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The Performance of DEWMA and EWMA for ZIB Model when Underlying Distribution is the Ratio of Two Poisson Means

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ABSTRACT

This research aims to present a double exponentially weighted moving average (DEWMA) control chart for a Zero Inflated Binomial model under the ratio distribution of the two Poisson means (ZIBpoi) by the Markov Chain approach. The objective is to compare the efficiency of DEWMA with EWMA procedure. The performance of DEWMA control chart gives an out-of-control signal quicker than EWMA control chart to small process shifts ($\delta \le 1\sigma$) for φ shifts and all cases for ξ shifts.

KEYWORDS: Average Run Length, zero-inflated binomial, Markov Chain, double exponentially weighted moving average

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1. INTRODUCTION

Generally, in the industrial production process, manufactured products must meet or exceed customer expectations. The goal of production aims to control the variation in product quality to a minimum. The Statistical Process Control (SPC), which employs statistical methods to monitor and control a process, is an effective monitoring technique widely used. SPC is comprised of seven tools: Pareto chart, histogram, process flow diagram, control charts, Scatter diagram, check sheets and cause and effect diagram. Control charts are an important tool for statistical process control. Their purpose is to monitor changes in the production process, whether it is in control or out of control, making it possible to quickly resolve quality issues when manufacturing malfunctions occur. Shewhart (1920s) developed the control charts tool to identify when a process was producing a good or a defective product. Shewhart control charts are suitable for detecting a large shift in the industrial process and are unable to detect a small shift in the process ($\delta > 1.5\sigma$), due the chart's lack of importance of historical data (memoryless) and the fact that production data may not always follow a normal distribution, making the Shewhart chart unsuitable due to its poor quality. In 1954, Page developed the cumulative sum (CUSUM) control chart, which became popular. Roberts (1959) presented the Exponentially weighted moving average (EWMA) control chart. The CUSUM control chart and EWMA are more efficient than traditional control charts for quick detection of small shifts ($\delta > 1.5\sigma$). Subsequently, Shamma & Shamma (1992) presented a Double Exponentially Weighted Moving Average (DEWMA) control chart. Since then, many other researchers have designed EWMA and DEWMA charts.

The extension of the EWMA technique to the DEWMA technique, for example, Mahmoud & Woodall (2010) conducted a study to compare some characteristics between the EWMA and the DEWMA. Alkahtani (2013) researched the robustness of DEWMA versus EWMA control charts in various skewed (Gamma) and symmetric non-normal (t) distributions to examine the effect of non-normality on the ARL. Rafael & Jay (2017) studied DEWMA for the individuals based on a linear prediction.

The ARL is mostly used to measure the performance of the control chart, Roberts (1959) presented the ARL for EWMA chart. Many methods used Monte Carlo (MC) simulation, which is a simple approach that is often used to test the validity of other methods. However, Monte Carlo simulations can be very expensive in computational time, and the Markov Chain Approach (MCA) can be less processed. The MCA was first presented by Brook & Evans (1972) to approximate the ARL. More references on the evaluation of characteristics by MCA are Champ & Woodall (1987); Calzada and Scariano (2003); Chang and Wu (2011); Chananet *et al.* (2014).

At present, count data with many zeros are common in a wide variety of disciplines. The zero-inflation model has become an important tool for analyzing count data with excess zeros. Lambert (1992) presented zero-inflated Poisson (ZIP) regression models. Next. Hall (2000) introduced the zero-inflated binomial (ZIB) model. Bualuang *et al.* (2017) introduced ARL formulas of MA control chart for ZIBpoi.

The remainder of this paper is organized as follows. In section 2, the methods included are the zero-inflated binomial model underlying distribution the ratio of two Poisson means, EWMA control chart, DEWMA control chart, and Average Run Length of EWMA and DEWMA control chart with Markov Chain approach. Section 3 discusses the output and performance of DEWMA versus EWMA control chart. Finally, the conclusions of EWMA and DEWMA are presented.

2. METHODS

2.1 Zero-inflated Binomial model

The zero-inflated binomial model when the underlying distribution is the ratio of two Poisson means (ZIB_{poi}) was proposed by Bualuang *et al.* (2017). Let $X_1, X_2, ..., X_{n-1}, X_n$ of random variables are independent and identically distributed, where X_i is the number of nonconforming items in sample i with parameter n, $p = \frac{\varphi}{\varphi + \xi}$ and θ. A zero-inflated binomial is to include a proportion $1-\theta$ of extra-zeros. Define parameters $\varphi > 0$ and $\xi > 0$ is two independent Poisson means. In the zero-inflated binomial when the underlying distribution is the ratio of two Poisson means (ZIB_{poi}), the density function $X \sim ZIB_{poi}(n, \varphi, \xi, \theta)$ can be written as follows:

$$f(x) = \begin{cases} \theta \binom{n}{x} \left(\frac{\varphi}{\varphi+\xi}\right)^x \left(1 - \frac{\varphi}{\varphi+\xi}\right)^{n-x} & ; x = 1,2,\ldots,n \\ (1-\theta) + \theta\left(1 - \frac{\varphi}{\varphi+\xi}\right)^n & ; x = 0 \end{cases} \quad (1)$$

The mean and variance of X given by

$$E(X) = n\theta\left(\frac{\varphi}{\varphi+\xi}\right) \quad (2)$$

and

$$Var(X) = \sigma^2 = n(n-1)\theta\left(\frac{\varphi}{\varphi+\xi}\right)^2 + n\theta\left(\frac{\varphi}{\varphi+\xi}\right) - \left[n\theta\left(\frac{\varphi}{\varphi+\xi}\right)\right]^2 \quad (3)$$

2.2 Exponentially weighted moving average control chart: EWMA

The EWMA control chart was proposed by Roberts (1959) (see also Lucas & Saccucci, 1990; Douglas, 2005; Noorosanna *et al.* 2011), which is a chart that can detect well a smaller than 1.5σ change in process parameters, the statistics of EWMA control chart is

$$Y_t = \lambda X_t + (1-\lambda)Y_{t-1} \qquad t = 1,2,\ldots \quad (4)$$

where X_t are observed values of the distribution at any time t, λ is a smoothing parameter have a value between 0 and 1, Y_t is the weighted average between current and previous observations at time t. The mean and variance given by

$$E(Y_t) = \mu_0 \quad (5)$$

and

$$Var(Y_t) = \sigma^2\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2t}\right] \quad (6)$$

The control limits are

$$LCL/UCL = \mu_0 \pm K_1\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2t}\right]} \quad (7)$$

where K_1 is the coefficient of control limit of EWMA control chart. When $t \to \infty$ has values closer to 1, then

$$LCL/UCL = \mu_0 \pm K_1\sigma\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \quad (8)$$

The ZIB_{poi} EWMA will signal and out-of-control when $Y_t < LCL$ or $Y_t > UCL$.

2.3 Double Exponentially Weighted Moving Average (DEWMA) control chart

The DEWMA control chart was developed from the EWMA control chart by Shamma & Shamma (1992) presented the control chart with the following DEWMA (Z_t) statistic as

$$Z_t = \lambda_2 Y_t + (1-\lambda_2) Z_{t-1} \qquad t = 1,2,\ldots \tag{9}$$

where

$$Y_t = \lambda_1 X_t + (1-\lambda_1) Y_{t-1} \qquad t = 1,2,\ldots \tag{10}$$

Equation 9 and 10, given $\lambda_1 = \lambda_2 = \lambda$. Thus, the statistic of DEWMA is

$$Z_t = \lambda Y_t + (1-\lambda) Z_{t-1} \tag{11}$$

where Y_t is the statistics of EWMA. The mean and variance given by

$$E(X_t) = E(Y_t) = E(Z_t) = \mu_0 \tag{12}$$

and

$$Var(Z_t) = \left[\lambda^4 \frac{1 + (1-\lambda^2) - (t^2 + 2t + 1)(1-\lambda)^{2t} + (2t^2 + 2t - 1)(1-\lambda)^{2t+2} - t^2 (1-\lambda)^{2t+4}}{\left(1 - (1-\lambda)^2\right)^3} \right] \sigma^2$$

When $t \to \infty$ has values closer to 1,

$$Var(Z_t) = \left[\frac{\lambda(2 - 2\lambda + \lambda^2)}{(1-\lambda)^3} \right] \sigma^2 \tag{13}$$

Therefore, the control limits of the DEWMA control chart are

$$LCL / UCL = \mu_0 \pm K_2 \sigma \sqrt{\frac{\lambda\left(2 - 2\lambda + \lambda^2\right)}{\left(2-\lambda\right)^3}} \tag{14}$$

where K_2 is the coefficient of control limit of DEWMA control chart. Given $LCL = 0$, because the DEWMA control chart was used to detect only positive changes on one side.

2.4 Average Run Length of DEWMA Control Chart

The ZIB_{poi} EWMA and ZIB_{poi} DEWMA control charts are plotted on the control charts and the process is considered out-of-control if the plotted point lies outside the LCL and UCL. The Average Run Length (ARL) is mostly used to measure the performance of control charts. ARL represents the average number of observations required to be monitored until an out-of-control process is detected for the first time. There are two criteria for ARL: ARL_0 for an in-control process and ARL_1 for an out-of-control process. The control chart with the lowest ARL_1 is considered the best chart. ARL has been evaluated by many researchers to compare the performance of control charts (Chananet *et al.*, 2014; Petcharat *et al.*, 2015; Phantu & Sukparungsee, 2020).

In this paper, develop ARL estimates using the Markov chain approach (MCA) for EWMA and DEWMA control chart. Our approach is based on a zero-state Markov chain approximation, proposed by Lucas & Saccucci (1990).

The observation is defined as x_j where $j = 1, 2, \ldots, k$ is in-control process and $j = k+1$ is out-of-control process

Let $Y_t = i$, the control chart in the control state V_i , which each the control chart can be described as a random move to states $V_0, V_1, \ldots, V_N$ where the V_N is an absorbing state to represents an out-of-control region above the control limit and V_0 is an initial state. The transition probability (P_{ij}) is the probability of moving from state i to state j where $i, j = 1, 2, \ldots, N$. We can write the transition matrix (P) and element of matrix (P_{ij}) as

$$\boldsymbol{P}_{ij} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1N} & | & P_{1,N+1} \\ P_{21} & P_{22} & \cdots & P_{2N} & | & P_{2,N+1} \\ \vdots & \vdots & \ddots & \vdots & | & \vdots \\ P_{N1} & P_{N2} & \cdots & P_{NN} & | & P_{N.N+1} \\ -- & -- & -- & -- & | & -- \\ 0 & 0 & \cdots & 0 & | & 1 \end{bmatrix}$$

or

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{R} & (\boldsymbol{I}_N - \boldsymbol{R})\boldsymbol{1}_N \\ \boldsymbol{0}_N^T & 1 \end{bmatrix} \tag{15}$$

where sub matrix $\boldsymbol{R}$ is the transition probability between in-control states with $N \times N$ dimension, $\boldsymbol{I}$ is the identity matrix $N \times N$, $\mathbf{1}$ is a column vector of ones $N \times 1$, and vector $\mathbf{0}$ with $1 \times N$ dimension, implies that the Markov chain cannot reach any in-control state starting from the absorbing state.

Table 1 ARL_1 of $DEWMA_{ZIB\_poi}$ versus $EWMA_{ZIB\_poi}$ by MCA Method where $\lambda = 0.05$, ARL_0 = 370.

shifts	φ shifts				ξ shifts			
	$\theta = 0.05$		$\theta = 0.10$		$\theta = 0.05$		$\theta = 0.10$	
	EWMA (K_1=1.3787)	DEWMA (K_2=0.5882)	EWMA (K_1=1.8036)	DEWMA (K_2=0.4607)	EWMA (K_1=1.3787)	DEWMA (K_2=0.5882)	EWMA (K_1=1.8036)	DEWMA (K_2=0.4607)
0.01	366.97	**364.75**	365.99	**362.70**	369.78	**369.40**	369.59	**368.95**
0.03	360.19	**354.04**	357.24	**348.38**	368.65	**367.52**	368.15	**366.42**
0.05	353.38	**343.93**	348.39	**334.96**	367.52	**365.65**	366.70	**363.91**
0.07	346.55	**334.37**	339.50	**322.39**	366.38	**363.80**	365.23	**361.42**
0.1	336.34	**320.99**	326.17	**304.96**	364.67	**361.03**	363.02	**357.72**
0.3	272.15	**253.59**	244.81	**221.79**	352.94	**343.30**	347.82	**334.13**
0.5	219.77	**210.31**	184.28	**173.32**	340.77	**326.67**	331.96	**312.33**
1	**138.58**	149.85	**103.01**	112.95	308.99	**289.41**	290.81	**264.95**
2	**74.82**	99.83	**51.04**	69.50	244.30	**229.79**	211.85	**194.60**

Note: The bold is minimal of ARL of control chart.

Table 2 ARL_1 of $DEWMA_{ZIB\_poi}$ versus $EWMA_{ZIB\_poi}$ by MCA Method where $\lambda = 0.10$, ARL_0 = 370.

shifts	φ shifts				ξ shifts			
	$\theta = 0.05$		$\theta = 0.10$		$\theta = 0.05$		$\theta = 0.10$	
	EWMA (K_1=1.1395)	DEWMA (K_2=1.2633)	EWMA (K_1=1.4192)	DEWMA (K_2=0.7812)	EWMA (K_1=1.1395)	DEWMA (K_2=1.2633)	EWMA (K_1=1.4192)	DEWMA (K_2=0.7812)
0.01	**361.62**	363.39	366.95	**365.40**	**368.75**	368.90	369.51	**369.33**
0.03	**345.32**	350.78	360.63	**355.85**	**365.87**	366.67	368.49	**367.76**
0.05	**330.08**	338.96	354.09	**346.26**	**363.00**	364.46	367.46	**366.17**
0.07	**315.82**	327.85	347.36	**336.68**	**360.16**	362.27	366.41	**364.57**
0.1	**296.10**	312.42	336.98	**322.46**	**355.95**	359.01	364.83	**362.16**
0.3	**202.22**	237.15	365.79	**240.10**	**329.14**	338.22	353.67	**345.64**
0.5	**148.61**	191.17	204.14	**183.36**	**304.44**	318.96	341.53	**328.61**
1	**87.31**	130.30	111.88	**110.46**	**250.87**	276.64	307.69	**285.71**
2	**47.75**	83.93	**51.73**	61.93	**171.90**	211.62	233.12	**208.81**

Note: The bold is minimal of ARL of control chart.

Table 3 ARL_1 of $DEWMA_{ZIB\_poi}$ versus $EWMA_{ZIB\_poi}$ by MCA Method where $\lambda = 0.05$, ARL_0 = 500

shifts	φ shifts				ξ shifts			
	$\theta = 0.05$		$\theta = 0.10$		$\theta = 0.05$		$\theta = 0.10$	
	EWMA (K_1=1.384)	DEWMA (K_2=0.42646)	EWMA (K_1=1.8313)	DEWMA (K_2=0.42632)	EWMA (K_1=1.384)	DEWMA (K_2=0.42646)	EWMA (K_1=1.8313)	DEWMA (K_2=0.42632)
0.01	397.60	**493.94**	497.14	**492.37**	499.72	**498.99**	499.89	**498.75**
0.03	491.66	**481.92**	490.11	**477.00**	499.01	**496.96**	498.80	**496.19**
0.05	485.45	**470.07**	482.50	**461.71**	498.29	**494.92**	497.69	**493.62**
0.07	478.99	**458.45**	474.37	**446.62**	497.56	**492.89**	496.56	**491.03**
0.1	468.88	**441.48**	461.34	**424.58**	496.44	**489.83**	494.82	**487.14**
0.3	394.87	**345.75**	363.10	**305.21**	488.30	**469.32**	482.00	**460.73**
0.5	323.09	**278.34**	274.13	**230.25**	478.97	**448.78**	467.12	**434.06**
1	194.97	**183.89**	145.13	**139.49**	450.53	**398.55**	422.28	**369.59**
2	**99.92**	111.88	**65.30**	80.04	375.77	**308.98**	315.93	**263.25**

Table 4 ARL_1 of $DEWMA_{ZIB\_poi}$ versus $EWMA_{ZIB\_poi}$ by MCA Method where $\lambda = 0.10$, ARL_0 = 500.

shifts	φ shifts				ξ shifts			
	$\theta = 0.05$		$\theta = 0.10$		$\theta = 0.05$		$\theta = 0.10$	
	EWMA (K_1=0.9884)	DEWMA (K_2=1.062)	EWMA (K_1=1.422)	DEWMA (K_2=0.847)	EWMA (K_1=0.9884)	DEWMA (K_2=1.062)	EWMA (K_1=1.422)	DEWMA (K_2=0.847)
0.01	**487.92**	489.69	497.24	**492.79**	**498.19**	498.44	499.82	**499.26**
0.03	**464.51**	469.73	490.69	**477.22**	**494.03**	494.90	498.79	**496.66**
0.05	**442.72**	451.16	483.69	**461.67**	**489.90**	491.38	497.75	**494.06**
0.07	**422.42**	433.84	476.28	**446.28**	**485.82**	487.90	496.69	**491.44**
0.1	**394.47**	409.99	464.47	**423.71**	**479.76**	482.73	495.06	**487.50**
0.3	**263.41**	297.33	374.12	**299.32**	**441.38**	450.01	483.23	**460.68**
0.5	**189.61**	231.80	289.97	**220.22**	**406.27**	420.06	469.70	**433.43**
1	**105.67**	149.49	150.12	**126.28**	**330.95**	355.64	429.05	**366.81**
2	**55.44**	90.71	**63.81**	68.50	**221.64**	260.62	328.67	**255.04**

Note: The bold is minimal of ARL of control chart.

An approximation of the ARL using the MCA to detect the mean changes of a process is in the interval of the lower control limit and upper control limit. The region of the in-control state is divided into *k* subintervals.The j^{th} subinterval of the upper control limit (U_j), j^{th} subinterval of the lower control limit (L_j) and the i^{th} subinterval of the midpoint (m_i) are given by:

$$U_j = h_L + \frac{j(h_U - h_L)}{k} \qquad (16)$$

$$m_j = h_L + \frac{(2i-1)(h_U - h_L)}{2k} \qquad (17)$$

$$L_j = h_L + \frac{(j-1)(h_U - h_L)}{k} \qquad (18)$$

Where, h_L is *LCL* and h_u = *UCL*.

Consequently, the transition probability equation (P_{ij}) can be rewritten as

$$P_{ij} = P(L_j \le Z_t \le U_j \,|\, Z_{t-1} = m_j) \qquad (19)$$

form eq. 4 and 10 replace to eq. 18.

$$P_{ij} = P(L_j < \lambda Y_t + (1-\lambda)Z_{t-1} < U_j \,|\, Z_{t-1} = m_j)$$

$$= P(L_j < \lambda Y_t + (1-\lambda)m_j < U_j)$$

$$= P(L_j < \lambda(\lambda X_t + (1-\lambda)m_j) + (1-\lambda)m_j < U_j)$$

$$= P\left(L_j < \lambda^2 X_t + (1-\lambda^2)m_j < U_j\right) \qquad (20)$$

Repalce eq.16 – 18 into eq. 20. This transition probability equation is

$$= P\left(h_L + \frac{(j-1)(h_U - h_L)}{k} < \lambda^2 X_t + (1-\lambda^2)\left(h_L + \frac{(2i-1)(h_U - h_L)}{2k}\right) < h_L + \frac{j(h_U - h_L)}{k}\right)$$

$$= P\left(\begin{matrix} h_L + \frac{(j-1)(h_U - h_L)}{k} - (1-\lambda^2)\left(h_L + \frac{(2i-1)(h_U - h_L)}{2k}\right) < \lambda^2 X_t < \\ h_L + \frac{j(h_U - h_L)}{k} - (1-\lambda^2)\left(h_L + \frac{(2i-1)(h_U - h_L)}{2k}\right) \end{matrix}\right)$$

$$= P\left(\lambda^2 h_L + \frac{(h_U - h_L)}{2k}2(j-1) - (1-\lambda^2)(2i-1) < \lambda^2 X_t < (\lambda^2 h_L + \frac{(h_U - h_L)}{2k}2j - (1-\lambda^2)(2i-1)\right)$$

$$= P\left(h_L + \frac{(h_U - h_L)}{2\lambda^2 k}2(j-1) - (1-\lambda^2)(2i-1) < X_t < (h_L + \frac{(h_U - h_L)}{2\lambda^2 k}2j - (1-\lambda^2)(2i-1)\right) \qquad (21)$$

2.5 Performance of DEWMA and EWMA control charts

The properties average run length of the $DEWMA_{ZIB\_poi}$ is analyzed within the Markov chain approach simulation study. Using Equation (10) (12) and (13) to analytical ARL, we set $\varphi_0 = 2$ and $\xi_0 = 12$, given ARL_0 = 370 and 500, varied with different shift sizes: 0.01, 0.03, 0.05, 0.07, 0.1, 0.3, 0.5, 1 and 2. We are interested only in the proportion increase, the shift increasing for φ_1 and ξ_1 decreasing, given θ and λ is 0.05, 0.10. The ARL_1 shown as tables 1 – 4 between $EWMA_{ZIB\_poi}$ versus $DEWMA_{ZIB\_poi}$. The simulation was repeated 100,000 times for each case by using R programming.

In table 1, consider the ZIB_{poi} model with $\lambda = 0.05$, θ = 0.05 and 0.10 at ARL_0 = 370. The numerical results DEWMA detect change more efficient than EWMA at the shift of parameter less than equal 0.5 for φ, and every shift of parameter for ξ.

In table 2, consider the ZIB_{poi} model with $\lambda = 0.10$, ARL_0 = 370. The numerical results

EWMA detect change more efficient than DEWMA for all the shift of parameter at θ=0.05, and DEWMA detect change more efficient than EWMA at θ=0.10. In table 3, consider the ZIB_{poi} model with $\lambda = 0.05$, θ= 0.05 and 0.10 at ARL_0 = 500. The numerical results DEWMA detect change more efficient than EWMA at the shift of parameter less than equal 1 for φ , and every shift of parameter for ξ .

In table 4, consider the ZIB_{poi} model with $\lambda = 0.10$, ARL_0 = 500. The numerical results EWMA detect change more efficient than DEWMA for all the shift of parameter at θ=0.05, and DEWMA detect change more efficient than EWMA at θ=0.10.

CONCLUSION

The paper presents DEWMA for ZIBpoi using with Markov chain approximation method. A table is provided to select the value of the chart limit coefficient that gives the desired in-control ARL. Numerical results indicate that DEWMA control chart for φ shifts gives an out-of-control signal quicker than EWMA control chart for small process shifts ($\delta \le 1\sigma$). When ξ shifts, the DEWMA control chart gives an out-of-control signal quicker than EWMA control chart for all cases. However, the DEWMA control chart is not chosen over the EWMA control chart when λ is greater than θ, as it gives an out-of-control signal slower than EWMA control chart.

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