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Research Article

The New Discrete Weighted Exponential Distribution and Application

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Abstract

This paper presents a new discrete weighted exponential distribution using Chakraborty's discrete method (2015). The probability mass function, cumulative distribution function, survival function, and some mathematical properties are all discussed. The maximum likelihood estimation is used to estimate the parameters of the new discrete weighted exponential distribution. Furthermore, the new distribution was applied to two data sets. In summary, the new distribution can be a count data distribution alternative.

Keywords: discrete weighted exponential, probability mass function, discrete method

Introduction

Statistical distributions are widely used, whether as models for population occurrences or in certain situations. Many studies have shown this. For example, some statistical distributions have been proposed for lifetime models that exhibit a non-constant failure rate function (Alqasllaf et al.,2015). The lifetime model or survival function was studied using an exponential distribution of interesting random variables. It is a special case of the gamma distribution. The exponential distribution can be used to model the lifetime. If random variable X is exponential distribution, its probability density function (pdf) is given by $f(x;\beta)=\frac{1}{\beta}e^{-x/\beta}$; $0<x<\infty$. β is scale parameter (Casella & Berger, 2001). Gupta and Kundu showed the exponential distribution of two parameters in 2009. They used Azzalini's (1985) concept to introduce a shape parameter of an exponential distribution, which they called a weighted exponential distribution, or WE distribution (Gupta & Kundu, 2009).

In 2011, Parriz, Iman, and Behzad showed how to estimate the parameters of a weighted exponential distribution using maximum likelihood estimation, which is an estimator that cannot be expressed explicitly (Parriz et al., 2011). In 2014, Fatemah, Ghitany, and Claudio used Monte Carlo simulation to assess the finite sample attributes of the weighted exponential distribution's difference parameter estimation approach, including maximum likelihood, moments, L-moment, ordinary least-squares, and weighted least-squares (Fatemah, Ghitany, & Claudio, 2014). In 2016, Shakila and Itrat developed a new weighted exponential distribution and applied it to data from 100 bank customers' waiting times (Shakila & Itrat, 2016). Oguntunde, Owoloko, and Balogun defined a new weighted exponential distribution based on Nasiru's content (2015) and the probability density function (pdf) of the weighted exponential distribution were $f(x;\lambda,\alpha)=(1+\lambda)\alpha e^{-(1+\lambda)\alpha x}$; $x>0$, $\alpha>0$ and $\lambda>0$ and applied it to four real data sets (Oguntunde et al., 2016). In 2018, Khongthip, Patummasut, and Bodhisuwan suggested a discrete weighted exponential distribution. A discrete distribution is derived from a matching continuous distribution using Roy's method of discretization. They demonstrated the probability mass function, survival function, probability generating function, and characteristic function (Khongthip et al., 2018). Mahdi, Omid, and Alireza developed and evaluated parameters for a discrete weighted exponential distribution (Mahdi et al., 2018).

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There are several discrete methods for producing a discrete distribution. In 2015, Chakraborty described the various ways of constructing discrete probability distributions as analogues of continuous probability distributions. For instance, the method based on pdf, the survival function, and the hazard function (Chakraborty, 2015).

The aims of this study were to create a new discrete weighted exponential distribution and to apply it to two data sets. Some mathematical characteristics have explicit expressions. The remainder of the paper is structured as follows. In section 2, the methods consisted of a discrete method and weighted exponential distributions. The results are shown in section 3. It consists of a new discrete weighted exponential distribution, mathematical properties, and parameter estimation. In section 4, the application for two data sets. Finally, the discussion and conclusions are in the last section, respectively.

Methods

Discrete method

There are several methods for obtaining a discrete distribution from a continuous one. In 2013, Lai showed the construction of a discrete lifetime distribution from a continuous one in his paper on the building of a discrete lifetime distribution. Chakraborty (2015) showed many methods to generate a discrete distribution of a continuous probability distribution. Methods based on the survival function (sf), hazard rate function, cdf, pdf, and two-stage composite methods.

This paper used a method based on continuous survival function. Chakraborty (2015) consider the definition of the discrete survival function defined as $S_Y(y) = P(Y \geq y)$ when Y is a discrete random variable and accordingly the cdf $F_Y(y) = P(Y \leq y)$ is related to the survival function as $S_Y(y) = 1 - F_Y(y-1)$ (Chakraborty, 2015).

If a continuous random variable X has the survival function $S_X(x)$, then the random variable $Y = \lfloor X \rfloor$ is the largest integer less or equal to X. The pmf of Y is

$$\begin{aligned} p(y) &= P(Y = y) \\ &= P(y \leq X < y+1) \\ &= F_X(y+1) - F_x(y) \\ &= S_X(y) - S_X(y+1) \quad ; y = 0,1,2,3,... \end{aligned} \tag{1}$$

Since for continuous random variable X, $P(X = x) = 0$ and $F_X(y) = 1 - S_X(y)$. The pmf will be in a compact form if the continuous survival function is in compact form. This method preserves the survival function that is $S_Y(y) = S_X(y)$.

The weighted exponential distribution

In 2009, Gupta and Kundu introduced the shape parameter of the exponential distribution (Gupta & Kundu, 2009). They applied the Azzalini concept, which is a lifetime distribution (Azzalini, 1985). The weighted exponential distribution (WE distribution) was called following the two parameters of the exponential distribution developed by Gupta and Kundu.

Definition 1: The random variable X is said to have weighted exponential distribution, with the shape and scale parameter as $\alpha > 0$ and $\lambda > 0$ respectively, if the pdf of X is

$$f(x) = \left(\frac{\alpha+1}{\alpha}\right)\lambda e^{-\lambda x}\left(1 - e^{-\alpha\lambda x}\right); \quad x > 0, \tag{2}$$

and 0 otherwise. We will denote it as weighted exponential(α, λ).

Properties of the weighted exponential distribution are given in the following: when $\alpha \to \infty$, the weighted exponential distribution converges to the weighted exponential with scale parameter 1 ($\lambda = 1$). As $\alpha \to 0$, the weighted exponential distribution converges to gamma distribution with shape parameter 2. When $\alpha = 1$, the weighted exponential distribution coincides with the generalized exponential distribution with shape and scale parameter 2 and λ respectively.

The cdf of the weighted exponential distribution can be written as

$$F(x) = \frac{\alpha+1}{\alpha}\left[1 - e^{-x} - \left(\frac{1}{1+\alpha}\right)\left(1 - e^{-(1+\alpha)x}\right)\right]; \quad x > 0, \alpha > 0 \tag{3}$$

(Gupta and Kundu, 2009).

In 2016, Oguntunde, Owoloko, and Balogun presented a weighted exponential distribution based on Nasiru's content (2015). The weighted family of distribution is given by

$$f(x) = Kg(x)\bar{G}(\lambda x),$$

where $g(x)$ is a pdf, $\bar{G}$ is the corresponding (or reliability) function, and K is a normalizing constant (Oguntunde et al., 2016).

The pdf of the weighted exponential distribution is given by

$$f(x) = (1+\alpha)\lambda e^{-(1+\alpha)\lambda x}; \quad x > 0, \lambda > 0. \tag{4}$$

The cdf of the weighted exponential distribution is given by

$$F(x) = 1 - e^{-(\lambda x + \alpha\lambda x)}; \quad x > 0, \lambda > 0, \tag{5}$$

where λ is scale parameter and α is shape parameter (Oguntunde et al., 2016).

The pdf of the weighted exponential distribution in Equation (4) is simpler and easier to handle as compared to the weighted exponential pdf in Equation (2).

Results

The new discrete weighted exponential distribution

The probability mass function, cumulative distribution function, survival function, some mathematical characteristics, and parameter estimates are all given in this section of the study.

Theorem 1: If Y be random variable of the new discrete weighted exponential distribution where parameter α and λ are scale parameter and shape parameter respectively. The pmf of Y is

$$p(y) = e^{-y(\lambda+\alpha\lambda)}\left[1 - e^{-(\lambda+\alpha\lambda)}\right], \tag{6}$$

where $y = 0,1,2,\ldots$ and $\alpha > 0, \lambda > 0$.

The corresponding cdf of new discrete weighted exponential distribution is given by

$$F(y) = 1 - \left[\frac{e^{-(\lambda+\alpha\lambda)(1+y)} - e^{-y(\lambda+\alpha\lambda)}}{1 - e^{(\lambda+\alpha\lambda)}}\right]; \quad y = 0,1,2,\ldots, \ \alpha > 0, \ \lambda > 0. \tag{7}$$

Proof: When X is random variable, the pdf of the weighted exponential distribution follows in Equation (4). The survival function is

$$\begin{aligned} S(x) &= 1 - F(x) \\ &= e^{-(\lambda x + \alpha\lambda x)}, \end{aligned} \tag{8}$$

and

$$\begin{aligned} S(x+1) &= e^{-(\lambda(x+1) + \alpha\lambda(x+1))} \\ &= e^{-(\lambda x + \lambda + \alpha\lambda x + \alpha\lambda)}. \end{aligned} \tag{9}$$

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From Equation (1), $Y=\lfloor X \rfloor$ is the largest integer less or equal to X then Equations (8) and (9) are plugged into Equation (1). We will obtain

$$\begin{aligned} p(y) &= P(Y=y) \\ &= S(y)-S(y+1) \\ &= e^{-(\lambda y+\alpha\lambda y)}-e^{-(\lambda y+\lambda+\alpha\lambda y+\alpha\lambda)} \\ &= e^{-y(\lambda+\alpha\lambda)}\left[1-e^{-(\lambda+\alpha\lambda)}\right]. \end{aligned}$$

The properties of probability function are $p(y)\ge 0$ for all y and $\sum_{\forall y} p(y)=1$. The following is a proof for the properties of a probability function.

Proof: We can see that $p(y)\ge 0$ for all y when $y=0,1,2,\ldots$ from the pmf of Y and we prove the properties of the probability function, $\sum_{\forall y} p(y)=1$, that following;

$$\begin{aligned} \sum_{\forall y} p(y) &= \sum_{\forall y} p(Y=y) \\ &= \sum_{y=0}^{\infty} e^{-y(\lambda+\alpha\lambda)}\left[1-e^{-(\lambda+\alpha\lambda)}\right] \\ &= \left[1-e^{-(\lambda+\alpha\lambda)}\right]\left[\frac{1}{1-e^{-(\lambda+\alpha\lambda)}}\right] \\ &= 1. \end{aligned}$$

Since $\sum_{y=0}^{\infty} e^{-y(\lambda+\alpha\lambda)}$ is the geometric series. So, we have the pmf of the new discrete weighted exponential distribution, which is shown in Equation (6). We can say that $p(y)$ is pmf. The pmf has the properties of a probability function.

Proof: The following is a proof of the cdf of a new discrete weighted exponential distribution.

$$\begin{aligned} F_Y(t) &= P(Y\le y) \\ &= \sum_{t=0}^{y} p(t) \\ &= \sum_{t=0}^{y} e^{-t(\lambda+\alpha\lambda)}\left[1-e^{-(\lambda+\alpha\lambda)}\right] \\ &= 1-\left[\frac{e^{-(\lambda+\alpha\lambda)(1+y)}-e^{-y(\lambda+\alpha\lambda)}}{1-e^{(\lambda+\alpha\lambda)}}\right]. \end{aligned}$$

So, the cdf of the new discrete weighted exponential distribution is shown in Equation (7).

Theorem 2: If Y is a random variable of the new discrete weighted exponential, then its survival function is

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$$S(y)=\frac{e^{-(\lambda+\alpha\lambda)(1+y)}-e^{-y(\lambda+\alpha\lambda)}}{1-e^{(\lambda+\alpha\lambda)}}\ , \tag{10}$$

where $y=0,1,2,\ldots$ and parameter $\alpha,\lambda>0$.

Proof: That Y is a random variable of the new discrete weighted exponential with cdf in Equation (7) and $S(y)$ is defined as the survival function of the new discrete weighted exponential. Thus,

$$\begin{aligned} S(y) &= 1-F(y) \\ &= 1-\left[1-\left(\frac{e^{-(\lambda+\alpha\lambda)(1+y)}-e^{-y(\lambda+\alpha\lambda)}}{1-e^{(\lambda+\alpha\lambda)}}\right)\right] \\ &= \frac{e^{-(\lambda+\alpha\lambda)(1+y)}-e^{-y(\lambda+\alpha\lambda)}}{1-e^{(\lambda+\alpha\lambda)}}. \end{aligned}$$

Figures 1 and 2 show some possible plots for the new discrete weighted exponential distribution's pmf and cdf. Figure 1 presents the pmf plots of Equation (6) for different the values of α and λ and the distribution is skewed to the right. Evidently, the scale parameter α changes according to the shape parameter λ. The pmf decreases as shape parameter λ increases.

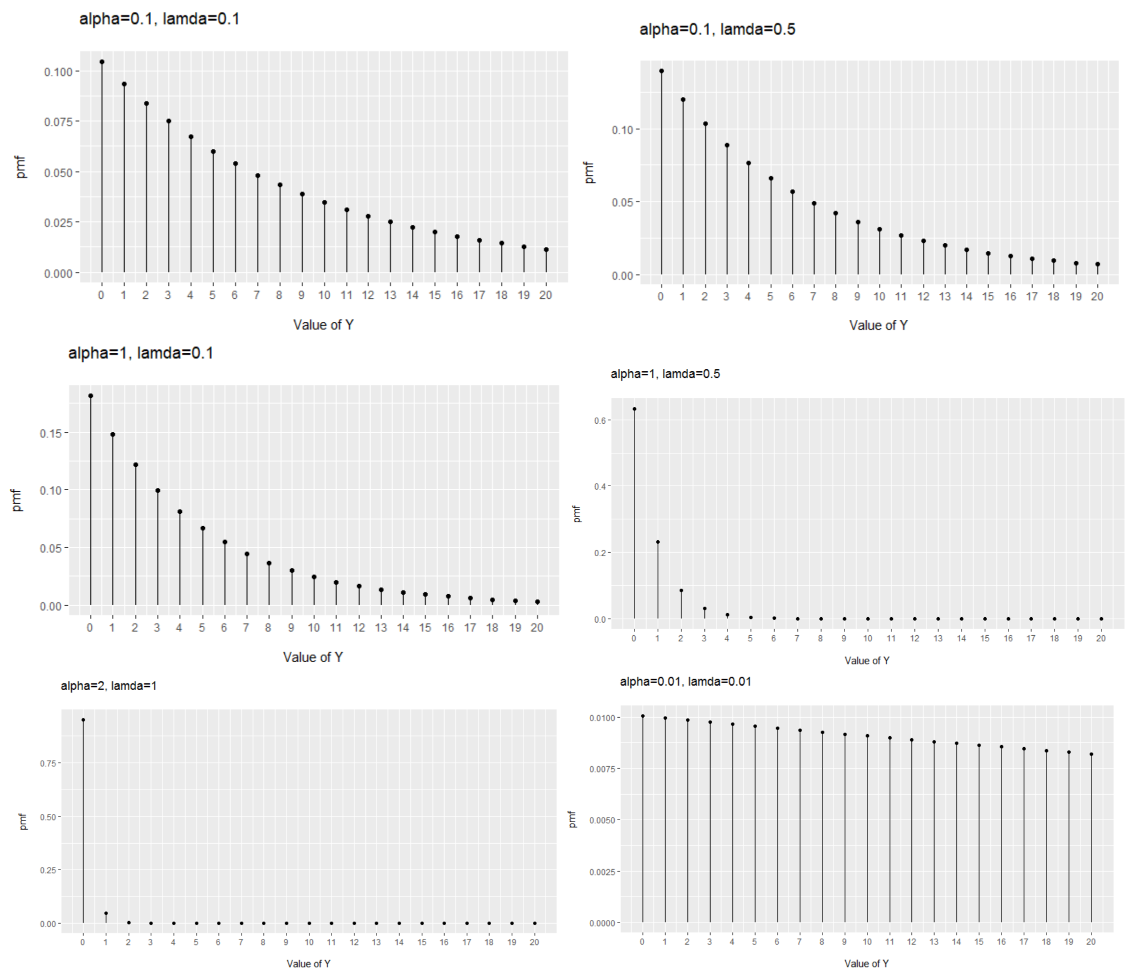

Figure 1 The pmf plots of the new discrete weighted exponential distribution

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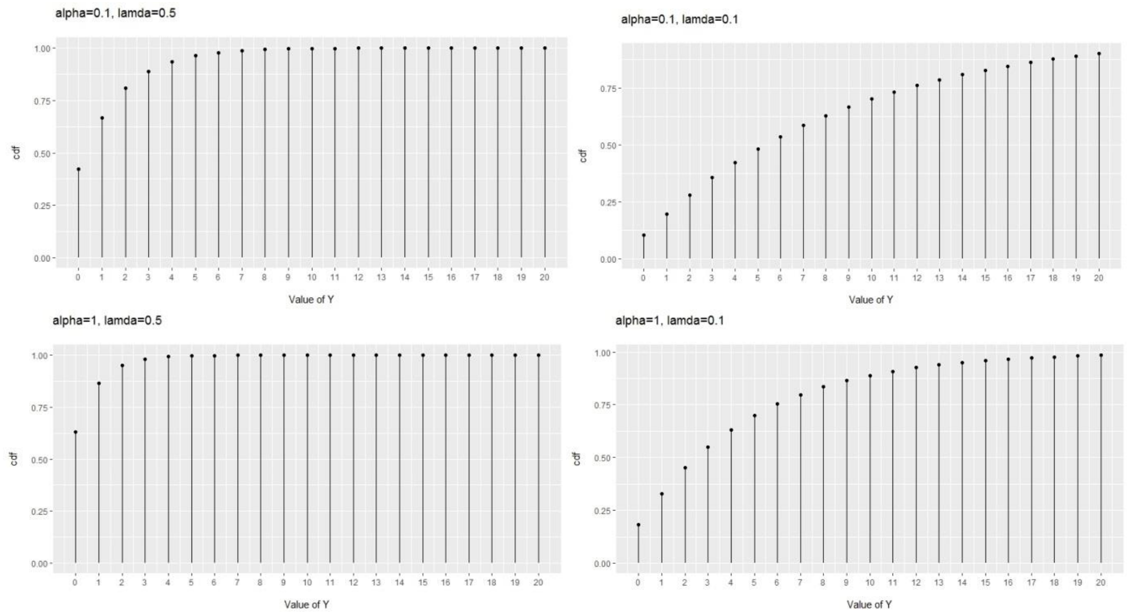

Figure 2 The cdf plots of the new discrete weighted exponential distribution

Figure 3 provides some possible plots for the survival function of the new discrete weighted exponential distribution.

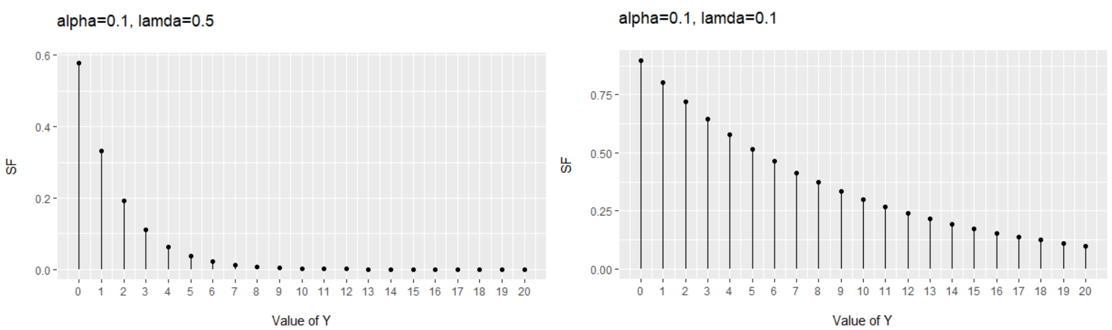

Figure 3 The survival function of the new discrete weighted exponential distribution

Mathematical properties

This section presented some mathematical properties, such as the probability generating function (pgf), expectation, variance, and moment generating function (mgf).

Probability generating function

The probability generating function is an essential component in statistical theory. The probability generating function of a discrete random variable is a power series representation of the random variable's probability mass function. A random variable's expectation and variance can also be obtained using this method.

Definition 2: Let Y be a random variable whose possible values are restricted to the non-negative integers, and $p(y)=P[Y=y]$, for $y=0,1,2,\ldots$. The probability generating function is then defined as

$$G(z)=E\left(z^{y}\right)=\sum_{y=0}^{\infty}p(y)z^{y}, \tag{11}$$

where $p(y)$ is the probability mass function of Y and $0\le z\le 1$. (Whittaher & Henderson, 2006)

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Theorem 3: Let Y be a random variable with the new discrete weighted exponential distribution, then the pgf is given by

$$G(z)=\frac{1-e^{-(\lambda+\alpha\lambda)}}{1-z\cdot e^{-(\lambda+\alpha\lambda)}}, \quad (12)$$

where $y=0,1,2,\ldots,\ 0\le z\le 1$ and $\alpha,\lambda>0$.

Proof: The pmf of the new discrete weighted exponential distribution is replaced in Equation (11) and then the pgf is

$$\begin{aligned}G(z)&=\sum_{y=0}^{\infty}p(y)z^{y}\\&=\sum_{y=0}^{\infty}e^{-y(\lambda+\alpha\lambda)}\left[1-e^{-(\lambda+\alpha\lambda)}\right]z^{y}\\&=\left[1-e^{-(\lambda+\alpha\lambda)}\right]\sum_{y=0}^{\infty}e^{-y(\lambda+\alpha\lambda)}z^{y}\\&=\frac{1-e^{-(\lambda+\alpha\lambda)}}{1-z\cdot e^{-(\lambda+\alpha\lambda)}}.\end{aligned}$$

Expectation, Variance and Moment generating function

Theorem 4: Let Y be a discrete random variable with probability generating function $G(z)$.Then

$$E(Y)=G'(1),$$

and $$E\{(Y)(Y-1)(Y-2)\ldots(X-k+1)\}=G^{k}(1)=\frac{d^{k}}{dz^{k}}G(z)\Big|_{z=1}$$

(Joel C. Miller, 2018).

The probability generating function can be used to determine the expectation and variance using Theorems 3 and 4.

The expectation of Y is

$$E(Y)=\frac{e^{-(\lambda+\alpha\lambda)}}{1-e^{-(\lambda+\alpha\lambda)}}, \quad (13)$$

and the variance of Y is

$$Var(Y)=\frac{e^{-(\lambda+\alpha\lambda)}}{\left(1-e^{-(\lambda+\alpha\lambda)}\right)^{2}}. \quad (14)$$

Proof:

$$\begin{aligned}G'(z)&=\frac{d}{dz}\left(\frac{1-e^{-(\lambda+\alpha\lambda)}}{1-z\cdot e^{-(\lambda+\alpha\lambda)}}\right)\\&=\frac{\left(1-e^{-(\lambda+\alpha\lambda)}\right)\cdot e^{-(\lambda+\alpha\lambda)}}{\left(1-z\cdot e^{-(\lambda+\alpha\lambda)}\right)^{2}}. \quad (15)\end{aligned}$$

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From Equation (15), setting $z=1$ following Theorem 4. we obtain

$$G'(1)=\frac{e^{-(\lambda+\alpha\lambda)}}{1-e^{-(\lambda+\alpha\lambda)}}. \tag{16}$$

So, $E(Y)=\frac{e^{-(\lambda+\alpha\lambda)}}{1-e^{-(\lambda+\alpha\lambda)}}$.

Similarly, to the expectation, the second derivative of $G(z)$ is

$$\begin{aligned}G''(z)&=\frac{d}{dz}\left(\frac{d}{dz}G(z)\right)\\&=\frac{d}{dz}\left(\frac{\left(1-e^{-(\lambda+\alpha\lambda)}\right)\cdot e^{-(\lambda+\alpha\lambda)}}{\left(1-z\cdot e^{-(\lambda+\alpha\lambda)}\right)^2}\right)\\&=\frac{2\left(1-e^{-(\lambda+\alpha\lambda)}\right)e^{-2(\lambda+\alpha\lambda)}}{\left(1-z\cdot e^{-(\lambda+\alpha\lambda)}\right)^3}.\end{aligned} \tag{17}$$

From Equation (17), setting $z=1$ we obtain

$$G''(1)=\frac{2e^{-2(\lambda+\alpha\lambda)}}{\left(1-e^{-(\lambda+\alpha\lambda)}\right)^2}. \tag{18}$$

Variance of Y can be derived from Theorem 4, Equation (16) and Equation (18). We obtain

$$\begin{aligned}Var(Y)&=G''(1)+G'(1)-\left(G'(1)\right)^2\\&=\frac{2e^{-2(\lambda+\alpha\lambda)}}{\left(1-e^{-(\lambda+\alpha\lambda)}\right)^2}+\frac{e^{-(\lambda+\alpha\lambda)}}{1-e^{-(\lambda+\alpha\lambda)}}-\frac{e^{-2(\lambda+\alpha\lambda)}}{\left(1-e^{-(\lambda+\alpha\lambda)}\right)^2}\\&=\frac{e^{-(\lambda+\alpha\lambda)}}{\left(1-e^{-(\lambda+\alpha\lambda)}\right)^2}.\end{aligned} \tag{19}$$

From Equation (19), we obtain variance of Y.

Definition 3: The moment generating function of the random variable Y is given by $E\left(e^{ty}\right)$ and is denoted by $M_Y t$, Hence

$$M_Y t=\sum_{\forall y}e^{ty}p(y),$$

if Y is discrete (Walpole et al., 2007).

From Definitions 2 and 3, set $z=e^t$ and We'll have the moment generating function.

$$G\left(e^t\right)=E\left(e^{ty}\right)=M_Y t,$$

$$M_Y(t)=\frac{1-e^{-(\lambda+\alpha\lambda)}}{1-e^{t-\lambda-\alpha\lambda}}. \tag{20}$$

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Parameter Estimation

The maximum likelihood estimation (MLE) method will be used to explain the parameter estimation method in this paper.

Let $Y_1, Y_2, Y_3, \ldots, Y_n$ be random samples drawn from the new discrete weighted exponential distribution with parameter α and λ as defined in Theorem 1. The likelihood function $L(y;\alpha,\lambda)$ is given by:

$$L(y;\alpha,\lambda) = \prod_{i=1}^{n} p(y_i)$$

$$\ln L(y;\alpha,\lambda) = \ln \prod_{i=1}^{n} p(y_i)$$

$$= \sum_{i=1}^{n} \ln\left\{ e^{-y_i(\lambda+\alpha\lambda)} \left[1 - e^{-(\lambda+\alpha\lambda)}\right]\right\}$$

$$= (\lambda+\alpha\lambda)\sum_{i=1}^{n}\{-y_i\} + n\ln\left[1-e^{-(\lambda+\alpha\lambda)}\right]. \tag{21}$$

By taking partial derivative Equation (21) with respected to parameter α and λ, we obtain

$$\frac{\partial}{\partial\alpha}\ln L(y;\alpha,\lambda) = \frac{\partial}{\partial\alpha}\left\{(\lambda+\alpha\lambda)\sum_{i=1}^{n}\{-y_i\} + n\ln\left[1-e^{-(\lambda+\alpha\lambda)}\right]\right\}$$

$$= -\lambda\sum_{i=1}^{n}\{y_i\} + \frac{n\lambda e^{-(\lambda+\alpha\lambda)}}{1-e^{-(\lambda+\alpha\lambda)}}. \tag{22}$$

$$\frac{\partial}{\partial\lambda}\ln L(y;\alpha,\lambda) = \frac{\partial}{\partial\lambda}\left\{(\lambda+\alpha\lambda)\sum_{i=1}^{n}\{-y_i\} + n\ln\left[1-e^{-(\lambda+\alpha\lambda)}\right]\right\}$$

$$= -(1+\alpha)\sum_{i=1}^{n}\{y_i\} + \frac{n(1+\alpha)e^{-(\lambda+\alpha\lambda)}}{1-e^{-(\lambda+\alpha\lambda)}}. \tag{23}$$

Then Equations (22) and (23) are set to zero respectively.

$$-\lambda\sum_{i=1}^{n}\{y_i\} + \frac{n\lambda e^{-(\lambda+\alpha\lambda)}}{1-e^{-(\lambda+\alpha\lambda)}} = 0 \tag{24}$$

$$-(1+\alpha)\sum_{i=1}^{n}\{y_i\} + \frac{n(1+\alpha)e^{-(\lambda+\alpha\lambda)}}{1-e^{-(\lambda+\alpha\lambda)}} = 0 \tag{25}$$

From Equations (24) and (25), we cannot obtain explicit expression of estimations, then numerical analysis such as the Newton-Raphson algorithm employs gives the maximum likelihood estimates of parameter α and λ.

Applications

This section considers two data sets to fit with the new discrete weighted exponential distribution and other discrete weighted exponential distributions, such as the discrete weighted exponential distribution demonstrated by Panpharisa et al. in 2018, which we will refer to as dWEP to make this paper easier to present, and the discrete weighted exponential distribution introduced by Mahdi et al. in 2018, which we will refer to as dWEM. R software was used to do the analysis for this study.

The Akaike information criterion (AIC), Bayesian information criterion (BIC), and Akaike information criterion corrected (AICc) are used to compare models. The AIC is the most often used fit statistic. Let L be the model likelihood, p is the number of parameters (predictors) in the model. The AIC is

$$AIC = -2\ln(L) + 2p\,.$$

The formulate of BIC is

$$BIC = -2\ln(L) + p\ln(n),$$

with p is number of parameters and n the number of observations in the model.

The formula for AICc depends on AIC. the formula for AICc is as follows;

$$AIC_c = AIC + \frac{2p(p+1)}{n-p-1},$$

where n denotes the number of observations and p denotes the number of parameters. (Mark J. Brewer et al., 2016)

Tables 1 and 3 provide the estimates of the parameters AIC, BIC, and AICc for two data sets.
Data set 1: The number of patients infected with influenza A/H1 2009 in Thailand between the first and fifteenth weeks of 2020. The data is accessible on the Thai National Influenza Center's website. The numbers are as follows: 15, 15, 19, 24, 24, 15, 12, 19, 19, 10, 15, 3, 0, 1, 3.

Table 1 shows the comparison of three models of discrete weighted exponential distribution. The result is that the value of AIC, BIC, and AICc of dWEM is the smallest in data set 1.

Table 1 Estimated parameters by maximum likelihood method for number of Influenza patient type A/H1 2009 in Thailand

Distribution	Parameter estimation	Criterion	Model function
1.New discrete weighted exponential	$\hat{\alpha}=10.0360$ $\hat{\lambda}=0.0067$	$LL=-53.963$ $AIC=111.926$ $BIC=118.758$ $AIC_C=112.926$	$p(y)=e^{-y(\lambda+\alpha\lambda)}\left[1-e^{-(\lambda+\alpha\lambda)}\right]$ $y=0,1,2,3,...$ and $\alpha,\lambda>0$
2. dWEP	$\hat{\alpha}=0.0010$ $\hat{\lambda}=0.1487$	$LL=-171.999$ $AIC=347.999$ $BIC=354.831$ $AIC_C=348.999$	$p(y)=\dfrac{1-e^{(\alpha+1)\lambda}+(\alpha+1)(e^{\lambda}-1)e^{(y+1)\alpha\lambda}}{\alpha e^{(\alpha+1)(y+1)\lambda}}$ $y=0,1,2,3,...$ and $\alpha,\lambda>0$
3. dWEM	$\hat{\alpha}=0.1292$ $\hat{\lambda}=0.3578$	$LL=-52.858*$ $AIC=109.716*$ $BIC=116.548*$ $AIC_C=110.716*$	$p(y)=\dfrac{\left\{(\alpha+\lambda+\alpha\lambda y)(1-e^{-\alpha})-\alpha\lambda e^{-\alpha}\right\}e^{-\alpha\lambda}}{\alpha+\lambda}$ $y=0,1,2,3,...$ and $\alpha>0$, $\lambda\ge 0$

Data set 2: The number of European red mites on apple leaves (Mahdi Rasekhi et al., 2018).

Table 2 The number of European redmites on apple leaves.

Count	0	1	2	3	4	5	6	7	≥ 8
Observed	70	38	17	10	9	3	2	1	0

As shown in Table 3, the AIC, BIC, and AICc of the new discrete weighted exponential distribution are lower than those of the other distributions.

Table 3 Estimated parameters by maximum likelihood method for number of European red mites on apple leaves.

Distribution	Parameter estimation	Criterion	Model function
1.New discrete weighted exponential	$\hat{\alpha}=3.8434$ $\hat{\lambda}=0.0120$	$LL=-34.585*$ $AIC=73.170*$ $BIC=77.959*$ $AIC_C=75.170*$	$p(y)=e^{-y(\lambda+\alpha\lambda)}\left[1-e^{-(\lambda+\alpha\lambda)}\right]$ $y=0,1,2,3,...$ and $\alpha,\lambda>0$
2. dWEP	$\hat{\alpha}=0.0022$ $\hat{\lambda}=1.1003$	$LL=-151.084$ $AIC=306.168$ $BIC=310.957$ $AIC_C=308.618$	$p(y)=\dfrac{1-e^{(\alpha+1)\lambda}+(\alpha+1)(e^{\lambda}-1)e^{(y+1)\alpha\lambda}}{\alpha e^{(\alpha+1)(y+1)\lambda}}$ $y=0,1,2,3,...$ and $\alpha,\lambda>0$
3. dWEM	$\hat{\alpha}=0.7356$ $\hat{\lambda}=0.1554$	$LL=-222.382$ $AIC=448.765$ $BIC=463.186$ $AIC_C=450.765$	$p(y)=\dfrac{\left\{(\alpha+\lambda+\alpha\lambda y)(1-e^{-\alpha})-\alpha\lambda e^{-\alpha}\right\}e^{-\alpha\lambda}}{\alpha+\lambda}$ $y=0,1,2,3,...$ and $\alpha>0$, $\lambda\geq 0$

Discussion

According to what has been published in other articles, the best model is the one with the lowest AIC, BIC, and AICc. As an outcome, it is worth highlighting that dWEM had the least AIC, BIC, and AICc values in the data set 1. Based on the applications offered in the first data set, dWEM is more extensible than the new discrete weighted exponential distribution and dWEP. However, with the second data set, the new discrete weighted exponential distribution clearly had the lowest AIC, BIC, and AICc values. Meanwhile, the suggested new discrete weighted exponential distribution is more tractable since it is easier to build. This result is similar to that of Oguntunde et al. (2016), who discovered that the weighted exponential distribution, which was expected to be more tractable than the weighted exponential distribution proposed by Gupta and Kundu (2009), did not outperform the weighted exponential distribution when applied to data on the remission time of blood cancer patients. Furthermore, the finding is similar to that of Adepoju et al. (2014), who discovered that the Kumaraswamy Nakagami distribution, which was supposed to be more tractable than the Beta Nakagami distribution, was not tractable.

Conclusion

A continuous weighted exponential distribution is used in this paper to develop the new weighted exponential distribution, which is created using the discrete method. The new weighted exponential distribution is shaped like a reversed J. In addition, this paper discusses the pmf, cdf, basic mathematical properties, and parameter estimation. The new weighted exponential distribution was applied to two data sets and compared to other discrete weighted exponential distributions, which discovered that the discrete weighted exponential distribution proposed by Mahdi et al. in 2018 is better for some data sets, but the new discrete weighted exponential distribution is more tractable because it is easier to construct.

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