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Research Article

Robustness of CUSUM-Tukey's Control Chart for Detecting of Changes in Process Dispersion

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Abstract

The aims of this paper have investigated the robustness of Cumulative Sum – Tukey's chart (CUSUM-Tukey) to detecting process dispersion when the assumption of process distribution has deviated from the normal distribution. The performance of control charts is commonly measured by an out of control Average Run Length (ARL_1) by given in control Average Run Length (ARL_0) = 370 and 500. The performance of CUSUM-Tukey is compared with Tukey's chart (Tukey) and Cumulative Sum control chart (CUSUM) where the best performance of the control chart will give the minimum value of ARL_1. The numerical results are figured out by the Monte Carlo simulation method to approximate ARL which found that the CUSUM-Tukey's control chart is superior to Tukey and CUSUM control charts for all case studies of distributions and all spans for calculating moving range.

Keywords: variation, nonparametric control chart, monitoring, Monte Carlo simulation.

Introduction

Statistical Process Control (SPC) is a collection of different technologies that help differentiate between the common cause and the special cause of variation in the response of a quality characteristic of interest in a process. One of the most effective techniques of SPC is the control chart which can be separate the common causes of the special causes of variation. It is used to monitor a changed parameter in process, such as scale, location and dispersion parameter. Shewhart (1931) first introduced the concept to use a control chart to bring the process back in a normal situation when it is out of control due to special cause variation. The Shewhart control chart normally accommodates just the current sample information, therefore they are sometimes called memoryless control charts. Similarly, Cumulative Sum (CUSUM) control chart (Page, 1954) and Exponentially Weighted Moving Average (EWMA) control chart (Roberts, 1959) are used to detect small to moderate shifts and also relax to normality assumption of the population. The structure of these charts allows them to use the past information along with current information which makes them namely memory or time-weighted control charts. In 2004, Alemi applied the standard form of the diagram constructing a control chart for individual observations with independent and identically distributed (i.i.d.) called Tukey's chart (Tukey). It is an effective alternative to the individual and moving range control chart (X-MR) for individual process monitoring when the data follow the skew distributions and/or have some outliers.

The monitoring of the process output requires early detection of the shifts of scale/location and dispersion parameters. To monitor those parameters, the stabilized dispersion parameter would be preferred rather than the scale/location parameter, the reason bring dependency on location structure on the dispersion parameter. Therefore, the dispersion parameter is of more immediate concern in process monitoring (Montgomery, 2009).

For efficient monitoring of process dispersion, Torng and Lee (2008) calculated the ARL values of the Tukey's chart under several distributions. For a small sample, the Tukey's chart performed very well when compared with other control charts. Torng et al. (2009) presented the economic design of the Tukey's chart. Lee (2011) applied the asymmetrical control limit (ACL) to the Tukey's chart. The ACL-Tukey's chart offered lower ARL values than the symmetrical control chart (SCL-Tukey) for skewed distributions. Sukparungsee (2012) evaluated the performance of Tukey's chart which its performance is superior to classical EWMA and Shewhart control charts. Lee et al. (2013) used the ACL to economic design Tukey to get the optimum performance. Sukparungsee (2013) also applied ACL to Tukey's chart to evaluate the performance under non-normal distributed data and Tukey's chart has better AARL performance for both cases of ACL-Tukey and SCL-Tukey. Khaliq et al. (2014) investigated the comparative analysis to judge the performance of Tukey's chart versus X/MR chart under the several probability models and Tukey's chart was the best choice in many cases. There are several studies presented that this chart is a good alternative to the traditional Shewhart and X/MR charts for monitoring when the data are skewed distribution. Khaliq et al. (2016) introduced the EWMA design for the Tukey's chart so-called mixed Tukey-EWMA chart. The concept of this chart was designed to have more sensitive for a small sustained shift in process location than the Tukey's chart. This design also performs better than the classical EWMA when the data follow the skewed distribution. Khaliq and Riaz (2017) designed the CUSUM structure of the Tukey's chart for small and sustained shifts. The performance of that chart is superior to the classical CUSUM chart when data follow the skewed distribution. For asymmetric distribution, this design has similar run length performance when compared with the classical CUSUM. Raiz et al. (2017) introduced mixed Tukey EWMA-CUSUM design which was more sensitive to small and moderate amounts of shift. Later, Mongkoltawat et al. (2017) presented Exponentially Weighted Moving Average-Tukey's chart (EWMA-Tukey) for moving range and range which was superior to EWMA and Tukey control charts for all magnitudes of changes. Thitisoowaranon et al. (2019) study the robustness of mixed CUSUM-Tukey's chart for detecting process dispersion which was superior to Tukey, CUSUM, and EWMA-Tukey for all cases of asymmetric distribution.

The control charting system is normally practiced in two distinct stages: Phase I (ARL_0) and Phase II (ARL_1). In Phase I, the key concern is to understand the process and to access process stability, making sure that the process is operating at the intended target under some natural causes of variation. Phase I also involves the estimation of the parameters as well as setting up or estimating the control limits. In Phase II, the control chart is used to monitor the process online to detect shifts happening in the process so that any corrective actions can be taken quickly. The performance of the charts based on moving range is evaluated by assessing the average run length (ARL) under normality and in the presence of various types of contaminations employing simulation.

Thus, this paper focuses on the robust CUSUM-Tukey control charts based on the moving range for monitoring the process dispersion parameter. Particularly, their design structures and performances are studied under different environments and in the presence of

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special causes in the dispersion parameter of the process. The remaining of the paper is organized as follows. Section 2 gives a review of the control chart and property. In Section 3, the performance evaluation and analysis are addressed. In Section 4, the detection properties of the CUSUM-Tukey's chart are verified by evaluating the average run length (ARL). Finally, Section 5 conclusion and summary of results are discussed.

Control chart and property

1. Tukey's control chart

In 2004, Alemi first purposed Tukey's control chart which is one of the nonparametric control charts by applying the principle of Box plot to obtain the control limits. The control limits of Tukey's chart are proposed by Torng and Lee (2008) as following

$$\begin{aligned} UCL &= F^{-1}(0.75) + K_1 \times IQR \\ LCL &= F^{-1}(0.25) - K_1 \times IQR \end{aligned} \tag{1}$$

where UCL and LCL are upper and lower control limits, respectively. The $F^{-1}(0.75)$ and $F^{-1}(0.25)$ are the third quartile (Q_3) and the first quartile (Q_1) and IQR is interquartile range $(IQR = Q_3 - Q_1)$. Usually, the value K_1 is a constant as a coefficient of control limits which given to 1.5 when the assumption of the process is the normal distribution (Ryan, 2000).

2. Cumulative Sum Control Chart

The CUSUM chart was introduced by Page (1954), and it is used cumulative deviation from the target value. It is a favorable tool to detect the small to moderate shifts. The CUSUM chart is based on the following two CUSUM statistics:

$$\begin{aligned} C_t^+ &= \max[0, (X_t - \mu_0) - k_1 + C_{t-1}^+] \\ C_t^- &= \max[0, -(X_t - \mu_0) - k_1 + C_{t-1}^-] \end{aligned} \tag{2}$$

where k_1 is the reference value, t is time or sample number. The C_t^+ and C_t^- are the upper and lower CUSUM statistics, respectively, and the initial value of C_0^+ and C_0^- are zero. The CUSUM statistics C_t^+ and C_t^- plotted against to upper control limit (K_2) and lower control limit is given to be zero, respectively.

3. Cumulative Sum – Tukey's Control Chart

The mixed control charts are combined CUSUM and Tukey's control chart by using the concept that the CUSUM control chart is represented as statistic while the parameter of mean and variance are substituted with nonparametric as quartile and interquartile, respectively. Then, the mixed CUSUM-Tukey statistics are denoted in the form of two statistics C_t^+ and C_t^- as:

$$\begin{aligned} C_t^+ &= \max[0, (x_t - Q_3^s) - k_2 + C_{t-1}^+] \\ C_t^- &= \max[0, -(Q_1^s - x_t) - k_2 + C_{t-1}^-] \end{aligned} \tag{3}$$

where k_2 is the reference value of CUSUM-Tukey's chart Q_1^s and Q_3^s are adjusted from Q_1 and Q_3 by Khaliq et al. (2017) as follows:

$$\begin{aligned} Q_3^s &= (Q_3 - 0.75Q_3) + 0.5Q_2 + 0.25Q_1 \\ Q_1^s &= (Q_1 - 0.75Q_1) + 0.5Q_2 + 0.25Q_3. \end{aligned} \tag{4}$$

The upper control limit of CUSUM-Tukey's chart is K_3 and lower control limit is given to be zero. The value of Q_1, Q_2 and Q_3 are defined as the first, second and third quartile, respectively.

Performance evaluation and analysis

A sequence of points plotted on a control chart until an out of control signal is identified is known as a run and a series of points in a run is named as Run Length (RL). Typically, RL is expected to be higher while out of control RL anticipated being as small as possible for in control process. Several measures based on RL are presented in the literatures to evaluate the performance of the control chart.

Average Run Length (ARL)

The ARL is generally used to assess the performance of control chart for specific shifts. This value refers to the average number of points plotted on a chart until an out of control signal is identified. There are two technical terms used in the control chart that are in control and out of control ARL. The in and out of control ARL are usually denoted by ARL_0 and ARL_1, respectively. The best performance in order to detect of a change will give the minimal value of ARL_1. The ARL can be calculated as:

$$ARL = \frac{\sum_{j=1}^{m} RL_j}{m}. \qquad (5)$$

Using Monte Carlo simulation with m replications, we computed the aforementioned RL properties of Tukey, CUSUM and CUSUM-Tukey control charts. The numerical result is reported on Tables 1 - 8 in the form of ARL_s when given ARL_0=370 and 500 by varying the magnitudes of shifts in dispersion parameter.

Performance measure evaluation and comparison

The performance of control charts is approximated by the Monte Carlo simulation technique with $m=10^5$ replications to order to get the RL properties for detection of a change in process dispersion based on the moving range with R programming. The comparative analysis of the CUSUM-Tukey's chart, Tukey and CUSUM charts are studied and addressed which the best performance of control charts will give the minimal value of ARL_1. The numerical results of ARL_s when observations are distributed as Lognormal(0,1) with the span value to calculate a moving range (MR) is given to 2, 3, 5 and 10 is presented on Table 1, Table 2, Table 3 and Table 4, respectively for the case of Lognormal distribution. Tables 5 - 8 shows the numerical results of the comparative performance of control charts when observations are Logistic(6,2) distributed with MR=2, 3, 5 and 10, respectively. These results are arranged in tabular using ARL_0 = 370 and 500, magnitudes of shift size (δ) are 1, 1.2, 1.4, 1.6, 1.8, 2, 2.5 and 3, and the number of observations are 2,000.

We have compared the performance of CUSUM-Tukey control chart of detection of change in process dispersion with Tukey and CUSUM control charts when the observations are distributed Lognormal and Logistic. The ARL_s of the control chart are approximated by the Monte Carlo simulation technique. The numerical results show that CUSUM-Tukey's chart is superior to the Tukey and CUSUM control charts for all cases. The comparative performance of Lognormal distribution with the span of MR = 2, 3, 5 and 10 also shown on Figures 1 – 4, respectively. On Figures 5 – 8, the performance of CUSUM-Tukey, Tukey and CUSUM control charts are compared underlying Logistic distribution with the span of MR = 2, 3, 5 and 10, respectively. Obviously, the performance of CUSUM-Tukey control chart is robust to Lognormal and Logistic distributions because the performance of CUSUM-Tukey can detect of change for process dispersion not only symmetric distributions but also asymmetric distributions.

Table 1. Comparison of ARL for Lognormal (0,1) and MR =2.

	ARL_0 = 370			ARL_0 = 500		
δ	Tukey K_1=9.77	CUSUM K_2=16.175	CUSUM-Tukey K_3=3.81	Tukey K_1=8.27	CUSUM K_2=14.08	CUSUM-Tukey K_3=1.655
1	370.66 ± 0.52*	370.14 ± 0.52	370.103 ± 0.52	500.31 ± 0.70	500.22 ± 0.59	500.04 ± 0.43
1.2	155.39 ± 0.22	125.21 ± 0.52	*57.673 ± 0.28*	200.91 ± 0.68	393.78 ± 0.55	*83.47 ± 0.28*
1.4	84.36 ± 0.12	78.36 ± 0.52	*19.174 ± 0.11*	157.67 ± 0.67	312.57 ± 0.54	*64.40 ± 0.20*
1.6	53.87 ± 0.08	45.69 ± 0.53	*9.752 ± 0.05*	102.61 ± 0.70	248.24 ± 0.52	*48.40 ± 0.13*
1.8	37.91 ± 0.05	28.86 ± 0.53	*4.485 ± 0.04*	86.58 ± 0.71	199.97 ± 0.48	*25.42 ± 0.10*
2	28.81 ± 0.04	23.71 ± 0.52	*1.475 ± 0.02*	74.57 ± 0.47	160.66 ± 0.43	*18.51 ± 0.83*
2.5	17.72 ± 0.03	13.72 ± 0.52	*0.675 ± 0.01*	65.18 ± 0.33	94.79 ± 0.33	*3.35 ± 0.02*
3	12.86 ± 0.02	8.26 ± 0.52	*0.232 ± 0.01*	49.33 ± 0.07	55.42 ± 0.26	*1.48 ± 0.01*

* standard deviation of run length.

Table 2. Comparison of ARL for Lognormal(0,1) and MR =3.

	ARL_0 = 370			ARL_0 = 500		
δ	Tukey K_1=9.87	CUSUM K_2=13.49	CUSUM-Tukey K_3=9.09	Tukey K_1=10.89	CUSUM K_2=15.10	CUSUM-Tukey K_3=10.321
1	370.75 ± 0.97	370.35 ± 0.53	370.49 ± 0.50	500.63 ± 0.61	500.54 ± 0.70	500.28 ± 0.71
1.2	330.49 ± 0.17	255.83 ± 0.22	*212.98 ± 0.19*	406.30 ± 0.51	378.63 ± 0.38	*293.50 ± 0.40*
1.4	270.92 ± 0.09	184.42 ± 0.12	*144.80 ± 0.06*	381.51 ± 0.35	246.32 ± 0.33	*150.17 ± 0.25*
1.6	247.33 ± 0.06	153.69 ± 0.08	*106.18 ± 0.03*	328.80 ± 0.32	192.37 ± 0.31	*97.51 ± 0.20*
1.8	195.27 ± 0.04	137.89 ± 0.05	*93.18 ± 0.02*	275.63 ± 0.29	111.15 ± 0.30	*57.33 ± 0.09*
2	128.11 ± 0.03	98.77 ± 0.04	*51.86 ± 0.01*	213.14 ± 0.24	91.26 ± 0.25	*32.65 ± 0.02*
2.5	118.75 ± 0.02	77.70 ± 0.03	*30.69 ± 0.01*	183.22 ± 0.19	81.24 ± 0.15	*11.15 ± 0.01*
3	94.26 ± 0.02	32.89 ± 0.02	*10.34 ± 0.00*	102.15 ± 0.15	42.31 ± 0.09	*3.24 ± 0.01*

Table 3. Comparison of ARL for Lognormal(0,1) and MR =5.

	ARL_0 = 370			ARL_0 = 500		
δ	Tukey K_1=8.54	CUSUM K_2=10.23	CUSUM-Tukey K_3=9.13	Tukey K_1=7.483	CUSUM K_2=13.21	CUSUM-Tukey K_3=8.23
1	370.82 ± 0.38	370.92 ± 0.31	370.55 ± 0.28	500.88 ± 0.64	500.52 ± 0.71	500.83 ± 0.68
1.2	268.54 ± 0.38	248.55 ± 0.32	*194.41 ± 0.20*	476.47 ± 0.62	340.23 ± 0.01	*202.21 ± 0.23*
1.4	215.03 ± 0.37	155.55 ± 0.33	*117.51 ± 0.12*	353.77 ± 0.61	241.41 ± 0.23	*119.69 ± 0.12*

1.6	191.21 ± 0.34	123.67 ± 0.35	*85.11 ± 0.08*	338.04 ± 0.60	173.85 ± 0.31	*86.95 ± 0.08*
1.8	143.34 ± 0.31	94.453 ± 0.376	*68.60 ± 0.06*	226.84 ± 0.60	120.34 ± 0.48	*40.29 ± 0.06*
2	101.00 ± 0.30	48.815 ± 0.38	*20.56 ± 0.05*	217.64 ± 0.57	90.09 ± 0.65	*28.89 ± 0.04*
2.5	98.15 ± 0.29	37.815 ± 0.38	*6.33 ± 0.03*	193.27 ± 0.55	55.88 ± 0.64	*6.85 ± 0.03*
3	77.38 ± 0.27	17.54 ± 0.38	*1.84 ± 0.03*	125.45 ± 0.48	16.47 ± 0.61	*1.41 ± 0.03*

Table 4. Comparison of ARL for Lognormal(0,1) and MR =10.

	ARL_0 = 370			ARL_0 = 500		
δ	Tukey K_1 =9.6	CUSUM K_2 =11.78	CUSUM-Tukey K_3 =10.02	Tukey K_1 =10.73	CUSUM K_2 =12.92	CUSUM-Tukey K_3 =8.785
1	370.81 ± 0.33	370.57 ± 0.53	370.21 ± 0.52	500.32 ± 0.38	500.43 ± 0.70	500.17 ± 0.61
1.2	342.01 ± 0.33	215.36 ± 0.29	*187.25* ± 0.32	421.87 ± 0.35	377.29 ± 0.38	*241.75* ± 0.33
1.4	301.02 ± 0.32	124.46 ± 0.15	*107.21* ± 0.22	401.55 ± 0.31	246.24 ± 0.35	*139.54* ± 0.30
1.6	294.75 ± 0.32	77.22 ± 0.12	*61.02* ± 0.19	342.76 ± 0.31	194.88 ± 0.33	*97.28* ± 0.27
1.8	251.34 ± 0.31	49.10 ± 0.11	*32.74* ± 0.15	254.64 ± 0.29	101.58 ± 0.31	*58.39* ± 0.24
2	195.36 ± 0.30	27.85 ± 0.09	*19.21* ± 0.12	175.36 ± 0.25	82.69 ± 0.24	*21.01* ± 0.22
2.5	157.22 ± 0.29	13.53 ± 0.06	*6.23* ± 0.08	113.33 ± 0.22	57.26 ± 0.20	*13.18* ± 0.14
3	112.36 ± 0.29	5.20 ± 0.02	*1.72* ± 0.03	100.18 ± 0.20	21.04 ± 0.15	*4.22* ± 0.10

Table 5. Comparison of ARL for Logistic(6,2) and MR =2.

	ARL_0 = 370			ARL_0 = 500		
δ	Tukey K_1 =2.503	CUSUM K_2 =17.83	CUSUM-Tukey K_3 =2.395	Tukey K_1 =2.64	CUSUM K_2 =10.5	CUSUM-Tukey K_3 =1.549
1	370.95 ± 0.53	370.60 ± 0.52	370.183 ± 0.03	500.89 ± 0.71	500.49 ± 0.22	500.00 ± 0.05
1.2	322.06 ± 0.51	203.14 ± 0.29	*136.771 ± 0.01*	465.77 ± 0.71	399.25 ± 0.16	*375.77 ± 0.04*
1.4	285.23 ± 0.53	111.56 ± 0.16	*82.334 ± 0.01*	403.58 ± 0.73	331.77 ± 0.12	*263.90 ± 0.03*
1.6	244.11 ± 0.53	61.13 ± 0.09	*38.826 ± 0.01*	359.40 ± 0.71	283.79 ± 0.09	*201.99 ± 0.03*
1.8	190.94 ± 0.53	33.55 ± 0.05	*21.551 ± 0.01*	296.38 ± 0.71	247.80 ± 0.08	*154.45 ± 0.02*
2	135.34 ± 0.53	18.43 ± 0.03	*13.954 ± 0.01*	231.46 ± 0.67	219.78 ± 0.06	*67.50 ± 0.02*
2.5	107.24 ± 0.53	4.11 ± 0.01	*4.004 ± 0.01*	177.25 ± 0.63	171.32 ± 0.04	*27.89 ± 0.02*
3	73.14 ± 0.53	0.92 ± 0.00	*0.248 ± 0.01*	146.53 ± 0.72	140.25 ± 0.03	*7.37 ± 0.01*

Table 6. Comparison of ARL for Logistic(6,2) and MR =3.

	ARL_0 = 370			ARL_0 = 500		
δ	Tukey K_1 =2.41	CUSUM K_2 =12.32	CUSUM-Tukey K_3 =5.104	Tukey K_1 =2.567	CUSUM K_2 =17.43	CUSUM-Tukey K_3 =5.021
1	370.66 ± 0.38	370.21 ± 0.53	370.34 ± 0.56	500.75 ± 0.76	500.16 ± 0.80	500.06 ± 0.75
1.2	344.69 ± 0.36	270.75 ± 0.33	*176.43 ± 0.19*	456.67 ± 0.03	460.19 ± 0.76	*387.82 ± 0.27*

1.4	335.53 ± 0.36	211.24 ± 0.31	*108.16 ± 0.10*	330.71 ± 0.01	328.68 ± 0.52	*240.55 ± 0.16*
1.6	306.68 ± 0.34	193.95 ± 0.30	*78.84 ± 0.07*	311.79 ± 0.01	305.47 ± 0.35	*204.75 ± 0.09*
1.8	255.41 ± 0.35	127.31 ± 0.28	*64.79 ± 0.05*	271.08 ± 0.00	296.60 ± 0.29	*171.88 ± 0.05*
2	201.16 ± 0.32	90.23 ± 0.21	*36.10 ± 0.04*	141.13 ± 0.00	157.71 ± 0.10	*122.12 ± 0.03*
2.5	197.43 ± 0.30	73.26 ± 0.20	*23.91 ± 0.03*	119.98 ± 0.00	132.56 ± 0.05	*82.97 ± 0.02*
3	135.00 ± 0.22	20.21 ± 0.12	*12.96 ± 0.02*	107.79 ± 0.00	92.57 ± 0.02	*36.89 ± 0.01*

Table 7. Comparison of ARL for Logistic(6,2) and MR =5.

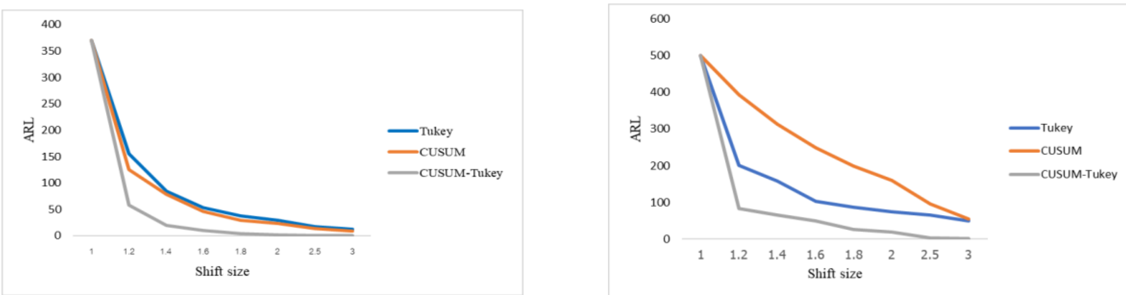
	ARL_0 = 370			ARL_0 = 500		
δ	Tukey K_1 =1.54	CUSUM K_2 =10.43	CUSUM-Tukey K_3 =4.701	Tukey K_1 =2.09	CUSUM K_2 =12.83	CUSUM-Tukey K_3 =4.925
1	370.45 ± 0.34	370.24 ± 0.35	370.12 ± 0.31	500.84 ± 0.80	500.41 ± 0.47	500.12 ± 0.35
1.2	330.22 ± 0.33	305.81 ± 0.31	*245.56* ± 0.31	427.52 ± 0.72	382.78 ± 0.41	*318.02* ± 0.31
1.4	284.38 ± 0.27	275.63 ± 0.30	*184.64* ± 0.28	316.74 ± 0.51	315.43 ± 0.38	*234.84* ± 0.27
1.6	217.41 ± 0.21	205.22 ± 0.24	*111.11* ± 0.20	308.10 ± 0.49	293.48 ± 0.36	*144.73* ± 0.16
1.8	199.81 ± 0.15	154.66 ± 0.16	*92.63* ± 0.15	291.48 ± 0.43	216.33 ± 0.28	*101.25* ± 0.14
2	136.91 ± 0.10	119.25 ± 0.10	*51.35* ± 0.12	171.25 ± 0.38	187.68 ± 0.21	*71.26* ± 0.11
2.5	99.10 ± 0.07	74.51 ± 0.09	*17.26* ± 0.08	145.32 ± 0.32	102.75 ± 0.20	*14.25* ± 0.08
3	65.24 ± 0.03	31.25 ± 0.06	*8.00* ± 0.02	102.05 ± 0.30	89.04 ± 0.16	*2.58* ± 0.02

Table 8. Comparison of ARL for Logistic(6,2) and MR =10.

	ARL_0 = 370			ARL_0 = 500		
δ	Tukey K_1=2.53	CUSUM K_2=15.45	CUSUM-Tukey K_3=6.432	Tukey K_1=2.7	CUSUM K_2=19.60	CUSUM-Tukey K_3=5.392
1	370.72 ± 0.39	370.22 ± 0.34	370.16 ± 0.37	500.49 ± 0.51	500.16 ± 0.70	500.12 ± 0.52
1.2	335.21 ± 0.34	245.09 ± 0.32	*210.41 ± 0.23*	434.04 ± 0.50	465.22 ± 0.62	*397.15± 0.51*
1.4	320.21 ± 0.32	228.03 ± 0.29	*131.40 ± 0.16*	401.26 ± 0.45	414.21 ± 0.61	*377.84± 0.45*
1.6	284.37 ± 0.31	209.63 ± 0.26	*114.48 ± 0.12*	365.39 ± 0.41	349.18 ± 0.52	*269.40± 0.38*
1.8	251.04 ± 0.29	179.86 ± 0.21	*78.85 ± 0.11*	326.95 ± 0.25	302.76 ± 0.41	*139.21± 0.32*
2	223.37 ± 0.21	145.56 ± 0.19	*34.98 ± 0.10*	295.01 ± 0.19	274.35 ± 0.32	*93.25 ± 0.27*
2.5	192.76 ± 0.15	99.92 ± 0.15	*13.57 ± 0.04*	186.48 ± 0.15	124.53 ± 0.29	*30.57 ± 0.14*
3	114.75 ± 0.09	83.20 ± 0.10	*4.03 ± 0.02*	94.17 ± 0.10	75.45 ± 0.24	*15.62 ± 0.09*

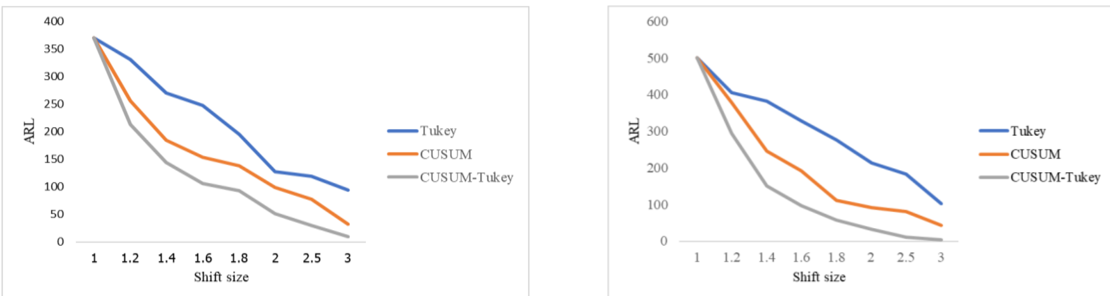
Conclusion

The comparative studies of the performance of Tukey, CUSUM and CUSUM-Tukey control charts for detecting a change in variation by using the moving range. The mixed CUSUM-Tukey control chart is superior to Tukey and CUSUM control charts for all magnitudes of shift in dispersion parameter on both cases of Lognormal(0,1) and Logistic(6,2) distributions. Therefore, the mixed CUSUM-Tukey control chart is robust to detect of dispersion parameter not only symmetry but also asymmetry distributions. In addition, the CUSUM-Tukey control chart robust to monitor the process dispersion when unknown population distributions and ARL_0 is changed to be 500. Because of the mixed CUSUM-Tukey control chart is a nonparametric control chart which parameter do not need to be known.



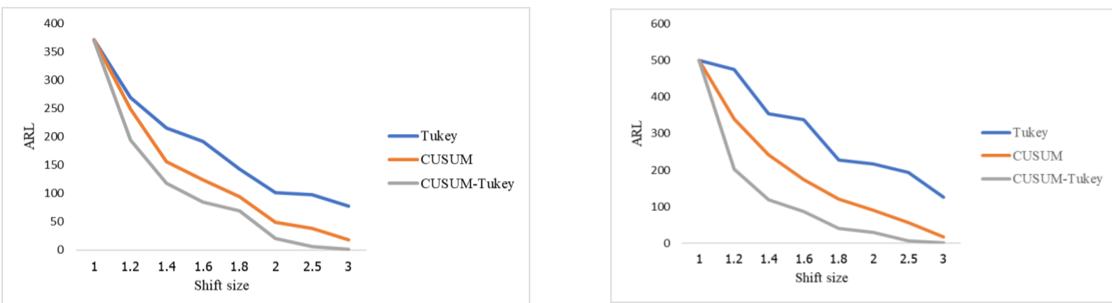
(a) ARL_0 = 370 (b) ARL_0 = 500

Figure 1. Comparison of ARL for Lognormal(0,1) when MR =2.



(a) ARL_0 = 370 (b) ARL_0 = 500

Figure 2. Comparison of ARL for Lognormal(0,1) when MR =3.



(a) ARL_0 = 370 (b) ARL_0 = 500

Figure 3. Comparison of ARL for Lognormal(0,1) when MR =5.

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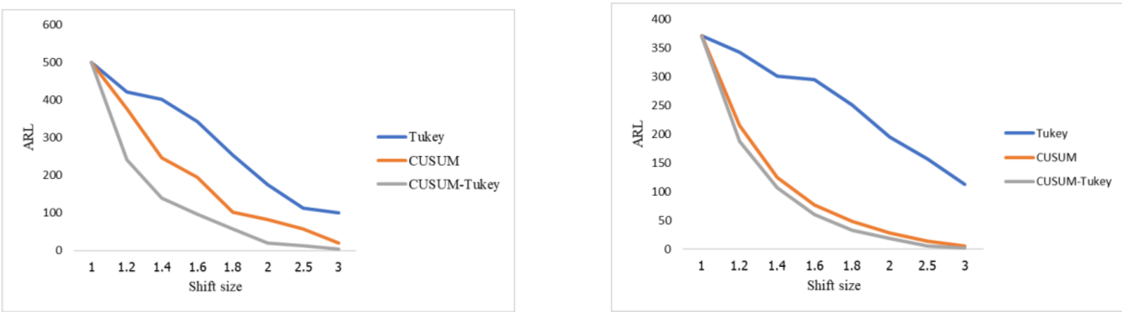
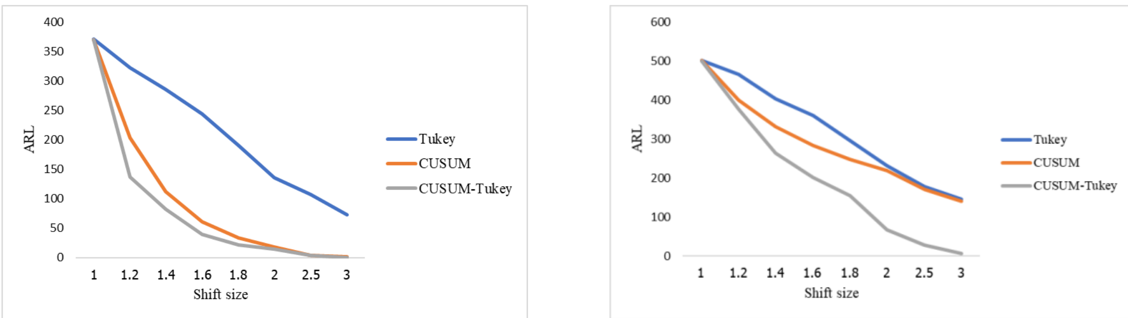


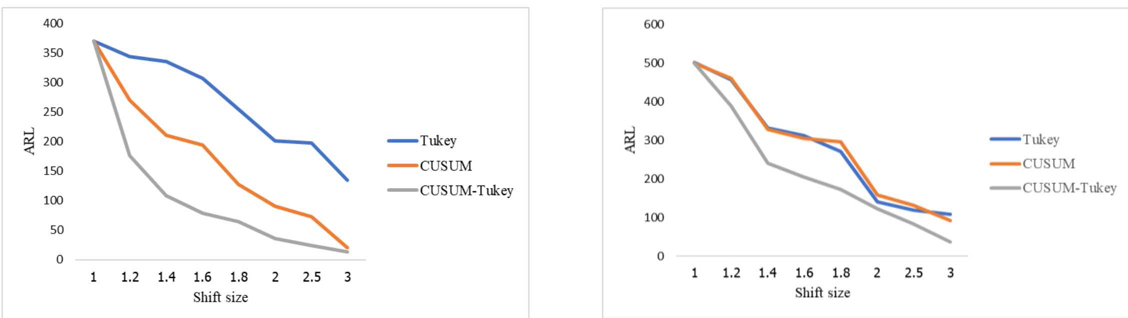
(a) ARL_0 = 370 (b) ARL_0 = 500

Figure 4. Comparison of ARL for Lognormal(0,1) when MR = 10.



(a) ARL_0 = 370 (b) ARL_0 = 500

Figure 5. Comparison of ARL for Logistic(6,2) when MR =2.



(a) ARL_0 = 370 (b) ARL_0 = 500

Figure 6. Comparison of ARL for Logistic(6,2) when MR = 3.

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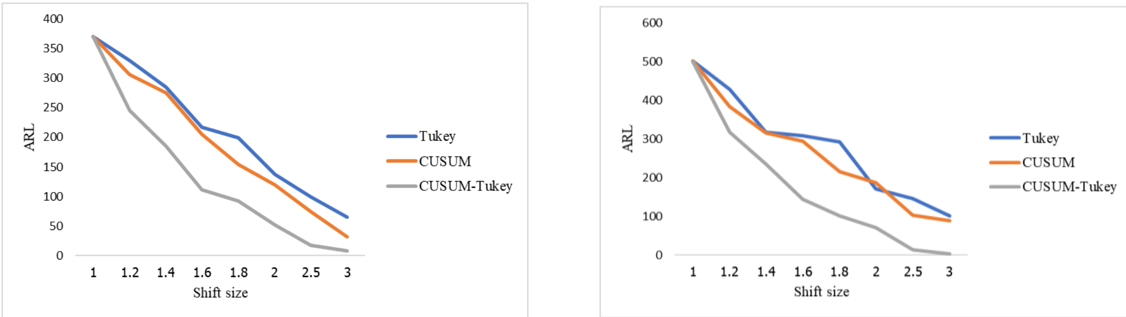
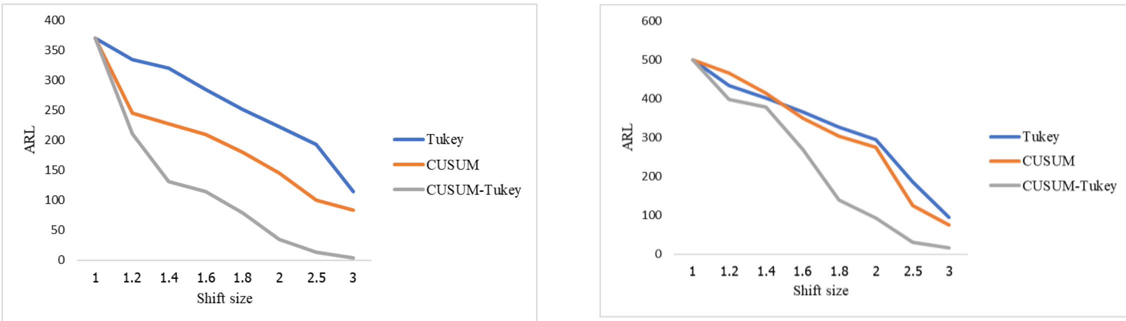


(a) ARL_0 = 370 (b) ARL_0 = 500

Figure 7. Comparison of ARL for Logistic(6,2) when MR = 5.



(a) ARL_0 = 370 (b) ARL_0 = 500

Figure 8. Comparison for ARL for Logistic(6,2) when MR = 10.

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