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Research Article

Fuzzy Cumulative Sum Control Chart for Monitoring Fuzzy Process

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Abstract

Today's market competition is increasing rapidly, product quality control is essential for manufacturers to maintain mass products with low production costs and high quality. Therefore, statistical process control (SPC) tools for manufacturing processes are become more emphasized because of their effectiveness of the control chart is to reduce the variability of the quality characteristic. The cumulative sum (CUSUM) control chart was developed to detect small shift in process mean. This paper based on the fuzzy set theory, we develop a fuzzy cusum control chart, namely Fuzzy CUSUM control chart to accommodate the fuzzy situation where fuzziness of vague sample data is taken into consideration. The CUSUM control chart and Fuzzy CUSUM control chart are used to control and monitor the mean of a process. The objective of this paper is to compare the ability and performance of the existing CUSUM control charts and the proposed fuzzy CUSUM control chart to detect shifts in a process. These data are generating by Monte Carlo Simulation technique. The criteria to evaluate the performance of control chart is average run length in situation process goes out-of-control. The result of comparison shows that Fuzzy CUSUM control chart is more sensitive than CUSUM control chart in the case of a process has small shifts in the mean.

Keywords: Cumulative Sum Control Chart, Fuzzy Cumulative Sum Control Chart, Average Run Length, Normal Distribution

Introduction

Statistical Process Control (SPC) is an importance tool in the production process in order to improve the quality of the production process. The SPC includes observation, evaluation, diagnosis, and implementation. Control charts are widely applied in SPC tools. Control charts were designed to monitor a process and detect shifts in mean and variance of quality characteristics to assure that the processes are performing in an acceptable manner. Control charts are prosperously applied in engineering, public health, economics, medicine and in other areas of applications. Traditional charts are based on a fundamental assumption that process data are statistically independent and normally distributed when the process is in control. The idea of a control chart was first proposed by Walter A. Shewhart (1931), which is called the Shewhart control chart. It is a quality control chart that is suitable for detecting large changes in the process. Recently, the cumulative sum (CUSUM) control chart presented by Page (1954), which is effective in detecting small shifts in the process.

In the consideration of real production process, the process is assumed that there are no doubts about observations. Such type of observations can be obtained by human judgments, evaluations and decisions. In real applications, the collected characteristic is a continuous random

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variable of a production process should include the variability caused by human judgments, measurement devices or environmental conditions. These variations result in uncertainty in the measurement system. In this paper, we discuss the application of the CUSUM control chart for monitoring in the mean of process. The CUSUM control chart are widely used in engineering and in health-care. The fuzzy set theory contributes its capability of systematic dealing with fuzzy data to monitor a manufacturing process with fuzzy sample data. The fuzzy control charts are more sensitive than Shewhart control chart. It is important to be able to evaluate the average run length (ARL) when observations are vagueness.

There are many researchers; Kanagawa et al (1993) propose a linguistic control chart for detecting change in mean and variance of process based on the estimation of probability distribution existing behind the linguistic data. GÜlbay and Kahraman (2007) present a fuzzy control chart using the fuzzy transformation method and direct fuzzy approach. Later, Wang and Hryniewicz (2013) proposed a fuzzy cumulative sum control chart using the membership function of fuzzy statistic. Recently, Muhammad et al (2018) developed an exponentially weighted moving average (EWMA) scheme when observations are fuzzy data for detecting small shifts in the process mean. Later, Amanda et al (2019) proposed Fuzzy Control chart for monitoring mean and range of the univariate process. The Shewhart control chart and the fuzzy control chart are compared based on the average run length (ARL) and extra quadratic loss (EQL). The performance of the control chart in monitoring the process mean is calculated using the average run length (ARL), the ARL is suggested as evaluation criteria. ARL is the expected number of samples that should occur before a sample shows the out-of-control condition. There are two characteristics of ARL: 1) the average number of sample taken from an in-control process until the control chart falsely signals out-of-control is denoted by ARL_0. An ARL_0 will be regarded as acceptable if it is large enough to keep the level of false alarms at an acceptable level and 2) the average number of observation that fall within the control limits before giving an alarm that the process is out-of-control is denoted by ARL_1.

In this paper, the fuzzy cumulative sum (Fuzzy CUSUM) control chart is proposed based on the triangular fuzzy number and compared with the cumulative sum (CUSUM) control chart via a simulation study. The performance of Fuzzy CUSUM control chart is investigated on the criteria of average run length. The paper is organized as follows: in Section 1, we introduce the statistical process control. In Section 2, the CUSUM control chart is described. In Section 3, the Fuzzy CUSUM control chart for the vagueness data is presented. The performance of Fuzzy CUSUM control chart is investigated by means of ARL and is compared to the CUSUM control chart in Section 4. Finally, conclusion of this research are discussed.

Cumulative Sum Control Chart

The traditional cumulative sum control chart was proposed by Page (1954). It is one of the most important tools in statistical process control, which are used to evaluate the performance of processes and it could be an effective tool for quickly identifying the change point of process. The CUSUM control chart is designed to detect small shifts effectively. Suppose that $\{X_t, t=1,2,..\}$ be the quality characteristic at time t based on Normal distribution.

The probability density function of the normal distribution is given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}; \quad -\infty < x < \infty$$

where μ is the mean of distribution

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σ^2 is the variance of distribution.

Let $X_1, X_2, ..., X_n$ be a sample of size n from the random variable, the sample mean at time t $\bar{X}_t$ can be calculated by

$$\bar{X}_t = \frac{1}{n}\sum_{j=1}^{n} X_j$$

The CUSUM statistics is defined as (Page 1954)

$$C_t = max\left(0, C_{t-1} + \bar{X}_t - a\right); t = 1,2,\ldots \quad (1)$$

where C_0 is the initial value
a is the reference value of CUSUM control chart.

Hence, the CUSUM control chart will be signal if $C_t > b,$ where b is the control limit of CUSUM control chart.

Fuzzy Cumulative Sum Control Chart

The fuzzy cumulative sum (Fuzzy CUSUM) control chart was proposed by Wang (2006) to detect a small shift in the process mean. Applying the Fuzzy CUSUM control chart for fuzzy data based on the concept of fuzzy random variables. The details of this control chart are given as follows. The fuzzy random variable with the sample size of n was represented by using a triangular fuzzy number denoted by $\tilde{X}_t = (X_{lt}, X_{ct}, X_{rt}).$ Therefore, the sample mean of the triangular fuzzy number are as follows:

$$\tilde{\bar{X}}_t = (\bar{X}_{lt}, \bar{X}_{ct}, \bar{X}_{rt}) \quad (2)$$

where $\bar{X}_{lt} = \frac{1}{n}\sum_{j=1}^{n} X_{lj},\ \bar{X}_{ct} = \frac{1}{n}\sum_{j=1}^{n} X_{cj},\ \bar{X}_{rt} = \frac{1}{n}\sum_{j=1}^{n} X_{rj}$

The statistics of triangular fuzzy number are defined as

$$\tilde{C}_{lt} = max\left(0, \tilde{C}_{lt-1} + \bar{X}_{lt} - K\right) \quad (3)$$

$$\tilde{C}_{ct} = max\left(0, \tilde{C}_{ct-1} + \bar{X}_{ct} - K\right) \quad (4)$$

$$\tilde{C}_{rt} = max\left(0, \tilde{C}_{rt-1} + \bar{X}_{rt} - K\right) \quad (5)$$

where $\tilde{C}_{l0}, \tilde{C}_{c0}, \tilde{C}_{r0} = 0$ are the initial values and K is the reference value.

The fuzzy CUSUM statistics ($\tilde{C}_t$) can be calculated by following recursion:

$$\tilde{C}_t = \frac{1}{2}\left(\tilde{C}_{lt} + \tilde{C}_{rt}\right) \quad (6)$$

Consequently, an alarm indicating an out-of-control process is declare when the fuzzy CUSUM statistics $\tilde{C}_t > h$ where h is the control limit of fuzzy CUSUM control chart.

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Comparison of Performance Control Chart

In this section, a numerical study is performed in order to study the performance of the CUSUM and Fuzzy CUSUM control chart for monitoring and controlling fuzzy data collected from a manufacturing process. The Monte Carlo simulation technique is used to generate samples from the normal distribution. The performance comparison of the control charts is evaluated by using the average run length (ARL). In case of in-control process, a large ARL value is desired, while a small ARL value is desired when the process is out-of-control.

The ARL approximation method is the Monte Carlo simulation technique (MC) which can be calculated by

$$ARL = \frac{\sum_{i=1}^{M} RL_i}{M} \tag{7}$$

where RL_i is the run length of the simulation time i

M is the number of the simulation study.

Let the process of quality characteristic follows a normal distribution with parameter μ and σ^2. In situation the process is in-control state, we let the normal parameter $\mu = \mu_0 = 10$ and the process is out-of-control state, the normal parameter $\mu = \mu_1 = \mu_0 + \delta\mu_0$ where δ is the magnitude of shift size; δ = 0.0, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 1.00 and 1.50 respectively. The ARL are estimated by running the proposed schemes using the R-language program with repeated 10,000 times. The ARL is fixed (ARL_0) = 370 and 500 respectively, when the process is in-control. The results of the simulation study are reported in Tables 1 - 4. The parameter value for a Fuzzy CUSUM control chart was chosen by given ARL_0 = 370 and 500.

In Table 1, we compare results of ARL_1 for fuzzy number process between CUSUM and Fuzzy CUSUM control charts. The values of parameter for CUSUM and Fuzzy CUSUM control charts were established by setting ARL_0 = 370 and the sample sizes (*n*) = 5.

Table 1 The Average Run Length (ARL) values for the CUSUM and the Fuzzy CUSUM control charts when given ARL_0 = 370 and *n* = 5

Shift size	Control Chart	
δ	CUSUM	Fuzzy CUSUM
0.00	370.01	370.63
0.10	322.54	150.22
0.20	279.56	89.18
0.30	246.14	63.42
0.40	219.46	49.23
0.50	197.49	40.31
0.60	179.56	34.00
0.70	164.62	29.41
0.80	152.00	25.78
0.90	140.98	22.98
1.00	131.55	20.74
1.50	98.57	13.73

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The ARL values of the CUSUM and the Fuzzy CUSUM control charts of fuzzy data with sample size $n = 5$ were reported for simulation studies in Table 1. Table 1 and Figure 1 shows that the ARL of the Fuzzy CUSUM control chart was lower than the CUSUM control chart, and the ARL value of the Fuzzy CUSUM control chart decreases rapidly as the shift size (δ) increases from 0.2-1.5.

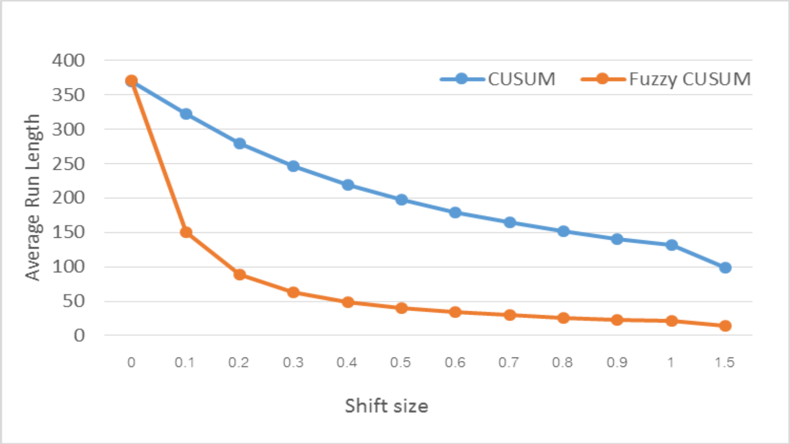

Figure 1 The Average Run Length (ARL) values for the CUSUM and the Fuzzy CUSUM control charts when given $ARL_0 = 370$ and $n = 5$

In Table 2 and Figure 2, we compare results of ARL_1 for fuzzy number process between CUSUM and Fuzzy CUSUM control charts. The values of parameter for CUSUM and Fuzzy CUSUM control charts were established by setting $ARL_0 = 370$ and $n = 10$.

Table 2 The Average Run Length (ARL) values for the CUSUM and the Fuzzy CUSUM control charts when given $ARL_0 = 370$ and $n = 10$

Shift size	Control Chart	
δ	CUSUM	Fuzzy CUSUM
0.00	370.00	370.68
0.10	310.73	148.90
0.20	266.62	89.18
0.30	233.36	62.05
0.40	207.30	48.19
0.50	186.57	40.02
0.60	169.56	33.77
0.70	155.38	29.04
0.80	143.35	25.41
0.90	133.07	22.85
1.00	124.19	20.54
1.50	93.01	13.64

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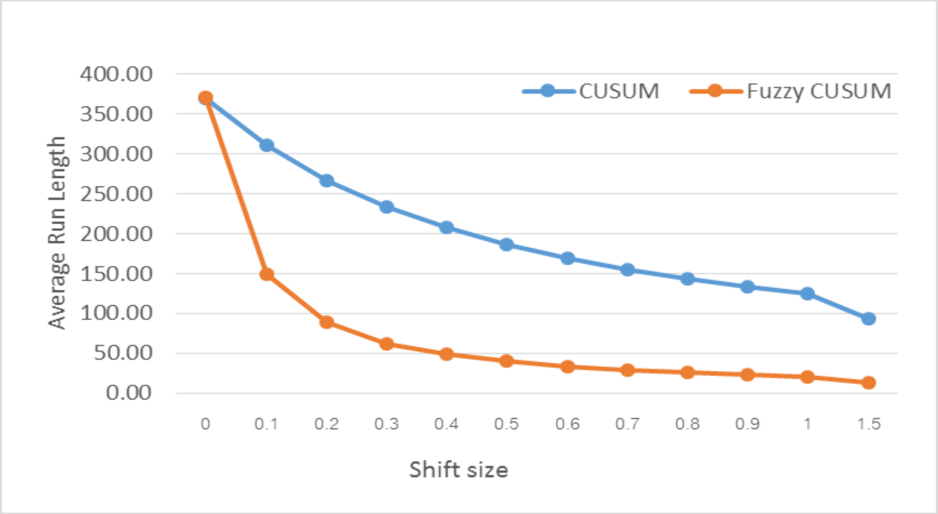

Figure 2 The Average Run Length (ARL) values for the CUSUM and the Fuzzy CUSUM control charts when given ARL_0 = 370 and n = 10

The results from the Table 2 and Figure 2, we find that the Fuzzy CUSUM control chart for mean detects all changes more quickly compared to the CUSUM control chart. In addition, it was found that the efficiency of the Fuzzy CUSUM control chart increased when the shift size was large, and the ARL value inversely proportional to the shift size.

In Table 3 and Figure 3, we compare results of ARL_1 for fuzzy number process between CUSUM and Fuzzy CUSUM control charts. The values of parameter for CUSUM and Fuzzy CUSUM control charts were established by setting ARL_0 = 500 and n = 5.

Table 3 The Average Run Length (ARL) values for the CUSUM and the Fuzzy CUSUM control charts when given ARL_0 = 500 and n = 5

Shift size	Control Chart	
δ	CUSUM	Fuzzy CUSUM
0.00	500.00	500.04
0.10	418.52	176.20
0.20	359.36	105.60
0.30	314.28	74.63
0.40	279.57	57.46
0.50	251.57	47.16
0.60	228.71	39.89
0.70	209.53	34.49
0.80	193.36	30.17
0.90	179.54	26.99
1.00	167.56	24.36
1.50	125.55	16.19

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The results from the Table 3 and Figure 3, we find that the Fuzzy CUSUM control chart is more sensitive to small process mean shifts than CUSUM control chart.

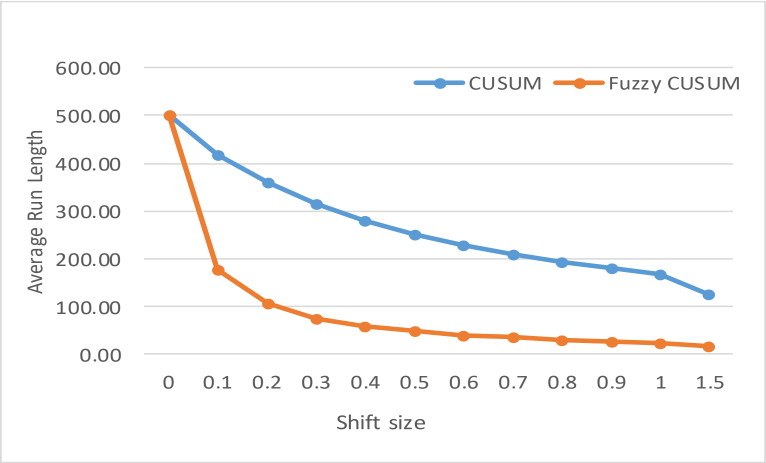

Figure 3 The Average Run Length (ARL) values for the CUSUM and the Fuzzy CUSUM control charts when given ARL_0 = 500 and n = 5

In Table 4, we compare results of ARL_1 for fuzzy number process between CUSUM and Fuzzy CUSUM control charts. The values of parameter for CUSUM and Fuzzy CUSUM control charts were established by setting ARL_0 = 500 and n = 10.

Table 4 The Average Run Length (ARL) values for the CUSUM and the Fuzzy CUSUM control charts when given ARL_0 = 500 and n = 10

Shift size	Control Chart	
δ	CUSUM	Fuzzy CUSUM
0.00	499.46	496.18
0.10	418.52	176.12
0.20	359.43	104.94
0.30	314.60	74.66
0.40	279.65	57.89
0.50	251.62	47.23
0.60	228.59	39.79
0.70	209.47	34.36
0.80	193.40	30.27
0.90	179.47	27.15
1.00	167.50	24.39
1.50	125.53	16.20

The results from the Table 4 and Figure 4, we find that the Fuzzy CUSUM control chart is the best control chart in the sense that it has minimizes the supremum of the conditional Average Run Length (ARL_1) when the process has a small shift size ($0.10 \le \delta \le 1.50$).

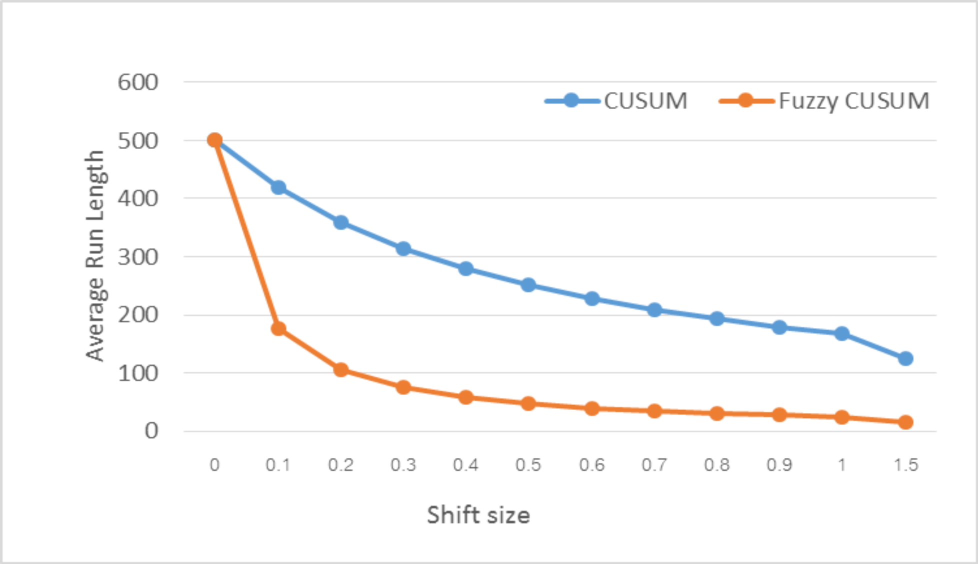

Figure 4 The Average Run Length (ARL) values for the CUSUM and the Fuzzy CUSUM control charts when given ARL_0 = 500 and n = 10

Conclusion

In this paper, we compared the performanace of the existing CUSUM control charts and the proposed Fuzzy CUSUM control chart to detect mean shifts in a process when the data are fuzzy number. Now, the comparison between CUSUM and Fuzzy CUSUM control charts from Tables 1-4 reveals that:

(1) The Fuzzy CUSUM control chart for mean detects shifts more quickly as compared to the CUSUM control chart in the situation of ARL_0 are 370 and 500.

(2) The performance of Fuzzy CUSUM control chart for mean performs better in case of small shifts (δ) for all sample sizes (n) = 5 and 10.

Hence, it is concluded that the Fuzzy CUSUM control chart for mean is more efficient in terms of ARLs values and have shown better performance than the CUSUM control chart for detecting small shifts ($0.10 \le \delta \le 1.50$).

Therefore, a natural extension of this work is that it may consider interesting connections to multivariate fuzzy issues in future works.

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