(1-\frac{S_f^*}{S_f}\right)\frac{dS_f}{dt}\\&\quad+\frac{\beta_{hf}A}{\left(\delta_f+\mu_f\right)\mu_h}\frac{dI_f}{dt}\end{aligned}$$

Substituting, the equations (1), (2), (4), and (5), we have

$$\begin{aligned}\frac{dV}{dt}&=\left(1-\frac{S_h^*}{S_h}\right)\left[A-\left((1-\omega)\beta_h S_h I_h+\beta_{hf}S_h I_f\right)-\mu_h S_h\right]\\&\quad+(1-\omega)\beta_h S_h I_h+\beta_{hf}S_h I_f-\left(\gamma_h+\delta_h+\mu_h\right)I_h\\&\quad+\frac{\beta_{hf}A}{\left(\delta_f+\mu_f\right)\mu_h}\left(1-\frac{S_f^*}{S_f}\right)\left[B-\beta_{fh}S_f I_h-(\delta_f+\mu_f)S_f\right]\\&\quad+\frac{\beta_{hf}A}{\left(\delta_f+\mu_f\right)\mu_h}\left[\beta_{fh}S_f I_h-(\delta_f+\mu_f)I_f\right],\\&=-\frac{\mu_h\left(S_h^*-S_h\right)^2}{S_h}-\frac{\beta_{hf}A}{\mu_h}\frac{\left(S_f^*-S_f\right)^2}{S_f}\\&\quad+\left[\frac{(1-\omega)\beta_h A}{\mu_h}+\frac{\beta_{hf}\beta_{fh}AB}{\mu_h\left(\delta_f+\mu_f\right)^2}-\left(\gamma_h+\delta_h+\mu_h\right)\right]I_h,\\&=-\frac{\mu_h\left(S_h^*-S_h\right)^2}{S_h}-\frac{\beta_{hf}A}{\mu_h}\frac{\left(S_f^*-S_f\right)^2}{S_f}-\left(\gamma_h+\delta_h+\mu_h\right)(1-R_0)I_h.\end{aligned}$$

The condition $\frac{dV}{dt}=0$ is true only if $S_h=S_h^*$, $S_f=S_f^*$, $I_h=0$, and the condition $\frac{dV}{dt}<0$ holds if $R_0\le 1$. Therefore, the disease-free point is the only positive invariant set in Ω. Consequently, if $R_0\le 1$ then the disease-free equilibrium point is globally asymptotically stable on Ω.

4.4 Global stability of endemic equilibrium point

Global stability analysis of endemic equilibrium (E_1) is explored via the construction of the Lyapunov function (see, e.g., Xu et al., 2020 and Elkaranshawy et al., 2021).

Theorem 2. If $R_0>1$ then the system has a unique endemic equilibrium point $E_1\left(S_h^*, I_h^*, R_h^*, S_f^*, I_f^*\right)$ which is global asymptotically stable on Ω.

Proof Consider the following Lyapunov function.

$$\begin{aligned}V&=S_h^*\left(\frac{S_h}{S_h^*}-1-\ln\left(\frac{S_h}{S_h^*}\right)\right)+I_h^*\left(\frac{I_h}{I_h^*}-1-\ln\left(\frac{I_h}{I_h^*}\right)\right)\\&\quad+\frac{S_h^*I_f^*}{S_f^*I_h^*}S_f^*\left(\frac{S_f}{S_f^*}-1-\ln\left(\frac{S_f}{S_f^*}\right)\right)\\&\quad+\frac{S_h^*I_f^*}{S_f^*I_h^*}I_f^*\left(\frac{I_f}{I_f^*}-1-\ln\left(\frac{I_f}{I_f^*}\right)\right).\end{aligned} \quad (10)$$

We have that

$$\begin{aligned}\frac{dV}{dt}&=\left(1-\frac{S_h^*}{S_h}\right)\frac{dS_h}{dt}+\left(1-\frac{I_h^*}{I_h}\right)\frac{dI_h}{dt}+\frac{S_h^*I_f^*}{S_f^*I_h^*}\left(1-\frac{S_f^*}{S_f}\right)\frac{dS_f}{dt}\\&\quad+\frac{S_h^*I_f^*}{S_f^*I_h^*}\left(1-\frac{I_f^*}{I_f}\right)\frac{dI_f}{dt}.\end{aligned}$$

Substituting, the equations (1), (2), (4), and (5) into the resulting equation, we have

$$\frac{dV}{dt}=\left(1-\frac{S_h^*}{S_h}\right)\left[A-\left((1-\omega)\beta_h S_h I_h+\beta_{hf}S_h I_f\right)-\mu_h S_h\right]$$
$$+\left(1-\frac{I_h^*}{I_h}\right)\left[(1-\omega)\beta_h S_h I_h+\beta_{hf}S_h I_f-(\delta_h+\gamma_h+\mu_h)I_h\right]$$
$$+\frac{S_h^* I_f^*}{S_f^* I_h^*}\left(1-\frac{S_f^*}{S_f}\right)\left[B-\beta_{fh}S_f I_h-\left(\delta_f+\mu_f\right)S_f\right]$$
$$+\frac{S_h^* I_f^*}{S_f^* I_h^*}\left(1-\frac{I_f^*}{I_f}\right)\left[\beta_{fh}S_f I_h-\left(\delta_f+\mu_f\right)I_f\right]. \qquad (11)$$

Rewrite some parameter in terms of E_1, from the system (1)-(5), we have

$$A=(1-\omega)\beta_h S_h^* I_h^*+\beta_{hf}S_h^* I_f^*+\mu_h S_h^*,$$
$$\delta_h+\gamma_h+\mu_h=\frac{(1-\omega)\beta_h S_h^* I_h^*+\beta_{hf}S_h^* I_f^*}{I_h^*},$$
$$B=\beta_{fh}S_f^* I_h^*+\left(\delta_f+\mu_f\right)S_f^*,$$
$$\delta_f+\mu_f=\frac{\beta_{fh}S_f^* I_h^*}{I_f^*}.$$

Substitute the parameter into (11), and it obtains

$$\frac{dV}{dt}=\left(1-\frac{S_h^*}{S_h}\right)\left[(1-\omega)\beta_h S_h^* I_h^*+\beta_{hf}S_h^* I_f^*+\mu_h S_h^*\right.$$
$$\left.-(1-\omega)\beta_h S_h I_h-\beta_{hf}S_h I_f-\mu_h S_h\right]+\left(1-\frac{I_h^*}{I_h}\right)$$
$$\times\left[(1-\omega)\beta_h S_h I_h+\beta_{hf}S_h I_f-\frac{(1-\omega)\beta_h S_h^* I_h^*+\beta_{hf}S_h^* I_f^*}{I_h^*}I_h\right]$$
$$+\frac{S_h^* I_f^*}{S_f^* I_h^*}\left(1-\frac{S_f^*}{S_f}\right)\left[\beta_{fh}S_f^* I_h^*+\left(\delta_f+\mu_f\right)S_f^*-\beta_{fh}S_f I_h\right.$$
$$\left.-\left(\delta_f+\mu_f\right)S_f\right]+\frac{S_h^* I_f^*}{S_f^* I_h^*}\left(1-\frac{I_f^*}{I_f}\right)\left[\beta_{fh}S_f I_h-\frac{\beta_{fh}S_f^* I_h^*}{I_f^*}I_f\right],$$
$$=-\frac{\mu_h\left(S_h-S_h^*\right)^2}{S_h}-\frac{\left(\delta_f+\mu_f\right)S_h^* I_f^*}{S_f^* I_h^*}\frac{\left(S_f-S_f^*\right)^2}{S_f}$$
$$+\left(2-\frac{S_h^*}{S_h}-\frac{S_h}{S_h^*}\right)(1-\omega)\beta_h S_h^* I_h^*$$
$$+\left(4-\frac{S_h^*}{S_h}-\frac{S_h I_f I_h^*}{S_h^* I_f^* I_h}-\frac{S_f^*}{S_f}-\frac{S_f I_h I_f^*}{S_f^* I_h^* I_f}\right)\beta_{hf}S_h^* I_f^*.$$

It is obtained that

$$2-\frac{S_h^*}{S_h}-\frac{S_h}{S_h^*}\le 0,\ 4-\frac{S_h^*}{S_h}-\frac{S_h I_f I_h^*}{S_h^* I_f^* I_h}-\frac{S_f^*}{S_f}-\frac{S_f I_h I_f^*}{S_f^* I_h^* I_f}\le 0,$$

since the geometric mean is less than or equal mean for $S_h,\ I_h,\ S_f,\ I_f>0$. From E_1, it is found that $S_h,\ I_h,\ S_f,\ I_f>0$ when $R_0>1$. Therefore, if $R_0>1$, then $\frac{dV}{dt}<0$.

Since $S_h-S_h^*$, $S_f-S_f^*$, $2-\frac{S_h^*}{S_h}-\frac{S_h}{S_h^*}$, , and $4-\frac{S_h^*}{S_h}-\frac{S_h I_f I_h^*}{S_h^* I_f^* I_h}-\frac{S_f^*}{S_f}-\frac{S_f I_h I_f^*}{S_f^* I_h^* I_f}$ equal to zero at E_1, then $\frac{dV}{dt}=0$.

Hence, if $R_0>1$. then the endemic equilibrium point (E_1) is globally asymptotically stable on Ω.

4.5 Sensitivity analysis

The necessity of investigating tells us how sensitive the basic reproductive number is with respect to its parameters. This will help us to know the parameters having the most significant impact on the outcome of the numerical results of the model. Sensitivity analysis tells us the importance of each parameter to disease transmission, which will help public health. Authorities must place priority on a well-posed investigation strategy for preventing and controlling the spread of disease in the population. The sensitivity indices are calculated using the technique of normalized forward (Samsuzzoha et al., 2013 and Ngoteya & Nkansah-Gyekye, 2015). The normalized forward sensitivity of the reproduction number (R_0) with respect to the parameter k is given by,

$$\Upsilon_k^{R_0}=\frac{\partial R_0}{\partial k}\times\frac{k}{R_0}. \qquad (12)$$

Using the formula presented in (12) and the baseline parameters, the numerical values for the sensitivity index are obtained. The sensitivity index is estimated with respect to each parameter. Results are shown in Table 1 and Table 2.

Table 1 Sensitivity indexes of the Conjunctivitis Model's parameter with respect to R_0 at E_0.

Parameters	Baseline Value	Sensitivity Indexes	Sign
ω	0.7	- 1.08971	Negative
μ_h	0.000256 per day	- 1.00039	Negative
A	0.00128 per day	+ 1.00000	Positive
β_{hf}	0.065 per day	+ 0.83298	Positive
β_{fh}	0.065 per day	+ 0.83298	Positive
B	0.5 per day	+ 0.83298	Positive
δ_h	1/2 per day	- 0.77747	Negative
δ_f	1/7 per day	- 0.56851	Negative
μ_f	0.125 per day	- 0.49744	Negative
β_h	0.086 per day	+ 0.46702	Positive
γ_h	0.1428 per day	- 0.22213	Negative

Table 1 shows that the most effective methods of decreasing R_0 evaluated at E_0 are ranked in order of effectiveness: (1) to increase ω, (2) to increase μ_h, (3) to decrease β_{hf}, (4) to decrease β_{fh}, (5) to decrease B, (6) to decrease A, (7) to increase δ_h, (8) to increase δ_f, (9) to increase μ_f, (10) to decrease β_h, and (11) to increase γ_h.

Table 2 Sensitivity indexes of the Conjunctivitis Model's parameter with respect to R_0 at E_1.

Parameters	Baseline Value	Sensitivity Indexes	Sign
μ_h	0.000256 per day	- 1.00040	Negative
A	0.00128 per day	+ 1.00000	Positive
δ_h	0.48 per day	- 0.77746	Negative
β_h	0.086 per day	+ 0.72442	Positive
β_{hf}	0.065 per day	+ 0.27558	Positive
β_{fh}	0.065 per day	+ 0.27558	Positive
B	0.5 per day	+ 0.27558	Positive
δ_f	1/20 per day	- 0.29395	Negative
μ_f	1/8 per day	- 0.67319	Negative
γ_h	1/2 per day	- 0.22213	Negative
ω	0.1	- 0.08049	Negative

From Table 2, we can see that the most effective methods of decreasing R_0 evaluated at E_1 are ranked in order of effectiveness: (1) to increase μ_h, (2) to decrease A, (3) to increase δ_h, (4) to decrease β_{hf}, (5) to decrease β_{fh}, (6) to decrease B, (7) to decrease β_h, (8) to increase δ_f, (9) to increase μ_f, (10) to increase γ_h, and (11) to increase ω.

Finally, we found that the most important parameters for preventing the disease are in order of effectiveness (1) increasing ω, by isolating patients from the community or preventing infection by maintaining hygiene, (2) increasing μ_h, by increasing the natural death rate of the human population, but it is not an acceptable method, (3) decreasing β_{hf}, by reducing the infection from mosquitoes to humans, such as using mosquito repellent or mosquito nets, etc. Moreover, the most important parameters for controlling the disease is (1) increasing μ_h, (2) to decrease A by quarantining the infected area, (3) increasing δ_h, by decreasing the duration of treatment or the time it takes to transmit conjunctivitis among infected individuals.

5. NUMERICAL RESULTS

In the present work, we use SIR-SI epidemic model with control measures. The numerical simulations are carried out to determine the impact of control measures on the conjunctivitis dynamics. The parameters used in the numerical simulations are given in Table 3.

Table 3 Parameter values in numerical simulations at disease-free state.

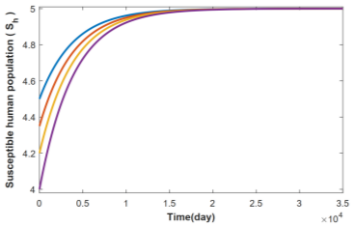
Parameters	Description	Value	Ref.
A	the recruitment rate of human population	0.00128 per day	Positive
β_h	the transmission rate of conjunctivitis from human to human	0.086 per day	Viriyapong & Khedwan, 2019
μ_h	the natural death rate of human population	0.000256 per day	
γ_h	the recovery rate of infected human	1/7 per day	
δ_h	the recovery of infected human who go to see the doctor	1/2 per day	
B	the recruitment rate of fly population	0.5 per day	Assumed
β_{hf}	the transmission rate of conjunctivitis from infected flies to susceptible human	0.065 per day	
β_{fh}	the transmission rate of conjunctivitis from infected human to susceptible flies	0.065 per day	
μ_f	the natural death rate of fly population	0.125 per day	
ω	the effective of patient's isolation	0.7	
δ_f	the efficacy of insecticide for protecting the flies	1/7 per day	

5.1 Stability of disease-free state

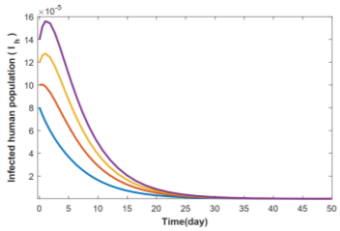
Using the values of parameters as shown in Table 3, the eigenvalues of the system (1) - (5) at disease-free equilibrium point and basic reproductive number are obtained as follow:

$$\lambda_{1,2} = -0.000256,\quad \lambda_3 = -0.000256,\quad \lambda_4 = -0.1573319,$$
$$\lambda_5 = -0.267857,\quad R_0 = 0.429501 < 1.$$

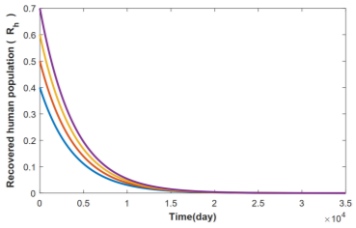
Since all eigenvalues are negative and the basic reproductive number is less than one, the disease-free equilibrium point, E_0, will be local asymptotically stable, as shown in Figure 2.



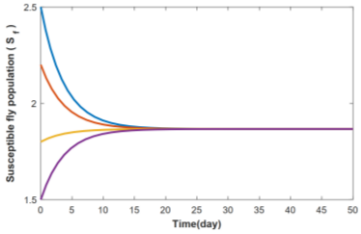
(1a) Susceptible human population



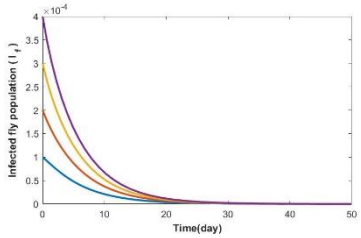
(1b) Infected human population



(1c) Recovered human population



(1d) Susceptible fly population



(1e) Infected fly population

Figure 2 Time series of (1a) susceptible human population, (1b) infected human population, (1c) recovered human population, (1d) susceptible fly population, (1e) infected fly population. The state variables approach DFE, $E_0 = (5, 0, 0, 1.86666, 0)$.

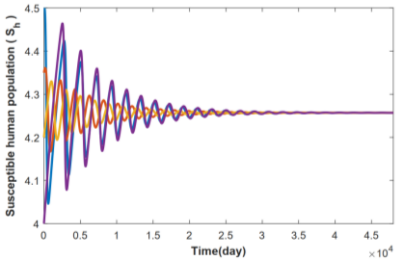
From Figure 2, graphs (1a)-(1e) presenting numerical solutions for susceptible human population, infected human population, recovered human population, susceptible fly population, and infected fly population show convergence to DFE, $E_0 = (5, 0, 0, 1.86666, 0)$ for four different sets of initial condition where $R_0 < 1$. Therefore, there will not be an epidemic of conjunctivitis in the future when the efficiency in isolating patients is 70% and the treatment period for conjunctivitis is 2 days, along with using insecticides every 7 days per time.

5.2 Stability of endemic state

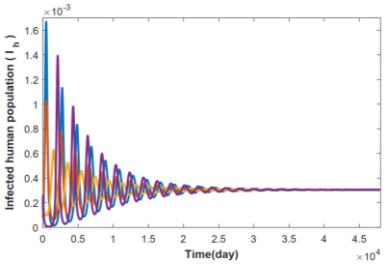
We change the values of the effective patient's isolation (ω) to 0.1, the recovery rate from treatment (δ_h) to 0.48 (Viriyapong & Khedwan, 2019), and the efficacy of insecticides for protecting the flies (δ_f) to 1/20, and keep the other values of parameters to be those given in Table 3. We obtain the eigenvalues of the system (1) – (5) at endemic equilibrium point and basic reproductive number as follows:

$$\lambda_1 = -0.000256,\ \lambda_2 = -0.468615,\ \lambda_3 = -0.175000,$$
$$\lambda_{4,5} = -0.000161 \pm 0.003221i,\quad R_0 = 1.174582 > 1.$$

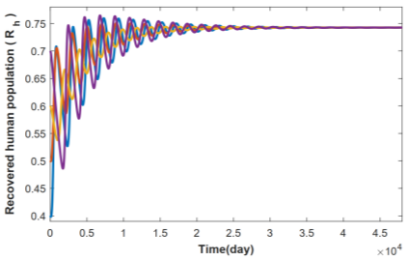
Since all real part of eigenvalues are negative and the basic reproductive number is more than one, the endemic state, E_1, will be local asymptotically stable, as shown in Figure 3.



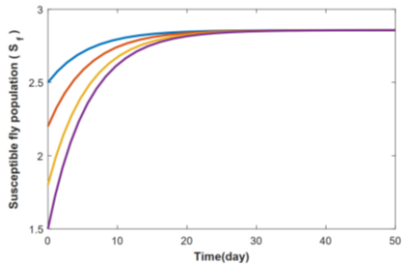
(2a) Susceptible human population



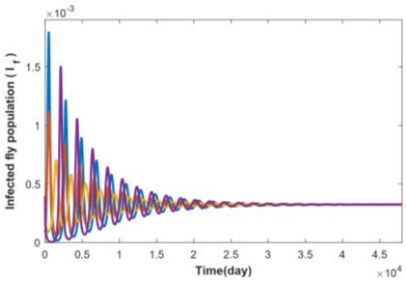
(2b) Infected human population



(2c) Recovered human population



(2d) Susceptible fly population



(2e) Infected fly population

Figure 3 Time series of (a) susceptible human population, (b) infected human population, (c) recovered human population, (d) susceptible fly population, (e) infected fly population. The state variables approach EE, $E_1 = (4.257058, 0.000305, 0.742636, 2.856819, 0.000324)$.

Figure 3 shows that if reducing patient separation efficiency is 10%, the duration of treatment for conjunctivitis is increased from 2 days to 3 days, and the duration of insecticide use is increased from 7 days per time to 20 days per time, then the graph (2a)–(3e) of numerical solutions convergence EE, $E_1 = (4.257058, 0.000305, 0.742636, 2.856819, 0.000324)$, and $R_0 = 1.174582 > 1$. That means that if we decrease the effectiveness of the patient's isolation to 10%, increase the recovery rate of treatment to 0.48 per day, and decrease the efficacy of insecticides for protecting the flies to 1/20 per day, then it is not sufficient to control the spread of disease.

5.3 Numerical results for infected human population with different values of ω, δ_h *and* δ_f

From Sections 4.1–4.2, it was found that the effectiveness of the patient's isolation, the recovery rate of treatment, and the effectiveness of insecticides in protecting flies affect the convergence of the equilibrium point and changes in the R_0. In Section 4.3, we are considering the relationship between infected human populations and values of ω, δ_h and δ_f as follows.

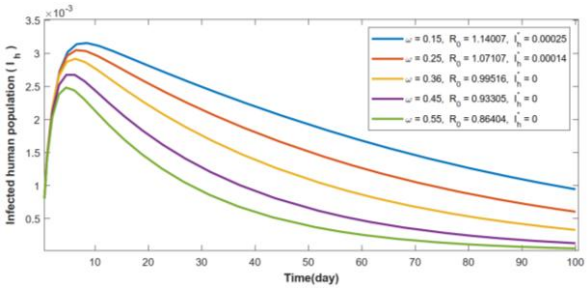


Figure 4 Time series of infected human populations with different values of ω .

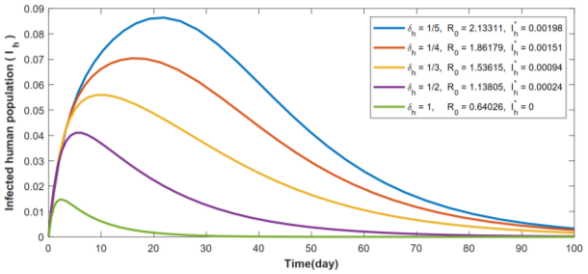


Figure 5 Time series of infected human populations with different values of δ_h .

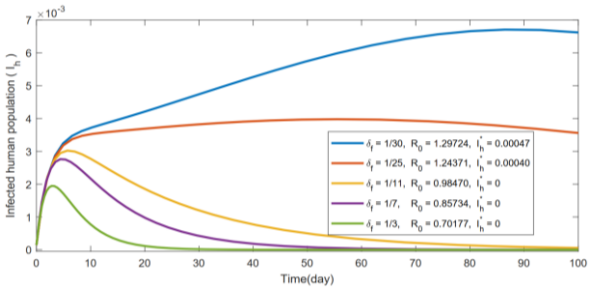


Figure 6 Time series of infected human populations with different values of δ_f .

Figure 4 indicates that the effectiveness of the patient's isolation has an effect on the infected human population when other parameters are set according to the EE point with the values of $\delta_h = 0.48$ $\delta_f = 1/20$. When the values of effectiveness of the patient's isolation (ω) are 0.15, 0.25, 0.36, and 0.55, the values of R_0 are 1.14007, 1.07107, 0.99516, 0.93305, and 0.86404, respectively, and the values of I_h^* are 0.00025, 0.00014, 0, 0, and 0, respectively. It can be observed that increasing the effectiveness of the patient's isolation affects infected human populations decreased. Similarly, Figures 5–6 show that the number of infected humans population decreases when the recovery rate of treatment increases and the effectiveness of insecticides in protecting flies increases.

6. CONCLUSION

In this paper, the SIR-SI (human-host) epidemic model of conjunctivitis which incorporates the effects of the effective of patient's isolation, treatment control and insecticide control is proposed and analyzed. We have analyzed the disease-free equilibrium (E_0), the endemic equilibrium point (E_1), and the basic reproductive number, $R_0=\frac{A\left((1-\omega)\beta_h\left(\delta_f+\mu_f\right)^2+\beta_f{}^2B\right)}{\mu_h\left(\delta_h+\gamma_h+\mu_h\right)\left(\delta_f+\mu_f\right)^2}$, where it becomes a threshold condition for the stability of the system at equilibrium points. The global asymptotically stability of the disease-free equilibrium point and endemic equilibrium point has been shown in Fig 2 and Fig 3, respectively. It has shown that the endemic equilibrium point is global asymptotically stable if $R_0>1$, and this means that the conjunctivitis will persist in community. It has also been shown that if $R_0<1$, the conjunctivitis will die out in the community. The sensitivity analysis is carried out by computing the sensitivity indexes at the disease-free equilibrium point. It can be found that the most sensitive parameter is the patient's isolation. It means that if most people have a high rate of patient isolation, then there are no conjunctivitis patients in the community. Every house must use insecticide to kill flies and maintain sanitation. The best intervention depends on how to control the density of people in the community. In addition, the sensitivity analysis also indicated that increasing the percentage of the effectiveness of the patient's isolation, the recovery rate of treatment, and the effectiveness of insecticides in protecting flies will reduce the basic reproductive number. Consequently, the disease-free equilibrium is stable instead of the endemic equilibrium point. It is concluded that when the effectiveness of the patient's isolation, the recovery rate of treatment, and the effectiveness of insecticides in protecting flies increase, the number of infected humans will decrease.

ACKNOWLEDGMENTS

The authors are grateful to the Department of Mathematics, Faculty of Science and Technology, Suratthani Rajabhat University, Surat Thani, Thailand for providing the facilities to carry out the research.

REFERENCES

Alkhudhari, Z., Al Sheikh, S., & Al-Tuwairqi, S. (2014). Global dynamics of a mathematical model on smoking. *International Scholarly Research Notices, 2014*, 1-7.

Chowell, G., Shim, E., Brauer, F., Diaz-Dueñas, P., Hyman, J. M., & Castillo-Chavez, C. (2006). Modelling the transmission dynamics of acute haemorrhagic conjunctivitis: application to the 2003 outbreak in Mexico. *Statistics in medicine, 25*(11), 1840-1857.

Diekmann, O., Heesterbeek, J. A., & Metz, J. A. (1990). On the definition and the computation of the basic reproduction ratio R0 in models for infectious diseases in heterogeneous populations. *Journal of mathematical biology, 28*(4), 365-382.

Elkaranshawy, H. A., Ezzat, H. M., & Ibrahim, N. N. (2021). Lyapunov function and global asymptotic stability for a new multiscale viral dynamics model incorporating the immune system response: Implemented upon HCV. *PloS one, 16*(10), 1-26.

Heffernan, J. M., Smith, R. J., & Wahl, L. M. (2005). Perspectives on the basic reproductive ratio. *Journal of the Royal Society Interface, 2*(4), 281-293.

Nana-Kyere, S., Banon, D. T., Marmah, S. N., & Kwarteng, D. (2019). Stochastic Optimal Control Model of Haemorrhagic Conjunctivitis Disease. *Journal of Advances in Mathematics and Computer Science, 31*(3), 1-19.

Ngoteya, F.N., & Nkansah-Gyekye, Y. (2015). Sensitivity Analysis of Parameters in a Competition Model. *Applied and Computational Mathematics, 4*(5), 363-368.

Samsuzzoha, M. D., Singh, M., & Lucy, D. (2013). Uncertainty and sensitivity analysis of the basic reproduction number of a vaccinated epidemic model of influenza. *Applied Mathematical Modelling, 37*(3), 903-915.

Sangthongjeen, S., Sudchumnong, A., & Naowarat, S. (2015). Effect of Education Campaign on Transmission Model of Conjunctivitis. *Australian Journal of Basic and Applied Sciences, 9*(7), 811-815.

Suksawat, J., & Naowarat, S. (2014). Effect of rainfall on the transmission model of conjunctivitis. *Advances in Environmental Biology, 8*(14), 99-104.

Thongtha, A., Modnak, C., & Thakjak, A. (2017). Optimal control for a mathematical model with transmission for house flies. *Pibul Research*, 28-36.

Thongtha, A., & Modnak, C. (2021). Optimal Control Strategy of Cholera Epidemic Dynamics from Vibrio and Flies Transmission. *NU. International Journal of Science, 18*(1), 15-27.

Uchenna, M., Offia, A., & Kafayat, E. (2019). Control model on transmission dynamic of conjunctivitis during harmattan in public schools. *Applied and Computational Mathematics, 8*(2), 29-36.

Unyong, B., & Naowarat, S. (2014). Stability analysis of conjunctivitis model with nonlinear incidence term. *Australian Journal of Basic and Applied Sciences, 8*(24), 52-58.

Van den Driessche, P., & Watmough, J. (2002). Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. *Mathematical biosciences, 180*(1-2), 29-48.

Viriyapong, R., & Khedwan, N. (2019). Effects of isolation by taking sick leaves of conjunctivitis infected individuals and treatment control on stability of mathematical modeling of conjunctivitis. *Science, Engineering and Health Studies, 13*(1), 20-28.

Xu, L., Lou, S., & Han, R. (2020). Global Stability for a Semidiscrete Logistic System with Feedback Control. *Discrete Dynamics in Nature and Society, 2020*, 1-7.