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Research Article

Non – Stationary Booking Demand in Two-Class Overbooking Model

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Abstract

Accurate demand forecasting is a crucial component of airline revenue management. However, it is cumbersome to find exact forecasting demand since the historical data, the censor data, do not reflect the actual demand. In order to obtain estimated demand, the Expectation-Maximization (EM) method is used to calculate the estimated demand. Furthermore, the previous research mainly focuses on the stationary demand, which does not represent an actual situation. So, this research engages in finding the optimal overbooking and booking limit of the Two-Class overbooking with non-stationary demand that maximizes the expected profit using numerical experiments. In addition, the performance of the Two-Class overbooking model with non-stationary demand is compared with the booking limit of the airline's policy. The optimal number of update booking limits with non – stationary demand using real data are concerned.

The simulation finds that the optimal booking calculated mostly is the booking limit. However, the booking limit is suited to the overbooking limit in some situations. Moreover, the expected profit from the optimal overbooking limit is greater than or equal to the airline's booking policy in all study situations. The Two-Class overbooking model yielded up to 21% more the expected profit margins than airline booking policies. In real data study, the Two-Class overbooking model is taken to calculate the optimal number of update booking limits when the demand is non – stationary and it is found that the 16-point update booking limit gives the maximum profit.

Keywords: Revenue Management, Overbooking, Two-Class overbooking, Non-stationary demand, Expectation-Maximization.

1. Introduction

Revenue management (RM) is a well-known application of mathematical modeling mostly employed in the airline industry to improve profitability with high fixed costs and low marginal costs. RM is defined as "selling the correct product to the correct customer at the correct time for the proper price" (Cross,1997). The aviation industry is an intensely competitive industry that constantly engages in overbooking to reinforce its revenues, taking advantage of the fact that customers cancel airfare reservations or do not show up for flights (Suzuki, 2006). The concept is straightforward. Airlines sell seats by creating different fare classes carrying different restrictions and prices.

The Air Transport Information Division of Airport of Thailand Public Company Limited (AOT) reported that the total air traffic, aircraft movement, and passenger numbers in Thailand increased by 21, 22, and 21 percent from 2015 to 2019. So, Thailand can increase revenue by roughly $200 million US.

Unfortunately, after the World Health Organization (WHO) declared the COVID-19 outbreak a pandemic since March 2020, daily life across the world has changed. The aviation industry has been one of the hardest-hit industries globally since the beginning of the outbreak. Some airlines would evolve during a crisis like no others bringing the industry into survival mode, diminished by a loss of traffic and revenues. While positive signs for recovery, COVID-19 remains an existential crisis for airports, airlines, and business sectors. The airline companies in Thailand are also affected by the pandemic outbreak; however, they could benefit from using RM strategies when the pandemic is better.

The Two-Class model, which concentrates on the booking control problem, dates back to Littlewood (1972). All booking requests show up at the time of service. Littlewood (1972) does not allow overbooking, which means there are no cancellations or no-shows. The Two-Class overbooking model proposed by Somboon and Amaruchkul (2016) is taken into account in this study. The model combines two strategies, overbooking and seat inventory, and proposes a static Two-Class overbooking model, in which low fare (Class-2) passengers arrive prior to high fare (Class-1) passengers. The airline incurs a penalty cost, any of which rejected booking requests. Also, the penalties are different for the two classes. The airline may overbook Class-2 customers. The two fare classes may have different show-up rates. We refer an interested reader to Somboon and Amaruchkul (2016) and also the literature cited therein for more details on the combined Two-Class overbooking model.

Accurate demand forecasting is a vital component in RM. The booking data of departure flights are used to forecast the demand for future departing flights. However, the previous research generates only the stationary demand, which does not represent an actual situation. A pattern in which demand is not constant for each time but varies due to seasonality, trend or other factors, the non-stationary demand, is considered. For example, a booking demand changes according to the number of days close to the travel date. The closer the travel date is, the more demand for booking. This research mainly focuses on finding the optimal overbooking and booking limit that maximizes the expected profit using numerical experiments. Moreover, the performance of the Two-Class overbooking model with
non-stationary demand is compared with the airline's policy booking limit. Finally, find the optimal number of update booking limits when the demand is non-stationary with real data.

The paper is organized, as follows: Section 2 briefly reviews the Two-Class overbooking model. Section 3 details the numerical setting and the procedure for simulation. Section 4 presents numerical results of the numerical study. Real data is applied in Section 5, and Section 6 presents conclude.

2. Two-Class Overbooking Model

In 2016, Somboon and Amaruchkul propose the two-class overbooking model by the following these information.

Let $\mathbb{Z}_+$ and $\mathbb{R}$ be the set of non-negative integers and the real number set respectively. Also, given t is the number of days until departure and suppose that $(y)^+ = \max(0, y)$ for $y \in \mathbb{R}$. Let $D(t)$ be the distribution function of a random variable at t days before departure. The quantile function of $D(t)$ can be written as

$$F^{-1}_{D(t)}(a) = \inf\{x : P(D(t) \le x) \ge a\}$$

Class-2 reservations are assumed to begin a booking before Class-1 reservations in this model. When a Class-i customer is accepted, the airline earns revenue p_i, $p_1 > p_2 > 0$, for each $i = 1,2$. If the airline rejects the request, it will incur a penalty cost g_i where $g_1 > g_2 > 0$. In this model, the penalty cost includes both opportunity cost and loss-of-goodwill. Loss of business reputation or the loss-of-goodwill determines the customer satisfaction that can be intangible and cumbersome to gauge in practice.

The opportunity cost is a measurement of future revenue loss based on what happens after the lost sales occur. If a customer is prone to return to make a request, the opportunity cost is the expected loss of revenue from this situation. On the other hand, if a customer never returns to make another reservation with the airline, the opportunity cost includes all future revenues.

In practice, an optimal booking limit is determined on a proportion of refund cost over time and change in show-up probability. It can be affected by overbooking limits that vary over time. Assume that $x(t) \in ¢_+$ is a Class-2 booking limit at the update booking limit t days prior to departure. It implies that the reservations for Class-2 are accepted up to $x(t)$. Overbooking is permitted if $x(t)$ is greater than capacity $k(t)$ at the update booking limit point t. Let $D_i(t)$ be the demand of Class-i booking at the update booking limit point t for each $i = 1,2$. The Class-2 reservations and rejected quantity are $\min(x(t), D_2(t))$ and $(D_2(t) - x(t))^+$ at the update booking limit point t, respectively.

After all Class-2 reservations have arrived, Class-1 customers begin their booking so that the available capacity for Class-1 customers will be $(k(t) - \min(x(t), D_2(t)))^+$. However, due to high penalty costs and high priority, we do not overbook Class-1 passengers, so they are accepted up to the remaining capacity. The number of Class-i reservations at the updated booking limit point t can be written as:

$$B_1(x(t)) = \min(k(t) - B_2(x(t)))^+, D_1(t)),\ B_2(x(t)) = \min(x(t), D_2(t)).$$

Some customers may not show up for or cancel their reservations before departure. Furthermore, we considered that both no-show and cancellation customers are the same in this model. Let $B_i(x(t)) = y_i$ be the number of Class-i reservations and $W_i(y_i)$ be the number of Class-i show-ups. Assume $W_i(y_i)$ is a Class-i show-up probability with a binomial distribution with parameters y_i and q_i, where $q_i \in (0,1]$ (Sawaki, 1989; Ringbom and Shy, 2002; and Somboon and Amaruchkul, 2016). Tasman Empire Airways demonstrated that using the binomial distribution is a suitable model for the show-up distribution (Thompson, 1961). Assume that r_i is a refund for no-show Class-i reservation for $i = 1,2$, where $g_i \in (0,1);\ r_i = g_i p_i$, which g_i is a proportion of the revenue cost.

Some passengers are denied at the departure if the number of show-up passengers exceeds capacity. As mentioned above that we do not overbook Class-1 customers, so all denied boarding passengers are Class-2. The compensation that the airline has to pay all passengers who were denied boarding is h, where $h > p_2$. However, it could include a higher fare class ticket for the next flight, cash vouchers, or hotel accommodations.

An optimal booking limit at the update booking limit point $x^*(t)$ that maximizes its expected profit is preferred:

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$$\pi(x(t)) = E\left[\sum_{i=1}^{2}[p_i B_i(x(t)) - r_i(B_i(x(t)) - W_i(B_i(x(t))))]\right] - E\left[h(W_2(B_2(x(t))) - \kappa(t))^+\right]$$
$$- E\left[g_2(D_2(t) - x(t))^+ + g_1(D_1(t) - (\kappa(t) - B_2(x(t)))^+)^+\right].$$

From the equation above, it can be seen that there are two uncertainty parts which are demand and the number of show-up customers. However, we are capable find a closed form of the optimal booking limit. Finding a closed form of the optimal booking limit can use the theorem below (Somboon and Amaruchkul, 2016).

Theorem 1. For $x = 0,1,...,\kappa-2$ and $x = \kappa,\kappa+1,...$, the expected profit function $\pi(x)$ is piecewise is unimodal in each piece. The expected profit $\pi(x)$ has a local maximum point x' on $x = 0,1,...,\kappa-2$ as

$$x' = \begin{cases} 0 & ; 0 \le \tau < P(D_1 > \kappa-1) \\ \kappa - F_{D_1}^{-1}(1-\tau) & ; P(D_1 > \kappa-1) \le \tau \le P(D_1 > 0) \\ \kappa-2 & ; P(D_1 > 0) < \tau \le 1 \end{cases}. \tag{1}$$

On the other hand, if $0 < \alpha_2/(h\theta_2) < \bar{F}(\kappa-1;\kappa,\theta_2)$, then the expected profit $\pi(x)$ has a local maximum point x'' for $x = \kappa,\kappa+1,...$. It can be written as

$$x'' = \arg\min\{x \in \{\kappa,\kappa+1,...\} : \bar{F}(\kappa-1;x,\theta_2) > \frac{\alpha_2}{h\theta_2}\}. \tag{2}$$

Otherwise, the expected profit function is increasing.

3. Numerical Illustration

As mentioned in the introduction, accurate demand forecasting is a key to gauging optimal booking and overbooking. It is cumbersome to find exact forecasting demand since the historical data, which is the censor data, do not reflect the actual demand. That is, the booking limit determines how many seats can be sold on a flight. A booking in a fare class is accepted by an airline until the booking limit is reached. The airline then stops selling seats in that fare class. It also ceases to collect valuable data. Demand for travel in that fare class may exceed the booking limit, but the data does not reflect this, as the booking limit is censored or "constrained". From this cause, true demand cannot obtain directly, which can only obtain estimated demand. In order to obtain estimated demand, the unconstraining method is used to calculate the estimated demand. In quantity-based RM, the Expectation – Maximization (EM) method is the most commonly used for correcting for constrained data with four basic steps: (i) replace missing values with estimated values, (ii) estimate parameters, (iii) re-estimate the missing values assuming the new parameter estimates are correct, (iv) re-estimate the parameters, and iterating until convergence (Talluri et al, 2004: 474-475).

Like Somboon and Amaruchkul (2016), we assume that the demand of seats for each class in a year follows Poisson distribution. The Poisson variable is assumed in numerical experiments; however, the proof in Theorem 1 does not need to assume Poisson distribution but holds for any non-negative random variable.

In this experiment, we generate non-stationary booking demand by dividing the booking demand into four quarters. Let λ_1 and λ_2 be initial parameters referring the seat demand of customers class-i; i =1,2 increasing 5 percent each quarter. We assume that λ_1 = 40, 50, 60, 70, and λ_2 = 90, 100, 110, 120, respectively. The plane's capacity is given by κ = 162 and the fare for class-i; i =1,2 is denoted by $p_1 = 3{,}043$ and $p_2 = 945$, respectively. r_i is the refund for

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passenger in each class where $r_i = 0.5p_i$ and $r_i = 0.8p_i$. The show-up probability θ_i for customer class-i; i =1,2 are 0.7 and 0.9, respectively. Also, assume that the compensation cost h = 2,000 Baht must be paid to all passenger when the number of passengers exceeds capacity.

The EM method is used to estimate the censored demand data then uncensored it using the unconstraining method. In this setting, we partition the demand into two classes by employing the demand function proposed by Komsan Suriya (2009) for airline industry in Thailand. Given that is the demand function.

Class-1 demand function

$$p_1 = 3,588 - 188.605q_1. \tag{3}$$

Class-2 demand function

$$p_2 = 1,763 - 188.605q_2. \tag{4}$$

where q_i is the demand for class-i, for i =1,2.

After substituting $p_1 = 3,043$ in (3) and $p_2 = 945$ in (4), we obtain the demand for Class-1 and Class-2 as, $q_1 = 4.6$ and $q_2 = 6.9$ million customers, respectively. The proportion of Class-1 (υ_1) and Class-2 (υ_2) passengers is

$$\upsilon_1 = 0.4 \text{ and } \upsilon_2 = 0.6. \tag{5}$$

We considered the proportions in (5) to divide the number of reservations into two classes.

Next, the booking and overbooking limit is gauged using the Two-Class overbooking model, then calculated the expected profit from the booking and overbooking limit. Repeat 1,000 iterations for all the steps and average the profit. The average profit between the Two-Class overbooking model and the airline's policy are compared.

4. Numerical Result

In this experiment, we simulate non-stationary booking demand detailed in the section 3. Then, we estimate censored booking demand data using EM method, and the Two-Class overbooking model is used to find the optimal overbooking for non-stationary demand. The expected profit in each situation can be summarized below.

4.1 The optimal overbooking and the expected profit of the Two-Class overbooking model for non-stationary booking demand.

Table 1 shows that the optimal booking demand when λ_1 =40 and λ_2 = 90 for all r_i and θ_i is the booking limit. Said that the airlines should accept Class-2 customers to make bookings as booking limit to achieve the highest expected profit.

Likewise, the optimal booking limit for λ_1 = 50 and λ_2 = 100 for all r_i and θ_i is the booking limit. The airlines have to open the reservation for Class-2 customers as booking limit to exceed the highest expected profit. The result as shown in Table 2.

Table 1 The optimal overbooking and the expected profit when λ_1 =40 and λ_2 =90

r_1	r_2	θ_1	θ_2	**Optimal booking limit**	**Expected profit**
1521.5	472.5	0.7	0.7	103	209,278.85
1521.5	472.5	0.7	0.9	104	217,153.85
1521.5	472.5	0.9	0.7	103	226,188.63
1521.5	472.5	0.9	0.9	103	234,063.63

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Table 1 The optimal overbooking and the expected profit when λ_1 =40 and λ_2 =90 (Cont.)

r_1	r_2	θ_1	θ_2	Optimal booking limit	Expected profit
1521.5	756	0.7	0.7	103	202,201.59
1521.5	756	0.7	0.9	104	214,801.59
1521.5	756	0.9	0.7	102	219,111.37
1521.5	756	0.9	0.9	103	231,711.37
2434.4	472.5	0.7	0.7	104	194,299.68
2434.4	472.5	0.7	0.9	105	202,174.68
2434.4	472.5	0.9	0.7	103	221,355.33
2434.4	472.5	0.9	0.9	104	229,230.33
2434.4	756	0.7	0.7	103	187,222.42
2434.4	756	0.7	0.9	105	199,822.42
2434.4	756	0.9	0.7	102	214,278.07
2434.4	756	0.9	0.9	103	226,878.07

While the result in both Table 1 and Table 2 are booking limit, the results in Table 3 show that the optimal booking limit for λ_1 = 60 and λ_2 = 110 for all r_i and θ_i also booking limit almost all the situations. There is one situation when r_1 **=** 2,434.4, r_2 **=** 472.5, θ_1 **=** 0.9 and θ_2 **=** 0.7 that is the overbooking. As a result, to reach the highest expected profit, the airline should open booking for Class-2 customers to book as same as optimal overbooking limit.

Table 2 The optimal overbooking and the expected profit when λ_1 =50 and λ_2 =100

r_1	r_2	θ_1	θ_2	Optimal booking limit	Expected profit
1521.5	472.5	0.7	0.7	94	238,360.92
1521.5	472.5	0.7	0.9	95	247,094.70
1521.5	472.5	0.9	0.7	93	258,224.33
1521.5	472.5	0.9	0.9	94	266,985.67
1521.5	756	0.7	0.7	93	231,836.35
1521.5	756	0.7	0.9	95	245,376.37
1521.5	756	0.9	0.7	93	251,370.72
1521.5	756	0.9	0.9	94	265,418.54
2434.4	472.5	0.7	0.7	95	221,204.43
2434.4	472.5	0.7	0.9	96	230,281.05
2434.4	472.5	0.9	0.7	94	252,450.86
2434.4	472.5	0.9	0.9	94	261,541.23
2434.4	756	0.7	0.7	94	214,174.52
2434.4	756	0.7	0.9	96	228,424.91
2434.4	756	0.9	0.7	93	245,561.12
2434.4	756	0.9	0.9	94	259,974.11

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Table 3 The optimal overbooking and the expected profit when λ_1 =60 and λ_2 =110

r_1	r_2	θ_1	θ_2	**Optimal booking limit**	**Expected profit**
1521.5	472.5	0.7	0.7	88	257,480.99
1521.5	472.5	0.7	0.9	89	267,740.80
1521.5	472.5	0.9	0.7	87	278,028.95
1521.5	472.5	0.9	0.9	88	288,288.76
1521.5	756	0.7	0.7	87	253,252.38
1521.5	756	0.7	0.9	89	269,292.18
1521.5	756	0.9	0.7	86	274,082.26
1521.5	756	0.9	0.9	88	290,122.06
2434.4	472.5	0.7	0.7	89	239,066.52
2434.4	472.5	0.7	0.9	90	249,321.08
2434.4	472.5	0.9	0.7	233	271,935.23
2434.4	472.5	0.9	0.9	88	282,015.11
2434.4	756	0.7	0.7	88	234,555.98
2434.4	756	0.7	0.9	90	250,593.68
2434.4	756	0.9	0.7	87	267,996.56
2434.4	756	0.9	0.9	88	283,848.41

Table 4 The optimal overbooking and the expected profit when λ_1 =70 and λ_2 =120

r_1	r_2	θ_1	θ_2	**Optimal booking limit**	**Expected profit**
1521.5	472.5	0.7	0.7	88	257,397.69
1521.5	472.5	0.7	0.9	89	267,316.26
1521.5	472.5	0.9	0.7	87	278,209.29
1521.5	472.5	0.9	0.9	88	288,127.85
1521.5	756	0.7	0.7	87	253,217.64
1521.5	756	0.7	0.9	89	268,709.34
1521.5	756	0.9	0.7	86	274,312.74
1521.5	756	0.9	0.9	88	289,804.44
2434.4	472.5	0.7	0.7	89	238,719.58
2434.4	472.5	0.7	0.9	90	248,638.14
2434.4	472.5	0.9	0.7	233	272,462.99
2434.4	472.5	0.9	0.9	180	281,909.05
2434.4	756	0.7	0.7	88	234,256.03
2434.4	756	0.7	0.9	90	249,747.73
2434.4	756	0.9	0.7	87	268,121.58
2434.4	756	0.9	0.9	88	283,424.28

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However, Table 4 shows that the optimal booking limit for $\lambda_1 = 70$ and $\lambda_2 = 120$ have both booking limit and overbooking limit. In this case, if the airline wants to succeed the maximum expected profit, the airline have to adjust the optimal booking for all r_i and θ_i; especially, in the situation that the booking limit is greater than the capacity. It means that the airline can only open booking for Class-2 customers.

The numerical study investigates that booking limit and overbooking limit provide the same expected profit in some situations. In this case, we recommend that the airline use a booking limit to sell both classes' flight tickets since the show-up probability of each class is uncertain in real situations. Generally, the show-up probability of Class-2 passengers is less than Class-1 passengers because of a cheaper flight fare, i.e., booking during promotion time. Class-2 passengers are most likely to terminate the fare without notice. Because of this reason, the airline should allow both Class-1 and Class-2 passengers to book flight tickets.

4.2 The comparison between the Two-class overbooking for both classes passengers to the airline's policy when the booking demand is non-stationary

For performance comparison, we assume that the expected profit of the airline's policy has a fixed booking limit for all periods, which are 9, 17, 41, 81, 122, and 171, which are 5%, 10%, 25%, 50%, 75%, and 105% of capacity, respectively. Otherwise, we investigate that the expected profit of the Two-Class overbooking model is greater than or equal to the airline's booking policy for all situations. The airline will get less advantage when the airline booking limit (ABL) is 9, followed by 17, 41, 81, 122, and 171, respectively.

Table 5 Loss Profit per Flight when λ_1 = 40 and λ_2 = 90

r_1	r_2	θ_1	θ_2	ABL	Loss Profit per Flight	Percentage of Loss Profit per Flight
2434.4	472.5	0.9	0.7	9	34,810.07	18.66
				17	31,030.07	16.30
				41	19,690.07	9.76
				81	1,903.07	0.87
				122	0.00	0.00
				171	0.00	0.00
1521.5	472.5	0.9	0.9	9	34,810.07	17.90
				17	31,030.07	15.66
				41	19,690.07	9.40
				81	1,903.07	0.84
				122	0.00	0.00
				171	0.00	0.00
2434.4	756	0.9	0.7	9	13,842.88	6.91
				17	12,330.88	6.11
				41	7,794.88	3.78
				81	680.08	0.32
				122	0.00	0.00
				171	0.00	0.00
2434.4	756	0.9	0.9	9	13,842.88	6.50
				17	12,330.88	5.75
				41	7,794.88	3.56
				81	680.08	0.30
				122	0.00	0.00
				171	0.00	0.00

Table 5 shows the loss profit per flight when λ_1 = 40 and λ_2 = 90 for all r_i and θ_i. It can be seen that the airline earns less advantage when the airline booking limit is 9, followed by 17, 41, 81, 122, and 171, respectively. Likewise, when $\lambda_1 = 50$ and $\lambda_2 = 100$ for all r_i, θ_i, and the airline booking limit, the airline losses its profit as shown in A.1 in Appendix (see others situation result in Appendix)

It concludes that if the airline open the observation for Class-2 customers in small amounts, the airline will get less profit. However, when the Two-Class overbooking model is applied, the model calculates the optimal booking limit suited for the situations. Consider the difference between the expected profit from the Two-Class overbooking model and the airline's booking policy; it can be seen that the differences in the profit are approximately 0-21%.

Practically, most airlines update their booking limit every month before departure for at least six months and then update every day in the last week before departure (Phillips, 2005). The following section will find the optimal number of update booking limits when real data demand is non – stationary.

5. Real Data Study

Passengers' information from two flights was received from the airline. The flights are from Bangkok to Phuket on every Sunday in 2014. Flight A is scheduled to depart for Sunday morning and Flight B is set for Sunday evening. The data includes both the number of reservations and the number of passengers who showed up at the departure time. Table 6 shows that the plane has 162 seats and that the airline's revenue is divided into eleven classes.

Table 6 Percentage of passengers from 2014 reclassified by revenue.

Class	Revenue	Percent [a]
Y	4,675	10.64
M	4,350	4.97
K	3,850	13.82
N	3,500	3.72
T	3,050	20.47
L	2,800	5.27
H	2,550	8.02
Q	2,100	4.47
V	1,900	9.23
G	1,690	12.58
B	945	6.71

Note: a. The Percentage of passengers in Table 6 was received from the airline company in Thailand where the name has not been disclosed to protect its and interviewees' privacy.

In this setting, the eleven classes were reclassified into two classes in order to use the Two-Class overbooking model. So, the revenue for Class-1 and Class-2 after rearrange is $p_1 = 3,043$ and $p_2 = 945$.

We established four different cases of updated booking limit points. Firstly, the booking limit is updated every day in the final week before departure; in this case, there is seven points update booking limit. Similarly, the booking limit is updated every month for the next six months and every day in the final week before departure. It can be assumed that there are thirteen points for updating the booking limit. The booking limit is then updated every three months before departure more than six months in advance, and the updated booking limit is considered the second case, giving this case fourteen updating points. Finally, we consider that the booking limit is updated every month and every day in the last week before departure, which can be counted as sixteen points to update the booking limit. Let b_i be the number of class-i booking at the departure time for $i=1,2$ and $W_i(b_i)$ be the number of class-i show-up customers. The profit can be written as

$$\hat{\pi} = \sum_{i=1}^{2}\left[p_i b_i - r_i (b_i - W(b_i))\right] - h\left(W_2(b_2) - \kappa\right)^{+}. \qquad (6)$$

Assume that t is the number of days before departure and x_t^* be the optimal booking limit at the updated booking limit points t before departure. The updated booking limit points are shown in Table 7.

Table 7 The Update Booking Limit Points for the Four Cases

Case	***t***
7 points	6, 5, 4, 3, 2, 1, 0
13 points	180, 150, 120, 90, 60, 30, 6, 5, 4, 3, 2, 1, 0
14 points	270, 180, 150, 120, 90, 60, 30, 6, 5, 4, 3, 2, 1, 0
16 points	330, 300, 270, 180, 150, 120, 90, 60, 30, 6, 5, 4, 3, 2, 1, 0

From the previous section, the Two-Classes overbooking can be used with the non – stationary demand using simulation technique. In this section, the real data is used to find the optimal number of update booking limits follow this steps:

1) Using real data, forecast demand $(\hat{\lambda})$ for the next flight using an exponential smoothing technique with a smoothing constant determined by minimizing the sum of squared errors.
2) Use the proportion of Class-1 and Class-2 passengers in (5) to divide the forecasted demand into two classes. The average demand of Class-1 and Class-2 are $\hat{\lambda}_{1,360}$ and $\hat{\lambda}_{2,360}$.
3) Generate the non – stationary demand for both Class-1 and Class-2 over 360 days, which is assumed to be a Poisson distribution with means $\hat{\lambda}_{1,360}$ and $\hat{\lambda}_{2,360}$, respectively.
4) Compute the optimal initial booking limit x_{360}^* using Theorem 1.
5) Compute the number of every day Class-1 and Class-2 bookings using x_{360}^*.
6) At update booking limit point *t*, recomputed the optimal booking limit x_t^* and forecast the two classes' demand $(\hat{\lambda}_{1,t}, \hat{\lambda}_{2,t})$ over the remaining days before departure.
7) Compute a new optimal booking limit x_t^* using the forecasted demand
8) Generate the number of show-up customers following binomial distribution with a mean equal to the number of class-i bookings and the show-up probability is θ_i, i = 1,2, then compute the profit using (6).
9) Repeat 1,000 iterations for all steps and average the profit.

In this experiment, the best solution of the optimal update booking limit point is 16, which gives the profit approximately 32% – 42% greater than 14 update booking limit points. Moreover, 7 and 13 points provide the same profit for all the situations. The optimal update booking limit of 13 point gives the highest profit in stationary demand situations, while the update

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booking limit of 16 point is more suitable in non-stationary demand situations. The result is shown in table 8.

Table 8 Profit of the Number of Update Booking Limit Points in different cases

θ_1	θ_2	**7 points**	**13 points**	**14 points**	**16 points**
0.7	0.7	234,015.76	234,015.76	236,970.50	323,450.99
0.7	0.8	238,740.76	238,740.76	241,448.38	325,017.75
0.7	0.9	241,859.89	241,859.89	244,772.03	326,587.10
0.7	0.95	244,222.39	244,222.39	246,992.78	327,351.43
0.8	0.7	251,067.90	251,067.90	254,572.05	354,704.28
0.8	0.8	255,822.95	255,822.95	259,015.01	356,275.45
0.8	0.9	260,547.95	260,547.95	263,492.42	357,835.87
0.8	0.95	262,437.95	262,437.95	265,458.02	358,598.11
0.9	0.7	268,274.24	268,274.24	272,197.11	385,984.22
0.9	0.8	272,526.74	272,526.74	276,399.52	387,536.86
0.9	0.9	277,251.74	277,251.74	280,876.93	389,096.20
0.9	0.95	277,874.48	277,874.48	281,841.54	389,847.83
0.95	0.7	279,104.43	279,104.43	282,587.31	401,629.10
0.95	0.8	281,928.39	281,928.39	285,674.59	403,193.53
0.95	0.9	286,653.39	286,653.39	290,152.00	404,747.25
0.95	0.95	288,543.39	288,543.39	292,117.60	405,500.98

6. Conclusion

Simulating the non-stationary booking demand data to calculate the optimal booking limit using the Two-Class overbooking model finds that the optimal booking calculated mostly is the booking limit. It means that seats are open to reserve for Class-2 customers as booking limit, while the rest of the seats were reserved for Class-1 customers. Otherwise, the booking limit is suited to the overbooking limit in some situations, that is, the open to Class-2 customers overbooking (in which case, there will be no seats left for Class-1 customers).

For comparison, the expected profit from the optimal overbooking limit is greater than or equal to the airline's booking policy in all study situations. The Two-Class overbooking model yielded up to 21% more the expected profit margins than airline booking policies. Using real data, the Two-Class overbooking model is then taken to gauge the optimal number of update booking limits when the demand is non – stationary. It finds that the update booking limit 16 point gives the maximum profit.

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Appendix

Table A.1 Loss Profit per Flight when λ_1 = 50 and λ_2 = 100

r_1	r_2	θ_1	θ_2	ABL	Loss Profit per Flight	Percentage of Loss Profit per Flight
2434.4	472.5	0.9	0.7	9	38,647.25	18.08
				17	34,867.25	16.02
				41	23,527.25	10.28
				81	4,681.06	1.89
				122	361.33	0.14
				171	361.33	0.14
1521.5	472.5	0.9	0.9	9	38,647.25	17.34
				17	34,867.25	15.38
				41	23,527.25	9.88
				81	4,681.06	1.82
				122	361.33	0.14
				171	361.33	0.14
2434.4	756	0.9	0.7	9	15,444.45	6.71
				17	13,932.45	6.01
				41	9,396.45	3.98
				81	1,857.97	0.76
				122	1,590.72	0.65
				171	1,590.72	0.65

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Table A.1 Loss Profit per Flight when λ_1 = 50 and λ_2 = 100 (Cont.)

r_1	r_2	θ_1	θ_2	ABL	Loss Profit per Flight	Percentage of Loss Profit per Flight
2434.4	756	0.9	0.9	9	15,312.84	6.26
				17	13,800.84	5.61
				41	9,264.83	3.70
				81	1,726.36	0.67
				122	1,459.11	0.56
				171	1,459.11	0.56

Table A.2 Loss Profit per Flight when λ_1 = 60 and λ_2 = 110

r_1	r_2	θ_1	θ_2	ABL	Loss Profit per Flight	Percentage of Loss Profit per Flight
2434.4	472.5	0.9	0.7	9	37,029.68	15.76
				17	33,249.68	13.93
				41	21,909.68	8.76
				81	3,009.68	1.12
				122	0.00	0.00
				171	0.00	0.00
1521.5	472.5	0.9	0.9	9	37,319.63	15.25
				17	33,539.63	13.50
				41	22,199.63	8.54
				81	3,299.63	1.18
				122	289.94	0.10
				171	289.94	0.10
2434.4	756	0.9	0.7	9	14,739.90	5.82
				17	13,227.90	5.19
				41	8,691.90	3.35
				81	1,131.90	0.42
				122	4,991.58	1.90
				171	4,991.58	1.90
2434.4	756	0.9	0.9	9	14,927.85	5.55
				17	13,415.85	4.96
				41	8,879.85	3.23
				81	1,319.85	0.47
				122	5,179.53	1.86
				171	5,179.53	1.86

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Table A.3 Loss Profit per Flight when λ_1 = 70 and λ_2 = 120

r_1	r_2	θ_1	θ_2	**ABL**	**Loss Profit per Flight**	**Percentage of Loss Profit per Flight**
2434.4	472.5	0.9	0.7	9	37,488.86	15.95
				17	33,708.86	14.12
				41	22,368.86	8.94
				81	3,468.86	1.29
				122	0.00	0.00
				171	0.00	0.00
1521.5	472.5	0.9	0.9	9	37,488.86	15.34
				17	33,708.86	13.58
				41	22,368.86	8.62
				81	3,468.86	1.25
				122	0.00	0.00
				171	0.00	0.00
2434.4	756	0.9	0.7	9	14,742.00	5.82
				17	13,230.00	5.19
				41	8,694.00	3.35
				81	1,134.00	0.42
				122	4,440.79	1.68
				171	4,440.79	1.68
2434.4	756	0.9	0.9	9	14,931.00	5.56
				17	13,419.00	4.97
				41	8,883.00	3.24
				81	1,323.00	0.47
				122	4,629.79	1.66
				171	4,629.79	1.66